Adaptive integral backstepping controller for linear induction motors

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ABSTRACT

Linear induction motors offer the possibility to perform a direct linear motion without the need of mechanical rotary to linear motion transformers. The main problem when controlling this kind of motors is the existence of undesirable behaviors such as end effect and parameter variations, which makes obtaining a precise plant model very complicated. This paper proposes an adaptive backstepping control technique with integral action based on Lyapunov stability approach, which can guarantee the convergence of position tracking error to zero despite of parameter uncertainties and external load disturbance. Parameter adaptation laws are designed to estimate mover mass, viscous friction coefficient and load disturbance, which are assumed to be unknown constant parameters; as a result the compensation of their negative effect on control design system. The performance of the proposed control design was tested through simulation. The numerical validation results have shown good performance compared to the conventional backstepping controller and proved the robustness of the proposed controller against parameter variations and load disturbance.

Keywords: Adaptive backstepping, Field oriented control, Integral action, Linear induction motor

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1. INTRODUCTION

Linear induction motors (LIMs) are widely used in industrial processes and transportation applications [1, 2]. The main advantage of the LIM is its ability to perform a direct linear motion without the need of any gear box or mechanical rotary to linear motion transformation [3].

The driving principle of the LIM is similar to that of a rotary induction motor (RIM), however, the control characteristics are more complicated and the parameters are time varying due to change of operating conditions, such as speed of mover, temperature and configuration of rail [4, 5].

Field oriented control (FOC) is considered as the most popular high performance control method for induction machines. The main idea of the FOC is to decouple the dynamics of thrust force and rotor flux to make the control as simple as in a separately excited DC machine [6-8]. Unfortunately, a good performance of FOC can only be achieved by a precise plant model and it can be disturbed by parameter variations, which presents a big issue, because unlike RIM the model of LIM presents more complexity. Moreover, there are significant parameter variations in the reaction rail resistivity, the dynamics of the air gap, slip frequency, phase unbalance, saturation of the magnetizing inductance, and end-effects phenomenon [9-12], causing a difficulty in achieving a good control performance. Many researches have been done in modeling the LIM performance taken into consideration time varying parameters and end effect phenomenon [3, 8-10, 13]. However, there are always some modeling uncertainties due to undesirable behaviors.
The advancement in nonlinear control field gave a variety of solutions in dealing with parameter variations and load disturbance. One of these solutions is backstepping control, this last is a nonlinear control technique that has been widely applied in controlling LIMs [12, 14]. The basic idea of backstepping is the decomposition of a high order nonlinear control design problem into smaller steps, in which every step introduces a reference to stabilize the next one using the so called “virtual control” variables, until we reach the real control input that insures the global stability based on lyapunov function [12, 14, 15]. In addition, the adaptive version of backstepping control guarantees the global stability of the system despite of parameter uncertainties and variations thanks to parameter update laws [15].

In this paper we propose an adaptive backstepping control design based on field oriented control to achieve a desired position trajectory under the assumption of unknown mover mass, friction coefficient and load disturbance. The rest of the paper is organized as follows: in section 2, we introduce the basics of indirect field oriented control. Section 3 presents the proposed adaptive backstepping controller. In section 4, we show the simulation results followed by discussion. Finally, a conclusion is drawn in section 5.

2. INDIRECT FIELD ORIENTED CONTROL

Figure 1 shows the construction of a LIM. The primary is simply a cut-open and rolled-flat rotary motor primary, and the secondary usually consists of a sheet conductor using aluminum with an iron back for the return path of the magnetic flux. For a zero relative velocity, the LIM can be considered as having an infinite primary, in which case the end effects may be ignored [12].

