

# The Control Structure for DC Motor based on the Flatness Control

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## Article Info

### Article history:

Received Jun 12<sup>th</sup>, 201x

Revised Aug 20<sup>th</sup>, 201x

Accepted Aug 26<sup>th</sup>, 201x

### Keyword:

DC Motor  
Flatness Cotroller  
PID Controller  
Control Structure  
Electric

## ABSTRACT

This article presents the new control structure for a Direct Current Motor (DC Motor) using the flatness-control principle. Basic on the mathematical model of DC Motors, the author demonstrates the application ability of the fatness-control theory to control the DC Motor, and then calculates the parameters and proposes the structure of the flatness-controller. The proposed structure is built and ran on Matlab-Simulink software to verify the system efficiency. The simulation results show that the quality of the control system is very good, especially in case of the flatness controller combined with PID controller to eliminate static error when the parameters of the DC Motor have not known accurately.

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## 1. INTRODUCTION

DC Motor is one of the traditional electric machines, is appeared in the late of 19th century. Because of the internal advantages of DC Motor such as simple control, large electromagnetic torque, the ability to adjust the speed with the wide range [1], DC Motors are still commonly used in industrial fields such as steel rolling, transportation, mining, defense, construction [2-4]. So improving the quality of DC Motor control system is essential. There are many studies to control DC Motors [5-8], the most popular is still the method using PID controllers. However, in many working modes of DC Motors, the nonlinear of DC Motor is high, which reduces the quality of control system. Therefore, the author proposes a suitable control system to improve the control quality of a DC Motor, which is a control system based on flatness principle.

The Flatness Control Theory is a new method control for nonlinear object [9], promising a high quality of control [10], being attracted by scientists around the world. Many researchers have come up with different definitions of flatness control systems, but in general, the flatness control is regarded as a useful tool for the nonlinear control system. The most specific of the flatness system is the existence of assemblage of  $z$ , through the  $z$  variables and the differential of the  $z$  variables, all state variables, and input variables can be determined. So we can determine in advance the trajectory of the input from the desired trajectory of the output.

## 2. THE STRUCTURE CONTROL BASED ON FLATNESS CONTROL THEORY

### 2.1. The basis of flatness control theory

Flatness control theory is applied to control a lot of nonlinear objects which have the status equation is written as follows [9, 12]:

$$\dot{x} = f(x, u) \quad (1)$$

With  $u = (u_1, u_2, \dots, u_m)^T$  is the input variables,  $x = (x_1, x_2, \dots, x_n)^T$  is the status variables.

The system (1) is called a flatness system if there are a set of variables  $z = (z_1, z_2, \dots, z_m)$  called the flat outputs, satisfying three conditions as follows:

- Existing a function  $h$ , which is satisfied:

$$z = h(x, u, \dots, u^{(\alpha)}) \quad (2)$$

- All input variables and state variables can be determined from the variable  $z$ , which means that there are function  $A$  and function  $B$ , which are satisfied:

$$x = A(z, \dot{z}, \dots, z^{(\beta)}) \quad (3)$$

$$u = B(z, \dot{z}, \dots, z^{(\beta+1)}) \quad (4)$$

- All variables of  $z$  are independently differential together, which means that there is not the function  $G$ , which is satisfied:  $G(z, \dot{z}, \dots, z^{(m)}) = 0$

There are many control systems that satisfy the properties of flatness systems such as electric motors, chemical reactors, cranes, transmission systems, eg.

## 2.2. Demonstrating the flatness of DC Motors

The structure of a separately excited DC Motor is shown in figure 1, which includes:

- The armature coil in the rotor
- The excitation coil in the stator

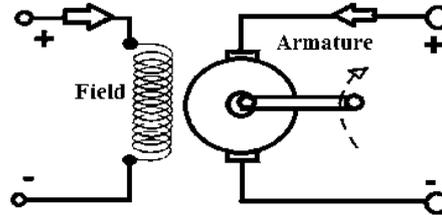


Figure 1. The structure of the separately excited DC Motor

To demonstrate a DC Motor is flatness system, the first we have to define the mathematical model of the DC Motor, in order to prove DC Motor satisfies the conditions of the flatness system.

