

Determination of all stability region of unstable fractional order system in case of bearing system

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Article Info

Article history:

Received Dec 18, 2018

Revised Mar 1, 2019

Accepted May 28, 2019

Keywords:

Control system

PID

RIM

Unstable fractional order
system

ABSTRACT

In this paper, we propose and investigate the optimal tuning PID controller of unstable fractional order system by desired transient characteristics using the real interpolation method (RIM). The research shows that the main advantages of this method are drawn as the followings: 1) Carrying out an investigation of the stable region of coefficients of a PID controller using D-decomposition method; 2) Applying the method to investigate an unstable fractional order system.

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1. INTRODUCTION

Due to the individual merits such as no contact, no wear, no lubrication and adjustable dynamics, magnetic bearings (MBs) are being widely applied to a variety of industrial products, especially the high-speed rotating machinery and precise positioning systems [1-3]. Considering the inherent nonlinearity and open-loop instability of MBs, the accurate modeling and control are two essential aspects of achieving high-performance requirements. Nowadays, PID controllers have received considerable attention in the last years both from an academic and industrial point of view [1-5]. In fact, in principle, they provide more flexibility in the controller design, concerning the standard PID controllers, because they have five parameters to select. However, this also implies that the tuning of the controller can be much more complicated. They have been successfully applied in practical applications such as motion control of manipulators and chaos control of electrical circuits. In these applications, it has been verified that PID controllers can improve the performance of traditional control system adopting integer order PID controllers. The most important advantage of the PID controllers is that they can afford broader possibilities offered by their new fractional order dynamics [4-7].

However, this also indicates that the tuning strategies of PID controllers are much more complicated. In the researches on the PID controllers, tuning of controller parameters has become a significant issue.

In general, the tuning methods for PID controllers are classified into analytical, numerical, and rule-based ones. In [6-7] the controller parameters have been analytically derived by solving nonlinear equations fulfilling the gain/phase crossover frequency and phase/gain margin specifications. The robustness to loop gain variations specification proposed in [8] has also been widely used to design PID and proportional-integral (PI) controllers. The merits of the analytical method are apparent; however, it is available only when the equations are few in number and simple. Therefore, it is challenging to obtain a complete PID controller for the MB system by solving five complicated nonlinear equations. As for the rule-based method, it can easily calculate the controller parameters based on empirical tuning rules, which can be observed in [9-21]. In this paper, we propose and investigate the optimal tuning PID controller of unstable fractional order system by desired transient characteristics using the real interpolation method (RIM). The main advantages of this method are drawn as the followings:

- 1) Carrying out an investigation of the stable region of coefficients of a PID controller using D-decomposition method.
- 2) Applying the method to investigate an unstable fractional order system.

The rest of this paper is organized as follows. In Section 2 the problem is formulated. Numerical results and discussions are described in Section 3. Conclusions are drawn in Section 4.

2. PROBLEM FORMULATION

In this section, Figure 1 illustrates the control system with the negative unity feedback. The mathematical description of this model is presented.

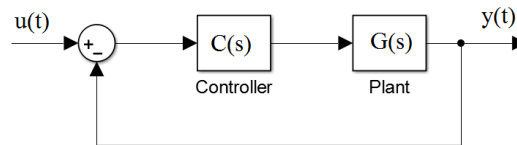


Figure 1. Control system with negative unity feedback

The transfer function of the PID controller can be formulated as

$$C(s) = K_p + \frac{K_i}{s} + K_d s \quad (1)$$

The transfer function of the plant (in the fractional form) is calculated as

$$G(s) = \frac{B_G(s)}{A_G(s)} = \frac{b_n s^{\beta_n} + b_{n-1} s^{\beta_{n-1}} + \dots + b_1 s^{\beta_1} + b_0 s^{\beta_0}}{a_m s^{\alpha_m} + a_{m-1} s^{\alpha_{m-1}} + \dots + a_1 s^{\alpha_1} + a_0 s^{\alpha_0}} \quad (2)$$

where $m, n \in N$, and $a_0, \dots, a_m, b_0, \dots, b_n, \alpha_m > \alpha_{m-1} > \dots > \alpha_1 > \alpha_0 = 0, \beta_n > \beta_{n-1} > \dots > \beta_1 > \beta_0 = 0$ are arbitrary real numbers $a_m \neq 0, b_n \neq 0$.