![Figure 1. Construction of a LIM](image)

The fifth order model of the LIM in the d-q axis reference frame modified from a three phase Y-connected rotary induction motor model is given as [12, 16]:

\[
\frac{di}{ds} = -\frac{1}{\sigma L_s} \left( R_s + \left( \frac{L_m}{L_r} \right)^2 R_r \right) i + \frac{\pi}{h} V e_i + \frac{L_m R_r}{\sigma L_s L_r} \varphi_{dr} + \frac{P l_m \pi}{\sigma L_s L_r h} \varphi_{qr} + \frac{1}{\sigma L_s} V ds
\]  

\[
\frac{di}{dt} = -\frac{\pi}{h} V e_i ds + \frac{1}{\sigma L_s} \left( R_s + \left( \frac{L_m}{L_r} \right)^2 R_r \right) i + \frac{P l_m \pi}{\sigma L_s L_r h} \varphi_{dr} + \frac{L_m R_r}{\sigma L_s L_r} \varphi_{qr} + \frac{1}{\sigma L_s} V ds
\]  

\[
\frac{d\varphi}{dt} = \frac{L m R}{L_r i} - \frac{R}{L_r} \varphi_{dr} + \frac{\pi}{h} (V - P) \varphi_{qr} 
\]  

\[
\frac{d\varphi}{dt} = \frac{L m R}{L_r q_s} - \frac{\pi}{h} (V - P) \varphi_{dr} - \frac{R}{L_r} \varphi_{qr} 
\]  

\[
F_v = K_f (\varphi_{dr} i_{qs} - \varphi_{dq} i_{ds}) = M \frac{dv}{dt} + f_{c,v} + F_L 
\]
where $R_s$ is stator resistance per phase, $R_r$ is rotor resistance per phase, $L_s$ is the stator inductance per phase, $L_r$ is the rotor inductance per phase, $L_m$ is the magnetizing inductance per phase, $P$ is the number of pole pairs, $h$ is the pole pitch, $\sigma = 1 - (L_m^2 / L_r L_s)$ is the leakage coefficient, $V_e$ is the synchronous linear velocity, $v$ is the mover linear velocity, $i_{ds}$ and $i_{qs}$ are the d-axis and q-axis stator currents, $\varphi_{dr}$ and $\varphi_{qr}$ are the d-axis and q-axis rotor fluxes, $V_{ds}$ and $V_{qs}$ are d-axis and q-axis stator voltages, $K_f = 3P d_m / (2L_s h)$ is a force constant, $F_e$ is the electromagnetic force, $F_L$ is the external load disturbance, $M$ is the total mass of the moving part and $f_c$ is the viscous friction and iron-loss coefficient.

The main idea of the field oriented control is to render the behaviour of the LIM similar to that of a separately excited DC machine, this last gives the ability to control the electromagnetic force and rotor flux independently [7], [17]. The decoupling is realized by forcing the secondary flux linkage to align with the d-axis. As a result we get,

$$\varphi_{eq} = 0, \quad \frac{d\varphi_{eq}}{dt} = 0$$

(6)

Using (6) and replacing in the rotor flux equations (3) and (4) we get,

$$\varphi_{dr} = \frac{L_m}{1 + \tau_s s} i_{ds}$$

(7)

where $s$ is the Laplace operator, and $\tau_s = L_r / R_r$ is the secondary time constant.

Replacing (6) in electromagnetic force equation (5) gives,

$$F_e = K_f \varphi_{dr} i_{qs}$$

(8)

Viewing equations (7) and (8) we can see that the rotor flux and the electromagnetic force can now be controlled separately using stator currents $i_{ds}$ and $i_{qs}$.

Using (6) and (4) the slip velocity can be written as

$$v_{sl} = v - \varphi_{dr} \frac{h L_m}{\pi L_s, \tau_s, \varphi_{dr}} i_{qs}$$

(9)

3. ADAPTIVE BACKSTEPPING CONTROL WITH INTEGRAL ACTION

The basic idea of the backstepping design is the decomposition of a complex nonlinear system into multiple single input-output first order subsystems, using the so-called “virtual control” variables [18], in which every control variable is forced to stabilize the previous subsystem generating a corresponding error which can be stabilized by convenient input selection via Lyapunov stability [14, 16, 18, 19]. The steps to design such controller for LIM position tracking are as follows:

Define the position tracking error

$$e_1 = d_{ref} - d$$

(10)

Its dynamic can be written as

$$\dot{e}_1 = \dot{d}_{ref} - \dot{d} = \dot{d}_{ref} - v$$

(11)

Consider the Lyapunov function candidate

$$V_1 = \frac{1}{2} e_1^2$$

(12)

Deriving $V_1$ and using the error dynamic equation (11) we obtain

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Choosing the velocity stabilising function as
\[ v_{ref} = k_1 e_1 + \dot{d}_{ref} \]  
(14)

where \( k_1 \) is a positive design constant to be determined later, gives
\[ \dot{V}_1 = -k_1 e_1^2 \leq 0 \]  
(15)

which guarantees the stabilisation of our closed-loop control.

