International Journal of Power Electronics and Drive Systems (IJPEDS)
Vol. 12, No. 1, May 2021, pp. 567∼575
ISSN: 2088-8694, DOI: 10.11591/ijpeds.v12.i1.pp567-575

Finite frequency H_∞ control design for nonlinear systems

Zineb Lahlou, Abderrahim El−Amrani, Ismail Boumhidi
LISAC Laboratory, Sidi Mohamed Ben Abdellah University, Fes, Morocco

Article Info

Article history:

Received Feb 17, 2020
Revised Jan 18, 2021
Accepted Feb 7, 2021

Keywords:

Finite frequency
LMIs
Nonlinear systems
T-S Model

ABSTRACT

The work deals finite frequency H_∞ control design for continuous time nonlinear systems, we provide sufficient conditions, ensuring that the closed-loop model is stable. Simulations will be gifted to show level of attenuation that a H_∞ lower can be by our method obtained developed where further comparison.

Corresponding Author:

Abderrahim El−Amrani
LISAC Laboratory
Sidi Mohamed Ben Abdellah University
Fes 30050, Morocco
Email: abderrahim.elamrani@usmba.ac.ma

1. INTRODUCTION

Fuzzy models [1] it generated widespread interest from engineers, mainly for renowned T-S systems my actually approach great category for non linear models. Then, the T-S systems is its universal approximation of a smooth non linear function by a family of IF and THEN non linear rules that represent the output/ input relationships of the models [2]-[11].

The interest of the literature mentioned above the H_∞ control design in the FF range. whereas, in such cases, standard design methods of full frequency range can provide conservatism. Nevertheless, in an actual application, the design characteristics are generally given in selector Frequency domains (see, [12]-[21]).

In this work, we develop new our method concerning FF design of nonlinears continuous systems. Using theadequate conditions are developed, ensuring that the closed loop system is stable. Numerical examples are provides to prove the effectiveness of FF propose method. **Notations** :

- $*$: Form symmetry
- $Q > 0$: Form positive
- $sym(M) > 0$: $M + M^*$
- I : form Identity
- $diag\{..\}$: Block diagonal form

2. T-S MODELS

Let's the continuous model is given by

$$\dot{x}(t) = \frac{\sum_{r=1}^{n} \sigma_r(t)(A_r x(t) + L_r u(t) + B_r v(t))}{\sum_{r=1}^{n} \sigma_r(t)}, y(t) = \frac{\sum_{r=1}^{n} \sigma_r(t)(C_r x(t) + E_r u(t) + D_r v(t))}{\sum_{r=1}^{n} \sigma_r(t)} \tag{1}$$

with

$$\lambda_r(t) = \prod_{j=1}^{p} N_{rs}(\mu_s(t))$$

and

$$\sigma_r = \frac{\lambda_r(t)}{\sum_{r=1}^{n} \lambda_r(t)}; \quad 0 \leq \lambda_r \leq 1 \quad and \quad \sum_{r=1}^{n} \lambda_r = 1 \tag{2}$$

and $\sigma = [\sigma_1, ..., \sigma_r]^*$, the T-S system can be rewritten as follows:

$$\begin{aligned} \dot{x}(t) &= A(\sigma)x(t) + L(\sigma)u(t) + B(\sigma)v(t) \\ y(t) &= C(\sigma)x(t) + E(\sigma)u(t) + D(\sigma)v(t) \end{aligned} \tag{3}$$

where

$$\{A(\sigma); B(\sigma); L(\sigma); B(\sigma); C(\sigma); E(\sigma); D(\sigma)\} = \sum_{r=1}^{n} \rho_r(t)\{A_r; L_r; B_r; C_r; E_r; D_r\} \tag{4}$$

3. PDC CONTROLLER SCHEME

The fuzzy control as follows:

$$u(t) = \sum_{s=1}^{n} \sigma_s K_s x(t) \tag{5}$$

then, we have the closed loop model:

$$\begin{aligned} \dot{x}(t) &= A_c(\lambda)x(t) + B(\lambda)v(t) \\ y(t) &= C_c(\lambda)x(t) + D(\lambda)v(t) \end{aligned} \tag{6}$$

with

$$A_c(\sigma) = A_c(\sigma) + L(\sigma)K(\sigma); \quad {}_c(\sigma) = C_c(\sigma) + E(\sigma)K(\sigma) \tag{7}$$

problem formulation Given: the state feedback in the form of (5) such that:

$$\int_{\mu \in \nabla} Y^*(\mu)Y(\mu)d\mu \leq \gamma^2 \int_{\mu \in \nabla} V^*(\mu)V(\mu)d\mu \tag{8}$$

with $\triangle$ is given in Table 1.