The mathematical equations of DC Motors are as follows:

- The equation of voltage:

$$u_a = R_a i_a + L_a \frac{di_a}{dt} + E \quad (\text{V}) \quad (5)$$

With  $u_a$  is the armature voltage;  $R_a, L_a$  is the armature resistance and inductance.

- The equation of the Electromotive:

$$E = K_E \omega = (L_{af} i_f) \omega \quad (\text{V}) \quad (6)$$

With  $K_E$  is the voltage coefficient,  $\omega$  is the rotor speed,  $L_{af}$  is the field armature mutual inductance,  $i_f$  is the field current.

- The motion equation of the DC Motor:

$$J \frac{d\omega}{dt} = T_e - T_L - B_m \omega - T_f \quad (\text{N.m}) \quad (7)$$

Where  $J$  is the inertia,  $B_m$  is the viscous friction coefficient,  $T_f$  is the coulomb friction torque,  $T_e$  is the electromechanical torque,  $T_L$  is the torque applied to the shaft.

- The equation of electromechanical torque:

$$T_e = K_T L_a = (L_{af} i_f) i_a \quad (\text{N.m}) \quad (8)$$

With  $K_T$  is the torque coefficient.

From the equations of DC Motors, changing into the state equations with the state variables are the armature current and the speed ( $x = (i_a, \omega)$ ,  $u = u_a$ ):

$$\begin{cases} \frac{di_a}{dt} = -\frac{R_a}{L_a}i_a - \frac{K_E}{L_a}\omega + \frac{1}{L_a}u_a \\ \frac{d\omega}{dt} = \frac{K_T}{J}i_a - \frac{B_m}{J}\omega - \frac{1}{J}T_L - \frac{1}{J}T_f \end{cases} \quad (9.a, b)$$

Selecting the flat output variable is  $z = \omega$ , it is easy to prove that the DC Motor is flatness under three conditions:

- The first condition is existing the function  $h$  satisfied:  $z = h(x, u, \dots, u^{(\alpha)})$ . From the equation (5.b), it is easy to see that the first condition is satisfied.
- The second condition, there are the function  $A$  and the function  $B$  satisfied:  $x = A(z, \dot{z}, \dots, z^{(\beta)})$ ,  $u = B(z, \dot{z}, \dots, z^{(\beta+1)})$ .

Transforming the equation 9.b, we have:

$$i_a = \frac{1}{K_T}(J\dot{\omega} + B_m\omega + T_L + T_f) = \frac{1}{K_T}(J\dot{z} + B_mz + T_L + T_f) = G1(z, \dot{z}) \quad (10)$$

So  $x = (i_a, \omega) = A(\omega, \dot{\omega})$

Transforming the equation 9.a, we have:

$$\begin{aligned} u_a &= \frac{R_a}{K_T}(J\dot{\omega} + B_m\omega + T_L + T_f) + \frac{L_a}{K_E}(J\ddot{\omega} + B_m\dot{\omega} + \dot{T}_L) + K_E\omega \\ &= \frac{R_a}{K_T}(J\dot{z} + B_mz + T_L + T_f) + \frac{L_a}{K_E}(J\ddot{z} + B_m\dot{z} + \dot{T}_L) + K_Ez = G2(z, \dot{z}, \ddot{z}) \end{aligned} \quad (11)$$

So  $u = u_a = B(z, \dot{z}, \ddot{z})$

So we can conclude that the second condition is satisfied

- The third condition, all variables of  $z$  are independently differential together, this is obvious because the flat outputs are selected with only one variable.

So we have concluded that DC Motors are flatness systems with flat output is  $z = \omega$

### 2.3. Designing the flatness controller

Based on the equations of DC Motors, we construct the stages of the flatness controller. From the equation (9.b), we design the speed controller (calculating the current  $i_a^*$ ). From the equation (9.a), we design the current controller (calculating the voltage  $u_a$ ).