However the velocity \( v \) is not a real input, it’s a “virtual control” input \([19, 20]\), so we define the following velocity tracking error:
\[ e_2 = v_{ref} - v \]  
(16)

To ensure the convergence of the position tracking error to zero at the steady state despite the presence of disturbance and modeling inaccuracy of the system we introduce an integral action to the desired velocity as follows \([15]\)
\[ v_{ref} = k_1 e_1 + \dot{d}_{ref} + k_1' e_1' \]  
(17)

where \( e_1' = \int e_1(t)dt \) is the integral action of the position tracking error and \( k_1' \) is a positive design constant.

The dynamics of the velocity tracking error give,
\[ \dot{e}_2 = \dot{v}_{ref} - \dot{v} = \dot{d}_{ref} + k_1 (\dot{d}_{ref} - v) + k_1' e_1' - \frac{F_c}{M} + \frac{f_c}{M} v + \frac{F_l}{M} \]  
(18)

Then the dynamics of \( e_1 \) can be rewritten as
\[ \dot{e}_1 = -k_1 e_1 + k_1' e_1' + e_2 \]  
(19)

Define another lyapunov function
\[ V_2 = \frac{1}{2} e_1^2 + \frac{1}{2} e_2^2 + \frac{k_1'}{2} e_1'^2 \]  
(20)

Its time derivative is
\[ \dot{V}_2 = e_1 \dot{e}_1 + e_2 \dot{e}_2 + k_1' \dot{e}_1' e_1 \]  
(21)

Using (19), (20) and (21) we get,
\[ \dot{V}_2 = -k_1 e_1^2 - k_2 e_2^2 + e_2 \left[ e_1 (1-k_1^2 + k_1') + k_1' k_1 e_1' + e_2 (k_1 + k_2) + \dot{d}_{ref} - \frac{F_c}{M} + Dv + L \right] \]  
(22)
where \( k_2 \) is a positive design parameter, \( D = f_c / M \) is the normalized friction coefficient and \( L = F_L / M \) is the normalized Load disturbance.

Now we can choose our force control input \( F_{e\_ref} \) to cancel the undesirable behaviours and guarantee achieving the desired position trajectory.

If we choose

\[
F_{e\_ref} = M\left[k_1(1-k_1^2 + k_1') + k_1'k_1e_1' + e_2(k_1 + k_2) + \ddot{e}_{ref} + Dv + L\right]
\]

we get

\[
\ddot{V}_2 = -k_1\dot{e}_1^2 - k_2\dot{e}_2^2 \leq 0
\]

which guarantees the stabilization of our control in the closed-loop.

However, the real values of parameters \( M, D \) and \( L \) are unknown and can’t be used, so we use their estimates \( \hat{M}, \hat{D} \) and \( \hat{L} \) instead:

\[
\hat{F}_{e\_ref} = \hat{M}\left[k_1(1-k_1^2 + k_1') + k_1'k_1e_1' + e_2(k_1 + k_2) + \ddot{e}_{ref} + \hat{D}v + \hat{L}\right]
\]

The next step is to extract the update laws of estimated parameters, for that purpose we define the following parameter estimation errors:

\[
\tilde{M} = M - \hat{M}
\]
\[
\tilde{D} = D - \hat{D}
\]
\[
\tilde{L} = L - \hat{L}
\]

Using these definitions and substituting the force control in (25) to the velocity tracking error in (18) the dynamics of \( e_2 \) become,

\[
\dot{e}_2 = \ddot{e}_{ref} + k_1(-k_1e_1 - k_1'e_1' + e_2) + k_1'e_1 - \frac{M - \hat{M}}{M} \beta + Dv + L
\]

where \( \beta = \left[k_1(1-k_1^2 + k_1') + k_1'k_1e_1' + e_2(k_1 + k_2) + \ddot{e}_{ref} + \hat{D}v + \hat{L}\right] \)

After few calculations we get,

\[
\dot{e}_2 = -e_1 - k_2\dot{e}_2 + \hat{D}v + \hat{L} + \frac{\hat{M}}{M} \beta
\]

Finally, we construct the final lyapunov function that includes all the system’s error signals in order to derive the estimates update laws,

\[
V_3 = \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2 + \frac{k_1'}{2}e_1'^2 + \frac{1}{2\delta_1M}\tilde{M}^2 + \frac{1}{2\delta_2}\tilde{D}^2 + \frac{1}{2\delta_3}\tilde{L}^2
\]

where \( \delta_1, \delta_2 \) and \( \delta_3 \) are positive design constants.