Table 1. Different frequency ranges

$-$	$Low frequency$	$Middle frequency$	$High frequency$
∇	$\lvert\mu\rvert \leq \bar{\mu}_l$	$\bar{\mu}_1 \leq \mu \leq \bar{\mu}_2$	$\lvert\mu\rvert \geq \bar{\mu}_h$
Π	$\begin{bmatrix} -S(\sigma) & R(\sigma) \\ R(\sigma) & \bar{\mu}_l^2 S \end{bmatrix}$	$\begin{bmatrix} -S(\sigma) & R(\sigma) + j\bar{\mu}_0 S(\sigma) \\ R(\sigma) - j\bar{\mu}_0 S(\sigma) & -\bar{\mu}_1\bar{\mu}_2 R \end{bmatrix}$	$\begin{bmatrix} S(\sigma) & R(\sigma) \\ R(\sigma) & -\bar{\mu}_h^2 S(\sigma) \end{bmatrix}$

4. MAIN RESULTS

4.1. Useful lemma

Lemma 4..1 Tuan, H. D *et al.*[22] If the following conditions are met:

$$\Omega_{rr} < 0 \qquad 1 \leq r \leq n \frac{1}{n-1}\Omega_{rr} + \frac{1}{2}[\Omega_{rs} + \Omega_{sr}] < 0; \quad 1 \leq r \neq s \leq n \tag{9}$$

and

$$\sum_{r=1}^{n}\sum_{s=1}^{n} \lambda_r \lambda_s \Omega_{rs} < 0 \tag{10}$$

Lemma 4..2 El-Amrani, A. *et al.* [23]. Let $\mathcal{T} \in \mathbb{R}^{n \times n}$ and $\mathcal{M} \in \mathbb{R}^{m \times n}$, so that the following conditions are equivalent:

1. $\mathcal{M}^{\perp *}\mathcal{T}\mathcal{M}^{\perp} < 0$
2. $\exists \mathcal{N} \in \mathbb{R}^{n\times m} : \mathcal{T} + sym[\mathcal{M}\mathcal{N}] < 0$

Lemma 4..3 Closed loop (6) is stable, if $R(\sigma) = R(\sigma)^* \in \mathbb{H}_n$, $0 < S = S^* \in \mathbb{H}_n$ such that

$$\begin{pmatrix} A_c(\sigma) & B(\sigma) \\ I & 0 \end{pmatrix}^* \Pi \begin{pmatrix} A_c(\sigma) & B(\sigma) \\ I & 0 \end{pmatrix} + \begin{pmatrix} C_c^T(\sigma)C_c(\sigma) & C_c^T(\sigma)D(\sigma) \\ D^T(\sigma)C_c(\sigma) & D^T(\sigma)D(\sigma) - \gamma^2 I \end{pmatrix} < 0 \tag{11}$$

with Π is given of Table 1.

4.2. Finite frequency analysis

Theorem 4..4 The fuzzy model (6) is stable, if $R(\sigma) \in \mathbb{H}_n$, $0 < S \in \mathbb{H}_n$, $0 < W(\sigma) \in \mathbb{H}_n$, $Z(\sigma) \in \mathbb{H}_n$, $H(\sigma) \in \mathbb{H}_n$ such that

$$\begin{bmatrix} -sym[Z(\sigma)] & W(\sigma) + Z(\sigma)A(\sigma) - H^*(\sigma) \\ \star & sym[H(\sigma)A_c(\sigma)] \end{bmatrix} < 0 \tag{12}$$

$$\begin{bmatrix} \Psi_{11}(\sigma) & \Psi_{12}(\sigma) + Z(\sigma)A_c(\sigma) - H^*(\sigma) & Z(\sigma)B(\sigma) & 0 \\ \star & \Psi_{22}(\sigma) + sym[H(\sigma)A_c(\sigma)] & H(\sigma)B(\sigma) & C_c^*(\sigma) \\ \star & \star & -\gamma^2 I & D^*(\sigma) \\ \star & \star & \star & -I \end{bmatrix} < 0 \tag{13}$$

• Low frequency (LF) range:

$$\Psi_{11}(\sigma) = -S(\sigma) - Z(\sigma) - Z^*(\sigma); \quad \Psi_{12}(\sigma) = R(\sigma); \quad \Psi_{22}(\sigma) = \bar{\mu}_l^2 S(\sigma) \tag{14}$$

• Middle frequency range (MF) range:

$$\Psi_{11} = -S(\sigma) - Z(\sigma) - Z^*(\sigma); \quad \Psi_{12} = R(\sigma) + j\bar{\mu}_0 S(\sigma); \quad \Psi_{22} = -\bar{\mu}_1\bar{\mu}_2 S(\sigma) \tag{15}$$

• High frequency (HF) range:

$$\Psi_{11}(\sigma) = S(\sigma) - Z(\sigma) - Z^*(\sigma); \quad \Psi_{12}(\sigma) = R(\sigma); \quad \Psi_{22}(\sigma) = -\bar{\mu}_h^2 S(\sigma)$$