- The equation of speed controller:

$$i_a^* = \frac{1}{K_T} \left( J \frac{d\omega^*}{dt} + B_m\omega^* + T_L + T_f \right) \quad (12)$$

- The equation of current controller:

$$u_a = R_a i_a^* + L_a \frac{di_a^*}{dt} + K_E \omega^* \quad (13)$$

The block diagram of flatness control system for DC Motor is shown in figure 2:

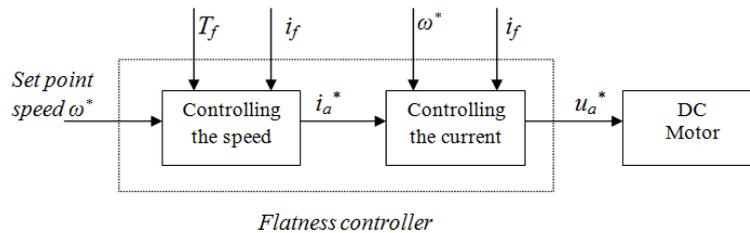


Figure 2. The block diagram of flatness control system for DC Motor

**2.4. Building the model simulation**

The control system diagram is constructed as shown in figure 3.

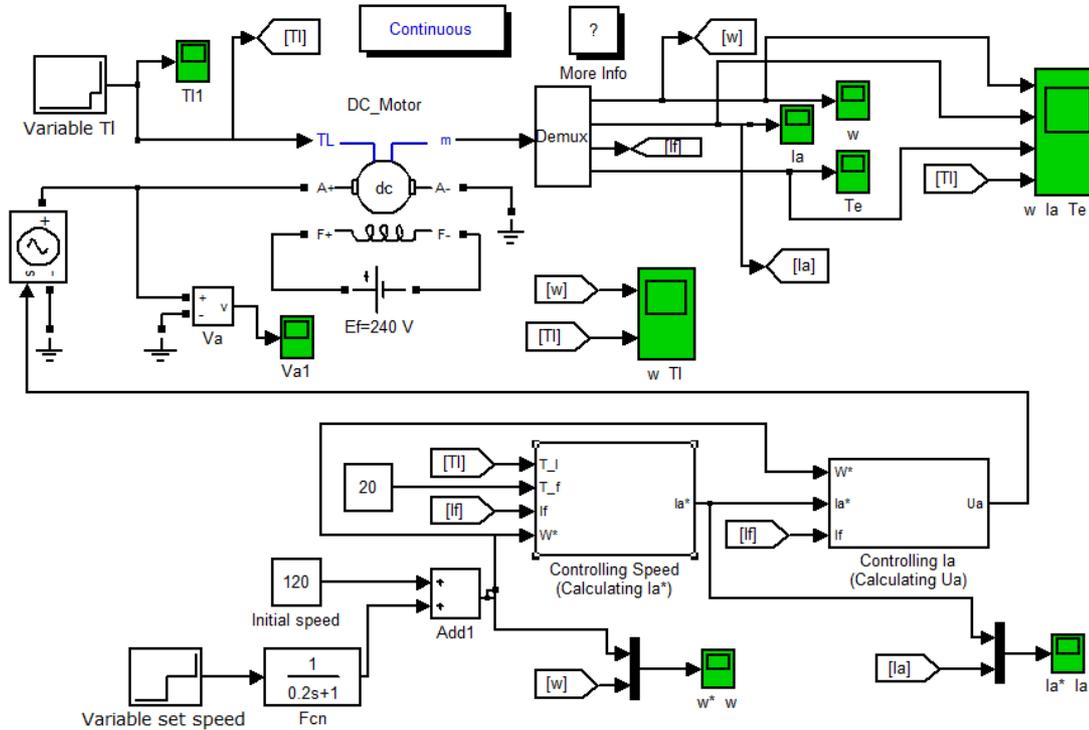


Figure 3. The simulation diagram of the control system with the flatness controller

In order to achieve objective results, in the diagram we use the DC Motor model available in the Matlab-Simulink library, the parameters of DC Motor are shown in table 1.