The transfer function of PID controller (1) can be rewritten in the rational form as the following

$$C(s) = \frac{B_C(s)}{A_C(s)} = \frac{K_d s^2 + K_p s + K_i}{s} \quad (3)$$

The characteristic equation in this analysis can be expressed as

$$W(s) = 1 + C(s)G(s) \quad (4)$$

Characteristic polynomial is formulated as

$$P(s) = sA_G(s) + (K_d s^2 + K_p s + K_i)B_G(s) \quad (5)$$

A gain-phase margin tester (GPMT) can be thought of as a “virtual compensator”, provides information for plotting the boundaries of constant gain margin and phase margin in a parameter plane [14]. The frequency independent GPMT is given as in [15]:

$$G_t(M, \phi) = M e^{-j\phi}. \quad (6)$$

For a given IOPID controller parameters K_p, K_i, K_d the closed-loop system is said to be bounded-input bounded-output (BIBO) stable if the quasi-polynomial $P(s, K_p, K_i, K_d)$ has no roots in the closed right-half of the s-plane (RHP). The stability domain S in the parameter space P with K_p, K_i, K_d being coordinates is the region that for $K_p, K_i, K_d \in S$ the roots of quasi-polynomial $P(s, K_p, K_i, K_d)$ all lie in open left-half of the s-plane (LHP). The boundaries of the stability domain S which are described by real root boundary (RRB), infinite root boundary (IRB) and complex root boundary (CRB) can be determined by the D-decomposition method [15], [16]. These boundaries are defined by the equations $P(0, K) = 0$, $P(\infty, K) = 0$ and $P(\pm j\omega, K) = 0$ for $\omega \in (0, \infty)$, respectively, where $P(s, K)$ is the characteristic function of the closed-loop system and K the vector of controller parameters.

2.1. Determining RRB

In applying the descriptions of stability boundaries of the stability domain S to the FOCE in (1), the RRB turns out to be simply a straight line given by

$$P(0, K_p, K_i, K_d) = 0 \Rightarrow K_i = 0, \quad (7)$$

for $s^{\rho_0} = 1$ in the transfer function of the plant in (2).

2.2. Determining IRB

The calculating of the IRB faces with more technical difficulties due to the fractional component. FOCE possesses an infinite number of roots, which cannot be calculated analytically in the general case. However, the asymptotic location of roots far from the origin is well known [17], which may lead to IRB. The objective of this section is to determine the stabilizing region in (K_p, K_i) plane with given K_d , and values for which the following complex polynomial is Hurwitz:

$$D(s) = sL(s)A_G(s) + (K_d s^2 + K_p s + K_i)M(s)B_G(s) \quad (8)$$

where $L(s)$ and $M(s)$ are given complex fractional order polynomials. When $L(s) = M(s) = 1$, the stabilization of Eq. (8) reduces to the standard IOPID stabilization.

From (8), we have $D(\infty) = 0$. Suppose when $s \rightarrow \infty$, $[L(s)A_G(s)]/[M(s)B_G(s)] \rightarrow cs^t$ where t and c are real and complex numbers respectively. We have [16]:

- (a) If $t \neq 1$, then the boundary does not exist.
- (b) If $t = 1$ and c is not real, then the boundary does not exist.
- (c) If $t = 1$ and c is real, then

$$\lim_{s \rightarrow \infty} \frac{D(s)}{sL(s)A_G(s)} = \lim_{s \rightarrow \infty} 1 + \frac{N(s)(K_p s + K_i + K_d s^2)}{sA_G(s)} = 1 + K_d c = 0 \quad (9)$$

$$K_d = -\frac{1}{c} \quad (10)$$

The RRB and IRB lines can be obtained from (7) and (10), respectively.

2.3. Determining CRB

To construct the CRB, we substitute $s = j\omega$ into (5) to obtain

$$P(j\omega) = A_C(j\omega)A_G(j\omega) + B_C(j\omega)B_G(j\omega) = 0 \quad (11)$$

Using D-decomposition to find stability [15]. In the s plane $j^\varphi = e^{j(\frac{\pi}{2})\varphi} = \cos(\frac{\pi}{2}\varphi) + j\sin(\frac{\pi}{2}\varphi)$, where φ is a real number. The formula (6) can be expressed into a formula with the separate real and imaginary components

$$P(j\omega) = [(K_i - K_d \omega^2)R_{BG}(\omega) - \omega I_{AG}(\omega)] + j\omega[K_p R_{BG}(\omega) + R_{AG}(\omega)] = 0, \quad (12)$$

where R_{BG} , R_{AG} , I_{AG} are the real component of $B_G(j\omega)$, the real and imaginary components of $A_G(j\omega)$, respectively.

Finally, by setting the real and imaginary parts equalized zero the formula (12) leads to the detail form:

$$\begin{cases} R_{AG}(\omega) + K_p R_{BG}(\omega) = 0 \\ (K_i - K_d \omega^2)R_{BG}(\omega) - \omega I_{AG}(\omega) = 0 \end{cases} \quad (13)$$

The solution in the (K_p, K_i) plane

This is a three-dimensional system in terms of the controller parameters K_p , K_i , K_d . In order to deal with two unknowns K_p , K_i , the value of K_d will be fixed to find the stability region in the (K_p, K_i) plane.