Proof 4..5 Let $\bar{A}(\sigma)$, $W(\sigma) = W(\sigma)^* > 0$ such that

$$\begin{bmatrix} A_c(\sigma) \\ I \end{bmatrix}^* \begin{bmatrix} 0 & W(\sigma) \\ W(\sigma) & 0 \end{bmatrix} \begin{bmatrix} A_c(\sigma) \\ I \end{bmatrix} < 0 \tag{16}$$

define:

$$\mathcal{T} = \begin{bmatrix} 0 & W(\sigma) \\ W(\sigma) & 0 \end{bmatrix}; \; \mathcal{N} = \begin{bmatrix} Z(\sigma) \\ H(\sigma) \end{bmatrix};$$

$$\mathcal{M} = \begin{bmatrix} -I & A_c(\sigma) \end{bmatrix}; \; \mathcal{M}^{\perp} = \begin{bmatrix} A_c(\sigma) \\ I \end{bmatrix} \tag{17}$$

let lemma 4.1., (16) and (17) are equivalent to:

$$\begin{bmatrix} 0 & W(\sigma) \\ W(\sigma) & 0 \end{bmatrix} + \begin{bmatrix} Z(\sigma) \\ H(\sigma) \end{bmatrix} \begin{bmatrix} -I & A_c(\sigma) \end{bmatrix} + \begin{bmatrix} -I & A_c(\sigma) \end{bmatrix}^* \begin{bmatrix} Z(\sigma) \\ H(\sigma) \end{bmatrix}^* < 0 \tag{18}$$

which is nothing but (12), let LF case :

$$\mathcal{T} = \begin{bmatrix} -S & R(\sigma) & 0 \\ \star & \bar{\mu}_l^2 S + C_c^*(\sigma)C_c(\sigma) & C_c^*(\sigma)D(\sigma) \\ \star & \star & -\gamma^2 I + D^*(\sigma)D(\sigma) \end{bmatrix};$$

$$\mathcal{M}^{\perp} = \begin{bmatrix} A_c(\sigma) & B(\sigma) \\ I & 0 \\ 0 & I \end{bmatrix}; \quad \mathcal{M} = \begin{bmatrix} -I & A_c(\sigma) & B(\sigma) \end{bmatrix};$$

$$\mathcal{N} = \begin{bmatrix} Z(\sigma)^T & H(\sigma)^T & 0 \end{bmatrix}^T \tag{19}$$

we have

$$\mathcal{T} \; + \; sym(\mathcal{N}\mathcal{M}) < 0 \tag{20}$$

using Lemma 4.1., we obtain (11).

4.3. Finite frequency design

Theorem 4..6 The fuzzy model (6) is stable, if $\tilde{R}(\sigma) \in \mathbb{H}_n$, $0 < \tilde{S} \in \mathbb{H}_n$, $0 < \tilde{W}(\sigma) \in \mathbb{H}_n$, $G(\sigma)$, $\bar{Z}(\sigma)$ such that:

$$\begin{bmatrix} -\bar{Z}^*(\sigma) - \bar{Z}(\sigma) & \tilde{W}(\sigma) + A(\sigma)\bar{Z}^*(\sigma) + B(\sigma)G^*(\sigma) - \beta\bar{Z}(\sigma) \\ \star & sym[\beta(A(\sigma)\bar{Z}^*(\sigma) + B(\sigma)G^*(\sigma))] \end{bmatrix} < 0 \tag{21}$$

$$\begin{bmatrix} \bar{\Psi}_{11}(\sigma) & \bar{\Psi}_{12}(\sigma) & B(\sigma) & 0 \\ \star & \bar{\Psi}_{22}(\sigma) & \beta B(\sigma) & \bar{Z}(\sigma)C^*(\sigma) + G(\sigma)E^*(\sigma) \\ \star & \star & -\gamma^2 I & D^*(\sigma) \\ \star & \star & \star & -I \end{bmatrix} < 0 \tag{22}$$

• $|\mu| \leq \bar{\mu}_l$

$$\bar{\Psi}_{11}(\sigma) = -\tilde{S}(\sigma) - \bar{Z}^*(\sigma) - \bar{Z}(\sigma);$$

$$\bar{\Psi}_{22}(\sigma) = \bar{\mu}_l^2 \tilde{S}(\sigma) + sym[\beta(A(\sigma)\bar{Z}^*(\sigma) + B(\sigma)G^*(\sigma))];$$

$$\bar{\Psi}_{12}(\sigma) = \tilde{R}(\sigma) - \beta\bar{Z}(\sigma) + A(\sigma)\bar{Z}^*(\sigma) + B(\sigma)G^*(\sigma).$$

• $\bar{\mu}_1 \leq \mu \leq \bar{\mu}_2$

$$\bar{\Psi}_{11}(\sigma) = -\tilde{S}(\sigma) - \bar{Z}^*(\sigma) - \bar{Z}(\sigma);$$

$$\bar{\Psi}_{22}(\sigma) = -\bar{\mu}_1\bar{\mu}_2\tilde{S}(\sigma) + sym[\beta(A(\sigma)\bar{Z}^*(\sigma) + B(\sigma)G^*(\sigma))];$$

$$\bar{\Psi}_{12}(\sigma) = \tilde{R}(\sigma) + j\bar{\mu}_0\tilde{S}(\sigma) - \beta\bar{U}(\sigma) + A(\sigma)\bar{Z}^*(\sigma) + B(\sigma)G^*(\sigma)$$