Table 1. The parameters of DC Motor

Ra(Ω)	La(H)	Rf(Ω)	Lf(H)	Laf(H)	J(kg.m <sup>2</sup> )	Bm(N.m.s)	Tf(N.m)
0.5	0.015	220	140	1.7	1.2	0.5	20

Based on the model and parameters of DC Motor, the control block is built as follows:

- The block of speed control is built based on the equation 12, shown in figure 4:

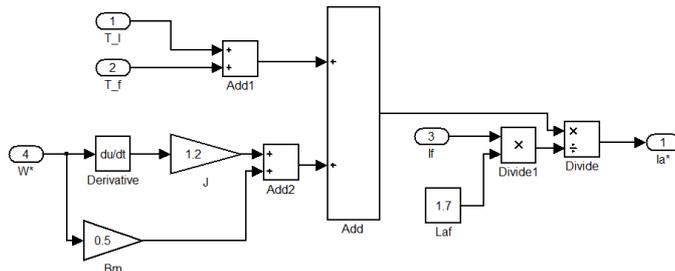


Figure 4. The speed control block

- The speed control block is built based on the equation 13, shown in figure 5:

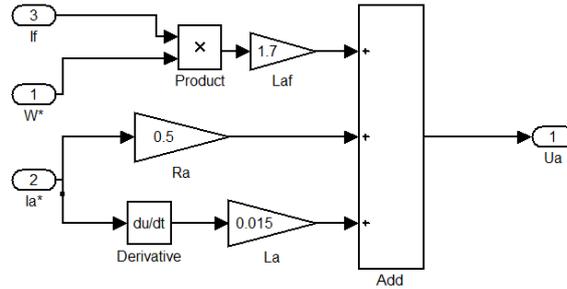


Figure 5. The current control block

**2.5. The simulated results**

Running the system with the initial set speed  $\omega = 120(rad / s)$ , then changing the value of set speed at time  $t = 1.5s$  and  $t = 3s$  ( $\omega = 150(rad / s)$  and  $\omega = 180(rad / s)$ ). Load initially with the torque of the shaft  $T_L = 50(N.m)$ , then at time  $t = 4.5(s)$  and  $t = 5.5(s)$ , we increase  $T_L$  to 100 (N.m) and 200 (N.m). The simulation results are shown in figure 6.a. The simulation results show that the change of speed values meet very good requirements.

For clarity, we study in more detail the graph of the set speed and actual speed in figure 6.b. From the graph 6.b, we see that the actual speed value is very close to the set speed value.

The change in speed depends on the load is shown in figure 6.c. Simulation results show that the system is very high quality, the speed of the Motor is almost unchanged when the torque changes.