Equation (11) rewrite in terms of two unknowns K_p , K_i .

$$\begin{cases} K_p = -R_{AG}(\omega) / R_{BG}(\omega) \\ K_i = \omega I_{AG}(\omega) / R_{BG}(\omega) + K_d \omega^2 \end{cases} \quad (14)$$

The above two equations trace out a curve in the (K_p, K_i) -plane representing the CRB, for fixed K_d , as ω run from 0 to ∞ .

The solution in the (K_p, K_d) plane

Next, to find the stability region in the (K_p, K_d) plane, the value of K_i will be fixed. Then formula (13) can be rewritten as:

$$\begin{cases} K_p = -R_{AG}(\omega) / R_{BG}(\omega) \\ K_d = [K_i - \omega I_{AG}(\omega) / R_{BG}(\omega)] / \omega^2 \end{cases} \quad (15)$$

The above two equations trace out a curve in the (K_p, K_d) -plane representing the CRB, for fixed K_i , as ω run from 0 to ∞ .

The solution in the (K_i, K_d) plane

Next, to find the stability region in the (K_i, K_d) plane, the value of K_p will be fixed. Then formula (13) can be rewritten as:

$$\begin{cases} K_p = -R_{AG}(\omega) / R_{BG}(\omega) \\ K_d = [K_i - \omega I_{AG}(\omega) / R_{BG}(\omega)] / \omega^2 \end{cases} \quad (16)$$

The above two equations trace out a curve in the (K_i, K_d) -plane representing the CRB, for fixed K_p , as ω run from 0 to ∞ .

3. NUMERICAL SAMPLE AND DISCUSSION

Given fractional order transfer function of an unstable bearing system [18] [19],

$$G(s) = \frac{6438.}{s^{2.76} + 330.04s^{1.81} - 84268.85s^{0.82} - 15869154.14} \quad (17)$$

Investigation of the stability region of the unstable fractional order transfer function of the bearing system (17) is carried out in the following parts.

3.1. Determining RRB

It is important to determine the stability region of the PID controller, tuned for the system (17). The RRB can be determined by (7) as the bellow

$$K_i = 0. \quad (18)$$

3.2. Determining IRB

The IRB can be determined according to (9)

$$\lim_{s \rightarrow \infty} \frac{\delta(s)}{sD(s)} = \lim_{s \rightarrow \infty} 1 + \frac{643.8(K_p s + K_i + K_d s^2)}{s(s^{2.76} + 330.04s^{1.81} - 84268.85s^{0.82} - 15869154.14)} = 1 \quad (19)$$

Not exist the IRB.

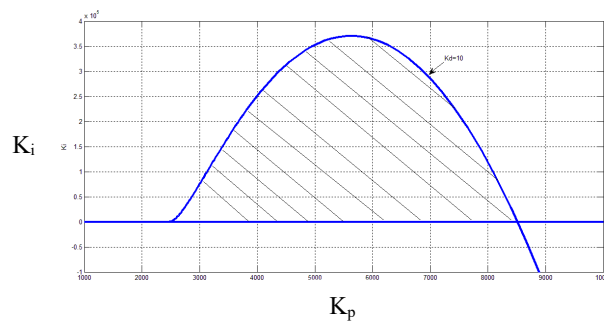


Figure 2. The CRB graph in the (K_p, K_i) plane and $K_d = 10$.

3.3. Determining CRB

The CRB can be determined by three conditions (13), (14) and (15). Figure 2 shows the stability region of the transfer function (17). The CRB in the (K_p, K_i) plane when $K_d = 10$ is shown in the Figure 2. The cross-line area is a feasible region for the coefficients of the PID controller. The cross-line region demonstrates stability region of K_p, K_i where $K_d = 10$. Obviously, the stability region of PID controller's coefficients lie in the first quarter of the coordinate [22-25].

Furthermore, Figure 3 shows a more feasible area with five values of K_d .

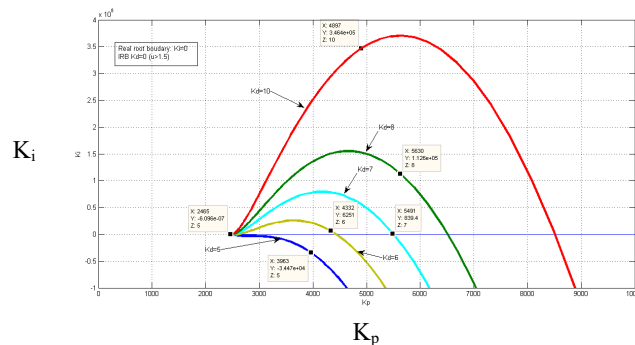


Figure 3. The CRB graphs in the (K_p, K_i) plane with 5 values of coefficients K_d .
Figure 3 shows that by increasing the value of the coefficient K_d , the feasible region is expanded.