Assuming that \( M, D \) and \( L \) are constant parameters, we can write,

\[
\dot{\hat{M}} = -\tilde{M}
\]
\[
\dot{\hat{D}} = -\tilde{D}
\]
Using (32), (33), (34) and the error dynamics in (19) and (30) we calculate the time derivative of (31)

\[
\dot{V}_3 = e_1 \ddot{e}_1 + e_2 \ddot{e}_2 + k_i e_i e_1 - \frac{1}{\delta_1 M} \ddot{M} M - \frac{1}{\delta_2} \ddot{D} D - \frac{1}{\delta_3} \ddot{L} L
\]

\[
= -k_i e_i^2 - k_2 e_2^2 + k_i e_i e_1 + e_2 \left( -e_1 + \frac{\ddot{M}}{M} \beta + \ddot{D} v + \ddot{L} \right) - \frac{1}{\delta_1 M} \ddot{M} M - \frac{1}{\delta_2} \ddot{D} D - \frac{1}{\delta_3} \ddot{L} L + e_i (-k_i e_i + e_2)
\]

We finally get

\[
\dot{V}_3 = -k_1 e_1^2 - k_2 e_2^2 + \frac{\ddot{M}}{M} \left( e_i \beta - \frac{1}{\delta_1} \ddot{M} \right) + \ddot{D} \left( e_i v - \frac{1}{\delta_2} \ddot{D} \right) + \ddot{L} \left( e_i - \frac{1}{\delta_3} \ddot{L} \right)
\]

In order to render the global Lyapunov function negative we choose the update laws of the parameter estimates as:

\[
\dot{M} = \delta_1 e_2 \beta
\]

\[
\dot{D} = \delta_2 e_2 v
\]

\[
\dot{L} = \delta_3 e_2
\]

The resulting Lyapunov function dynamics give

\[
\dot{V}_3 = -k_i e_i^2 - k_2 e_2^2 \leq 0
\]

which guarantees the stability of the global system and the achievement of the desired position trajectory despite of load disturbance and undesirable behaviors.

The previous controller designing steps are summed up in the diagram in Figure 2:

![Adaptive backstepping controller scheme](image)

**Figure 2. Adaptive backstepping controller scheme**

4. SIMULATION AND DISCUSSION

To investigate the performance of the proposed adaptive backstepping controller with integral action for the LIM position tracking, we run numerical simulation in which the effectiveness of the proposed controller under different operating conditions is tested (external load disturbance, friction and mover mass variation).

The obtained results are compared to those of conventional backstepping control technique. The LIM parameters used for simulation are shown in Table 1.
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Table 1. LIM parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rs</td>
<td>3.4 (Ω)</td>
</tr>
<tr>
<td>Rr</td>
<td>1.95 (Ω)</td>
</tr>
<tr>
<td>Ls</td>
<td>0.1078 (H)</td>
</tr>
<tr>
<td>Lr</td>
<td>0.1078 (H)</td>
</tr>
<tr>
<td>Lm</td>
<td>0.1042 (H)</td>
</tr>
<tr>
<td>P</td>
<td>2</td>
</tr>
<tr>
<td>M</td>
<td>5.47 (Kg)</td>
</tr>
<tr>
<td>Fc</td>
<td>26.36 (Nm.s/rd)</td>
</tr>
</tbody>
</table>

4.1. Case 1 (known parameters)

First, we present the simulated results for a periodic step reference with known parameters and no load disturbance. The controller design parameters and update laws are chosen as follows: \( k_1 = 10, k_1' = 0.1, k_2 = 80, \delta_1 = 0.001, \delta_2 = 0.8, \delta_3 = 500 \). The position tracking response is shown in Figure 3a. The applied electromagnetic force is shown in Figure 3b. The stator current \( I_{qs} \) is shown in Figure 3c. And Figure 3d. shows the rotor fluxes.