• $|\mu| \geq \bar{\mu}_h$

$$\bar{\Psi}_{11}(\sigma) = \tilde{S}(\sigma) - \bar{Z}^*(\sigma) - \bar{Z}(\sigma);$$

$$\bar{\Psi}_{12}(\sigma) = \tilde{R}(\sigma) - \beta\bar{Z}(\sigma) + A(\sigma)\bar{Z}^*(\sigma) + B(\sigma)G^*(\sigma);$$

$$\bar{\Psi}_{22}(\sigma) = -\bar{\mu}_h^2\tilde{S}(\sigma) + sym[\beta(A(\sigma)\bar{Z}^*(\sigma) + B(\sigma)G^*(\sigma))].$$

therefore :

$$K(\sigma) = (\bar{Z}^{-1}(\sigma)G(\sigma))^* \tag{23}$$

Proof 4..7 First, for matrix variable $H(\sigma)$ in theorem 4.2., let $H(\sigma) = \beta Z(\sigma)$, after that by replacing (7) into (12) and (13), respectively, moreover, let,

$$\bar{Z}(\sigma) = Z^{-1}(\sigma);\ G(\sigma) = \bar{Z}(\sigma)K^*;\ \tilde{S}(\sigma) = Z^{-1}(\sigma)S(\sigma)Z^{-*}(\sigma);$$

$$\tilde{R}(\sigma) = Z^{-1}(\sigma)R(\sigma)Z^{-*}(\sigma);\ \tilde{W}(\sigma) = Z^{-1}(\sigma)W(\sigma)Z^{-*}(\sigma)$$

Multiplying (12) by $diag\{Z^{-1}(\sigma),\ Z^{-1}(\sigma)\}$, and (13) by $diag\{Z^{-1}(\sigma),\ Z^{-1}(\sigma),\ I,\ I\}$, we have (12) and (13) are equivalent (21) and (22), respectively. **Theorem 4..8** The fuzzy model (6) is stable. If $R_r \in \mathbb{H}_n$, $0 < S \in \mathbb{H}_n$, $0 < W_r \in \mathbb{H}_n$, $\bar{Z}_s$, G_s such that

$$\tilde{\Psi}_{rr} < 0; \quad \tilde{\Upsilon}_{rr} < 0; \quad 1 \leq r \leq n \tag{24}$$

$$\frac{1}{r-1}\tilde{\Psi}_{rr} + \frac{1}{2}[\tilde{\Psi}_{rs} + \tilde{\Psi}_{sr}] < 0; \quad 1 \leq r \neq s \leq n \tag{25}$$

$$\frac{1}{r-1}\tilde{\Upsilon}_{rr} + \frac{1}{2}[\tilde{\Upsilon}_{rs} + \tilde{\Upsilon}_{sr}] < 0; \quad 1 \leq r \neq s \leq n \tag{26}$$

where

$$\tilde{\Psi}_{rs} = \begin{bmatrix} \tilde{\Psi}_{11rs} & \tilde{\Psi}_{12rs} & B_r & 0 \\ \star & \tilde{\Psi_{22rs}} & \beta B_r & \bar{F}_s C_r^* + G_s E_r^* \\ \star & \star & -\gamma^2 I & D_r^* \\ \star & \star & \star & -I \end{bmatrix};$$

$$\tilde{\Upsilon}_{rs} = \begin{bmatrix} -\bar{Z}_s^* - \bar{Z}_s & W_r + A_r\bar{Z}_s^* + B_rG_s^* - \beta\bar{Z}_s \\ \star & sym[\beta(A_r\bar{Z}_s^* + B_rG_s^*)] \end{bmatrix}$$

• $|\mu| \leq \bar{\mu}_l$

$$\tilde{\Psi}_{11rs} = -\tilde{S}_s - \bar{Z}_s^* - \bar{Z}_s;$$
$$\tilde{\Psi}_{12rs} = \tilde{R}_r - \beta\bar{Z}_s + A_r\bar{Z}_s^* + B_rG_s^*;$$
$$\tilde{\Psi}_{22rs} = \bar{\mu}_l^2\tilde{S}_s + sym[\beta(A_r\bar{Z}_s^* + B_rG_s^*)].$$

• $\bar{\mu}_1 \leq \mu \leq \bar{\mu}_2$

$$\tilde{\Psi}_{11rs} = -\tilde{S}_s - \bar{Z}_s^* - \bar{Z}_s;$$
$$\tilde{\Psi}_{12rs} = \tilde{R}_r + j\bar{\mu}_0\tilde{S}_s - \beta\bar{U}_s + A_r\bar{Z}_s^* + B_rG_s^*;$$
$$\tilde{\Psi}_{22rs} = -\bar{\mu}_1\bar{\mu}_2\tilde{S}_s + sym[\beta(A_r\bar{Z}_s^* + B_rG_s^*)].$$

• $|\mu| \geq \bar{\mu}_h$

$$\tilde{\Psi}_{11rs} = \tilde{S}_s - \bar{Z}_s^* - \bar{Z}_s;$$
$$\tilde{\Psi}_{12rs} = \tilde{R}_r - \beta\bar{Z}_s + A_r\bar{Z}_s^* + B_rG_s^*;$$
$$\tilde{\Psi}_{22rs} = -\bar{\mu}_h^2\tilde{S}_s + sym[\beta(A_r\bar{Z}_s^* + B_rG_s^*)].$$

The matrices gains are obtained by:

$$K_s = (\bar{Z}_s^{-1}G_s)^*, \qquad 1 \leq s \leq n \tag{27}$$

Proof 4..9 by applying the Lemma 4.1., we have Theorem 4.3.