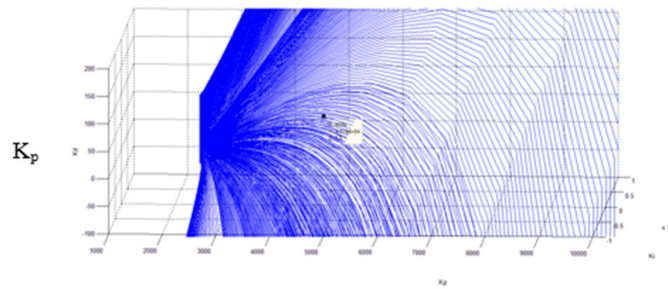


Figure 4. The CRB of the PID coefficients

In Figure 4. The distribution of K_p, K_i, K_d in the 3-D system is represented. In more details, we can investigate the stability region of the PID controller's coefficient separately in the 2-D coordinate plane. In the Figure 4.

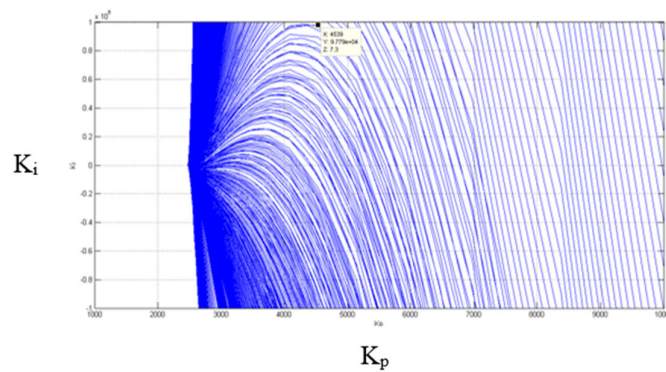


Figure 5. The CRB in the (K_p, K_i) plane

Figure 5 illustrates the stability region of K_p and K_i . Obviously, the stability region is covered the first and fourth quarters of the coordinate system. From the Figure 5, there is a significance that the coefficient $K_p \geq 2465$. Furthermore, The Figure 6 show the stability region in the K_p, K_d plane. The values of the coefficient K_p, K_d are located in the first and fourth quarters, and of course $K_p \geq 2456$.

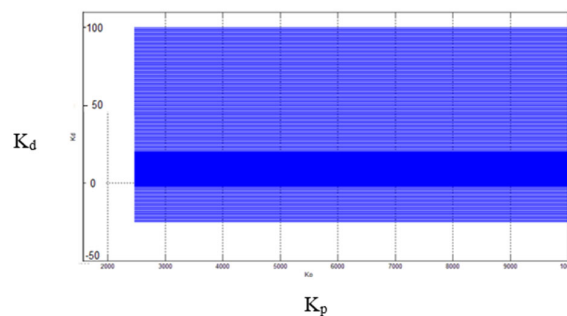


Figure 6. The CRB in the (K_p, K_d) plane

The CRB in the (K_i, K_d) plane shows that the distribution of K_i and K_d

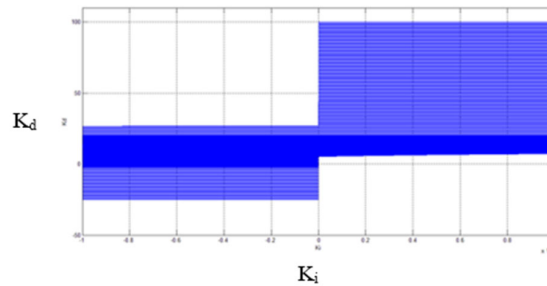


Figure 7. The CRB in the (K_i, K_d) plane

The results simply lead to the conclusion

$$\begin{cases} K_p \geq 2465 \\ K_i > 0 \end{cases} \quad (20)$$

Three parameters K_p, K_i, K_d of the PID controller are determined in four figure 4-7 [22-25].

4. CONCLUSION

In this paper, we propose and investigate optimal tuning PID controller of unstable fractional order system by desired transient characteristics using the real interpolation method (RIM). The research shows that the main advantages of this method are drawn as the followings: 1) Carrying out an investigation of the stable region of coefficients of a PID controller using D-decomposition method; 2) Applying the method to investigate an unstable fractional order system.

ACKNOWLEDGEMENTS

This research was supported by National Key Laboratory of Digital Control and System Engineering (DCSELAB), HCMUT, VNU-HCM, Vietnam.

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