The obtained results show good performance for both conventional and adaptive backstepping controllers when the parameters are known and invariant. The position tracking error converges to zero with fast response time about 0.5s, the decoupling between electromagnetic force and rotor flux is realized, and we can see that the rotor flux is kept constant in the steady state (Figure 3d) while the electromagnetic force is being controlled by the variation of quadratic stator current \( I_{qs} \) (Figure 3b, Figure 3c.).

Next, we test the robustness of the proposed control scheme under the existence of parameter uncertainties. For that purpose we devide the next simulation into three steps, each step presents a possible parameter variation as Case 2, 3, 4.

4.2. Case 2 (load disturbance)

Case 2: a constant force disturbance of 10N is injected at \( t=5s \) and removed at \( t=7s \). The obtained results are presented in Figure 4.
Figure 4. Adaptive backstepping position control of LIM with applied load force of 10N, (a) position tracking response, (b) zoom in position, (c) electromagnetic force, (d) zoom in electromagnetic force, (e) stator current $I_{qs}$, (f) rotor fluxes.

4.3. Case 3 (1.5×D)
Case 3: friction coefficient variation equals to 1.5×D. The obtained results are presented in Figure 5.
Figure 5. Adaptive backstepping position control of LIM with 1.5×D, (a) position tracking response, (b) zoom in position, (c) electromagnetic force, (d) stator current Iqs, (e) rotor fluxes.

4.4. Case 4 (2×M)
Case 4: mover mass variation equals to 2×M. The results are presented in Figure 6.
Figure 6. Adaptive backstepping position control of LIM with 2×M, (a) position tracking response, (b) zoom in position, (c) electromagnetic force, (d) stator current Iqs, (e) rotor fluxes.

The simulated results prove that adaptive backstepping controller is insensitive against load disturbance and can reject it with nearly no overshoot, in contrary of conventional backstepping where the control is sensitive causing a failure in position tracking with a non-null static error (Figure 4a. Figure 4b.). The electromagnetique force generated by the adaptive controller has risen by 10N after applying the load force in order to cancel the disturbance (Figure 4c. Figure 4d.).

In case 3, where the friction coefficient has risen by 50%, we can see that the response time for conventional backstepping has risen slightly to 0.7s, while for the adaptive controller it hasn’t been affected (Figure 5a. Figure 5b.), that shows that the proposed technique is insensitive against friction force variation too.

In the case of mover mass change to double value, we can observe that the adaptive backstepping controller is always insensitive and can achieve the desired position trajectory with fast response time and no overshoot, while the conventional backstepping controller response has overshot before it could achieve the desired trajectory causing a delay in response time as well (Figure 6a. Figure 6b.).

The previous numerical validations show good performance and prove the robustness of the adaptive backstepping controller against sudden changes thanks to parameter update lows based on Lyapunov stability, which guarantees the stability of the global system despite of load disturbance and parameter variations. The estimated parameters are shown in Figure 7. We can observe from the figure that the estimated parameters converge to the real values with acceptable response time and error range allowing the controller to track the desired trajectory despite of parameter changes.

Figure 7. Estimated parameters, (a) load force disturbance, (b) viscous friction coefficient, (c) mover mass.
5. CONCLUSION

This paper has demonstrated the application of an adaptive backstepping controller with integral action to track a position reference for LIM. First, the indirect field oriented control was used to decouple the control of electromagnetic force and rotor flux. Then, an adaptive backstepping controller was designed and parameter update laws were extracted, to achieve a desired position reference under the assumption of parameter uncertainties, and the existence of external load force disturbance. The effectiveness of the proposed controller was tested through numerical simulation under different operating conditions. The obtained results show good tracking performance compared to the conventional backstepping controller, and prove the ability of the adaptive backstepping design to reject external load disturbance without overshoot, and its insensitivity against sudden parameter variations thanks to parameter update laws. Finally, the robustness of the adaptive backstepping controller was successfully confirmed.

REFERENCES


Adaptive integral backstepping controller for linear induction motors (Omar Mahmoudi)