5. SIMULATIONS

5.1. Example 1

Consider fuzzy system (3) with two rules [24]:

$$A_1 = \begin{bmatrix} 0 & 1 \\ 17.2941 & 0 \end{bmatrix}; \; A_2 = \begin{bmatrix} 0 & 1 \\ 12.6305 & 0 \end{bmatrix};$$

$$E_1 = \begin{bmatrix} 0 \\ -0.1765 \end{bmatrix}; \; L_1 = L_2 = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix};$$

$$E_2 = \begin{bmatrix} 0 \\ -0.0779 \end{bmatrix}; \quad B_1 = B_2 = 0.1;$$

$$C_1 = C_2 = \begin{bmatrix} 1 & 1 \end{bmatrix}; \quad D_1 = D_2 = 0 \tag{28}$$

and

$$N_2(x_1) = (\frac{1}{1 + \exp(-7(x_1 - \frac{\pi}{4}))})(\frac{1}{1 + \exp(-7(x_1 + \frac{\pi}{4}))});$$
$$N_1(x_1) = 1 - N_2(x_1). \tag{29}$$

let

$$v(t) = \begin{cases} 2 & 2 \leq t \leq 3 \\ 2 & 5 \leq t \leq 6 \\ 0 & others \end{cases} \tag{30}$$

We propose in table 2 shows the values of γ obtained in different frequency ranges. By Theorem 4.3., the controller gains are given by:

• Low frequency (LF) range (with $\beta_1 = 0.0502$ and $\gamma = 0.2507$):

$$K_1 = \begin{bmatrix} 168.2205 & 22.9439 \end{bmatrix}; \quad K_2 = \begin{bmatrix} 353.5002 & 115.2592 \end{bmatrix} \tag{31}$$

• Middle frequency (MF) range (with $\beta_1 = 0.5025$ and $\gamma = 0.8355$):

$$K_1 = \begin{bmatrix} 186.8988 & 36.1978 \end{bmatrix}; \quad K_2 = \begin{bmatrix} 378.9459 & 109.7698 \end{bmatrix} \tag{32}$$

• High frequency (HF) range (with $\beta_1 = 0.2478$ and $\gamma = 0.5702$):

$$K_1 = \begin{bmatrix} 203.4092 & 43.1392 \end{bmatrix}; \quad K_2 = \begin{bmatrix} 356.0428 & 101.8833 \end{bmatrix} \tag{33}$$

Table 2. Obtained γ by different domains

Frequency ranges	*methods*	γ
EF ($0 \le \mu \le \infty$)	[16]	Infeasible
EF ($0 \le \mu \le \infty$)	**Theorem 4.3**. ($\tilde{\mathbf{S}}_{\mathbf{s}} = \mathbf{0}$)	**1.1789**
LF ($\lvert\mu\rvert \le 0.7$)	*Theorem 2 in* [16]	1.3598
LF ($\lvert\mu\rvert \le 0.7$)	**Theorem 4.3**.	**0.2507**
MF ($1 \le \mu \le 5$)	*Corollary 1 in* [16]	1.3010
MF ($1 \le \mu \le 5$)	**Theorem 4.3**.	**0.8355**
HF ($\lvert\mu\rvert \ge 6$)	*Corollary 2 in* [16]	-
HF ($\lvert\mu\rvert \ge 6$)	**Theorem 4.3**.	**0.5702**
MF ($628 \le \mu \le 6283$)	*Corollary 1 in* [16]	1.5786
MF ($628 \le \mu \le 6283$)	**Theorem 4.3**.	**0.9245**
HF ($\lvert\mu\rvert \ge 6283$)	*Corollary 2 in* [16]	-
HF ($\lvert\mu\rvert \ge 6283$)	**Theorem 4.3**.	**0.2102**

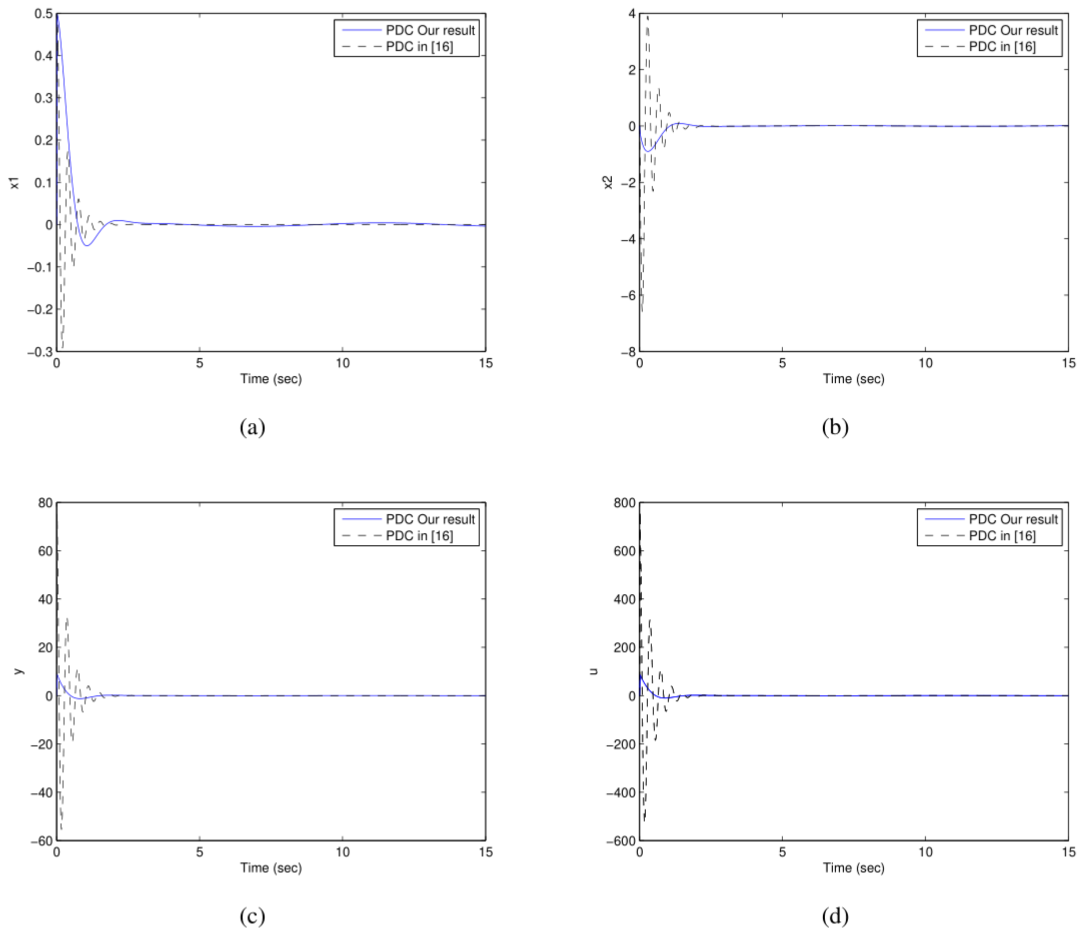

Figure 1. Trajectories of $x_i(t)$, $i = 1, 2$, $u(t)$ and $y(t)$ for LF $\lvert\mu\rvert \le 0.7$ range, (a) state $x_1(t)$, (b) state $x_2(t)$, (c) estimation controlled output $y(t)$, (d) estimation controlled output $u(t)$

5.2. Example 2

Let the fuzzy system (3) [25], where:

$$A_1 = \begin{bmatrix} -1 & -1.155 \\ 1 & 0 \end{bmatrix}; A_2 = \begin{bmatrix} -1 & -1.155 \\ 1 & 0 \end{bmatrix};$$

$$L_1 = \begin{bmatrix} 1.4387 \\ 0 \end{bmatrix}; \;\; L_2 = \begin{bmatrix} 0.5613 \\ 0 \end{bmatrix}; B_1 = B_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix};$$

$$C_1 = C_2 = \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}; E_1 = E_2 = \begin{bmatrix} 0 \\ 2 \end{bmatrix}; \quad D_1 = D_2 = \begin{bmatrix} 0 \\ 2 \end{bmatrix} \tag{34}$$

and

$$N_2(x_1(t)) = 0.5 - \frac{x_1^3(t)}{6.75};$$

$$N_1(x_1(t)) = 1 - N_2(x_1(t)); \;\; x_1(t) \in \left(\; 1.5 \quad 1.5 \; \right) \tag{35}$$

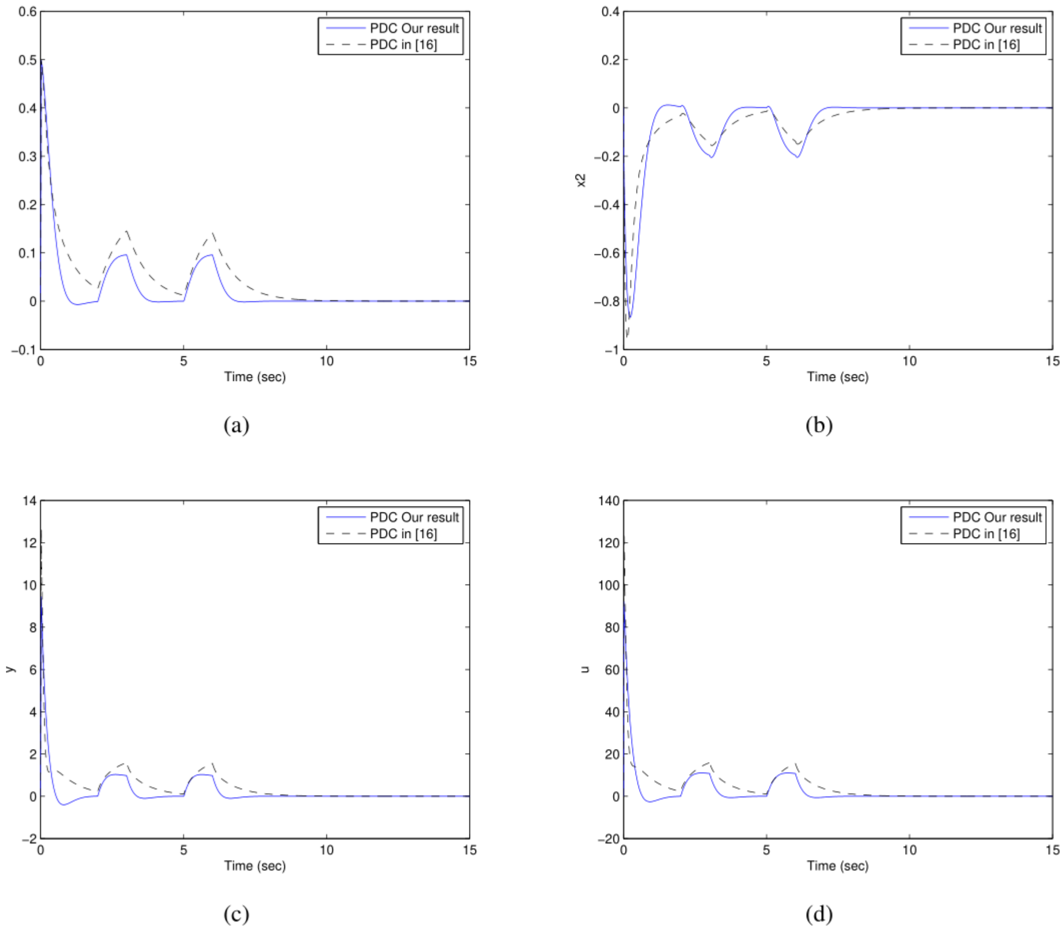

Figure 2. Trajectories of $x_i(t)$, $i = 1, 2$, $u(t)$ and $y(t)$ for MF $1 \leq |\mu| \leq 5$ range, (a) state $x_1(t)$, (b) state $x_2(t)$, (c) estimation controlled output $y(t)$, (d) estimation controlled output $u(t)$

Table 3. H_∞ performance bounds γ by different domains

$Frequency\ ranges$	$methods$	γ
$0 \leq \mu \leq \infty$	[16]	Infeasible
$0 \leq \mu \leq \infty$	**Theorem 4.3.** $(\mathbf{\tilde{S}_s = 0})$	**1.4517**
$\lvert\mu\rvert \leq 628$	[16]	1.2215
$\lvert\mu\rvert \leq 628$	**Theorem 4.3.**	**0.7514**
$628 \leq \mu \leq 6283$	[16]	1.025
$628 \leq \mu \leq 6283$	**Theorem 4.3.**	**0.5274**
$\lvert\mu\rvert \geq 6283$	[16]	-
$\lvert\mu\rvert \geq 6283$	**Theorem 4.3.**	**0.3854**

The FF case of $u(t)$ is assumed to satisfy 100 Hz; $[100\ 1000]$ Hz and 1000 Hz, i.e.,. $|\mu| \leq 628$ rad/s; $628 \leq \mu \leq 6283$ rad/s; and $|\mu| \geq 6283$ rad/s, respectively for $u(t)$ and $\beta = 1$.

6. CONCLUSION

We sent the FF state feedback design. To reduce the closed-loop system and establish less conservative results, we have considered two practical examples has been provides to show the feasibility of tuning FF H_∞ fuzzy control design method.

REFERENCES

[1] Takagi, T., & Sugeno, M., "Fuzzy identification of systems and its applications to modeling and control," *IEEE transactions on systems, man, and cybernetics*, vol. SMC-15, no. 1, pp. 116-132,1985.

[2] Zhao, T., & Dian, S., "Fuzzy dynamic output feedback H_∞ control for continuous-time TS fuzzy systems under imperfect premise matching." *ISA transactions*, vol. 70, pp. 248-259, 2017.

[3] Luo, J., Li, M., Liu, X., Tian, W., Zhong, S., & Shi, K., "Stabilization analysis for fuzzy systems with a switched sampled-data control," *Journal of the Franklin Institute*, vol. 357, no. 1, pp. 39-58, 2020.

[4] Liu, Y., Guo, B. Z., & Park, J. H., "Non-fragile H_∞ filtering for delayed Takagi–Sugeno fuzzy systems with randomly occurring gain variations," Fuzzy Sets and Systems, vol. 316, pp. 99-116, 2017.

[5] Qi, R., Tao, G., & Jiang, B., "Adaptive Control of T–S fuzzy systems with actuator faults," In *Fuzzy System Identification and Adaptive Control*, Springer, Cham, pp. 247-273, 2019.

[6] Su, X., Wu, L., Shi, P., & Song, Y. D., "H_∞ model reduction of Takagi–Sugeno fuzzy stochastic systems," *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, vol. 42, no. 6, pp. 1574-1585, 2012.

[7] Robert E. S., Iwasaki T., & Dimitri E, *A unified Algebraic approach to linear control design*, London, UK: Taylor & Francis, 1997.

[8] Ngo, Q. V., Chai, Y., & Nguyen, T. T., "The fuzzy-PID based-pitch angle controller for small-scale wind turbine," *International Journal of Power Electronics and Drive Systems*, vol. 11, no. 1, pp. 135-142, 2020.

[9] M'hamed, L., Roufaida, A., & Nawal, A. A. M., "Sensorless control of PMSM with fuzzy model reference adaptive system," *International Journal of Power Electronics and Drive Systems*, vol. 10, no. 4, pp. 1772-1780 , 2019.

[10] Attia, H. A., Gonzalo, F. D. A., & Stand-alone, P. V., "system with MPPT function based on fuzzy logic control for remote building applications," *International Journal of Power Electronics and Drive Systems*, vol. 10, no. 2, pp. 842-851, 2019.

[11] Miqoi, S., El Ougli, A., & Tidhaf, B., "Adaptive fuzzy sliding mode based MPPT controller for a photovoltaic water pumping system," International Journal of Power Electronics and Drive Systems, vol. 10, no. 1, pp. 414-422, 2019.

[12] El-Amrani, A., El Hajjaji, A., Boumhidi, I., & Hmamed, A., "Improved finite frequency H_∞ filtering for Takagi-Sugeno fuzzy systems," *International Journal of Systems, Control and Communications*, vol. 11, no. 1, pp. 1-24, 2020.

[13] El Hellani, D., El Hajjaji, A., & Ceschi, R., "Finite frequency H_∞ filter design for TS fuzzy systems: New approach," *Signal Processing*, vol. 143, pp. 191-199, 2018.

[14] El-Amrani, A., El Hajjaji, A., Hmamed, A., & Boumhidi, I., "Finite drequency filter design for TS fuzzy continuous systems," *2018 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE)*, 2018, pp. 1-7.

[15] El-Amrani, A., Boumhidi, I., Boukili, B., & Hmamed, A., "A finite frequency range approach to H_∞ filtering for TS fuzzy systems," *Procedia computer science*, vol. 148, pp. 485-494, 2019.

[16] Wang, H., Peng, L. Y., Ju, H. H., & Wang, Y. L., "H_∞ state feedback controller design for continuous-time T–S fuzzy systems in finite frequency domain," *Information Sciences*, vol. 223, pp. 221-235, 2013.

[17] El-Amrani, A., Boukili, B., El Hajjaji, A., & Hmamed, A., "H_∞ model reduction for T-S fuzzy systems over finite frequency ranges," *Optimal Control Applications & Methods*, vol. 39, no. 4, pp. 1479-1496, 2018.

[18] El-Amrani, A., Hmamed, A., Boukili, B., & El Hajjaji, A., "H_∞ filtering of TS fuzzy systems in Finite Frequency domain," *2016 5th International Conference on Systems and Control (ICSC)*, 2016, pp. 306-312.

[19] Duan, Z., Shen, J., Ghous, I., & Fu, J., "H_∞ filtering for discrete-time 2D T–S fuzzy systems with finite frequency disturbances," *IET Control Theory & Applications*, vol. 13, no. 13, pp. 1983-1994, 2019.

[20] Xie, W. B., Han, Z. K., Wu, F., & Zhu, S., (). "H_∞ observer–controller synthesis approach in low frequency for T–S fuzzy systems," IET Control Theory & Applications, vol. 14, no. 5, pp. 738-749, 2020.
[21] Wang, M., Feng, G. G., & Jianbin, Q., "Finite frequency fuzzy output feedback controller design for roesser-type two-dimensional nonlinear systems," in *IEEE Transactions on Fuzzy Systems*, 2020.
[22] Tuan, H. D., Apkarian, P., Narikiyo, T., & Yamamoto, Y., "Parameterized linear matrix inequality techniques in fuzzy control system design," *IEEE Transactions on fuzzy systems*, vol. 9, no. 2, pp. 324-332, 2001.
[23] de Oliveira, M. C., & Skelton, R. E., "Stability tests for constrained linear systems," In *Perspectives in robust control*, London: Springer,2001, pp. 241-257.
[24] Liu, X., & Zhang, Q., "Approaches to quadratic stability conditions and H_∞ control designs for TS fuzzy systems," *IEEE Transactions on Fuzzy systems*, vol. 11, no. 6, pp. 830-839, 2003.
[25] Wu, H. N., & Zhang, H. Y., "Reliable mixed L_2/H_∞ fuzzy static output feedback control for nonlinear systems with sensor faults," *Automatica*, vol. 41, no. 11, pp. 1925-1932, 2005.