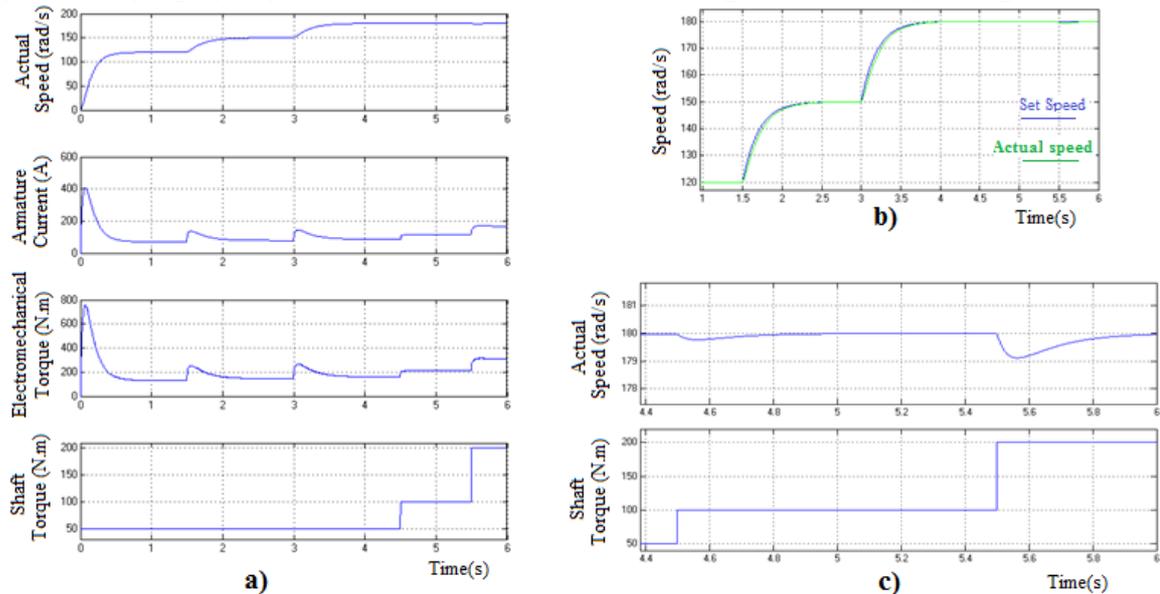


Figure 6. The simulated results with the flatness controller

**3. THE PROBLEM OF THE PARAMETER ERROR OF THE MOTOR MODEL**

The simulation results in part 2 show that the quality of the control system is perfect, because of the assumption that we know exactly the parameters of the DC Motor. However, in reality, it is impossible to know the parameters of DC exactly, so the quality of control system is not perfect as above. To check the control quality when DC parameter is not known exactly, we assume that some parameters of the DC Motor are changed, the parameter of the DC Motor, after changed is shown in Table 2:

Table 2. The parameters of DC Motor after changed

Ra(Ω)	La(H)	Rf(Ω)	Lf(H)	Laf(H)	J(kg.m <sup>2</sup> )	Bm(N.m.s)	Tf(N.m)
0.55	0.016	220	145	1.6	1.3	0.6	20

Running the system model, the response of the actual speed and set speed is shown in figure 7. The results show that there is a difference between the set speed and the actual speed. To eliminate this difference, we add a PID controller connected in parallel with the flatness controller. The parallel control diagram is shown in figure 8. The simulation model with the additional PID controller is shown in figure 9.

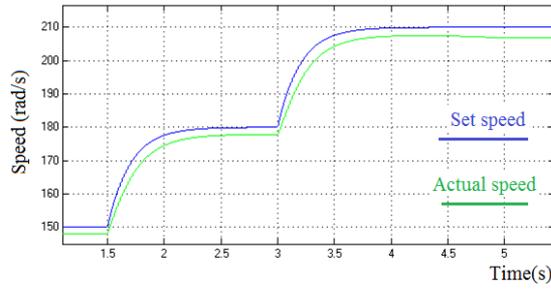


Figure 7. The simulated results when the DC parameter is not known exactly

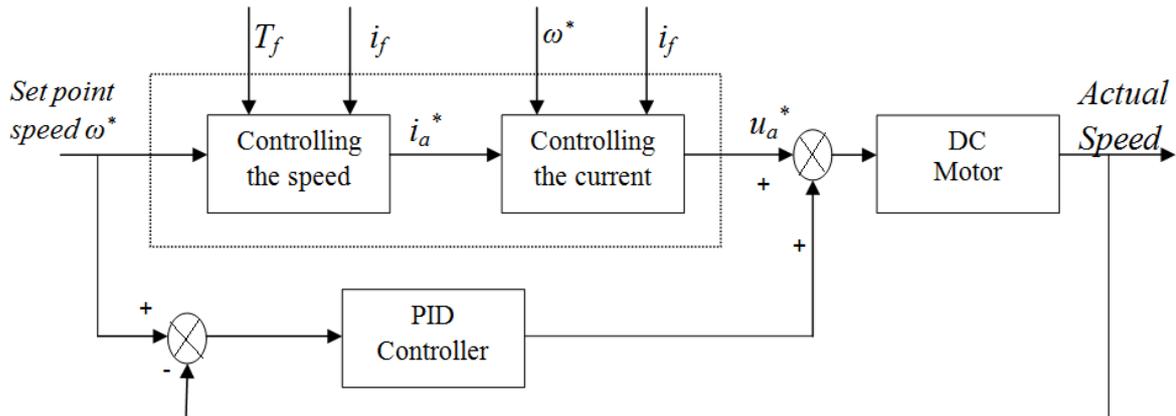


Figure 8. The parallel control diagram

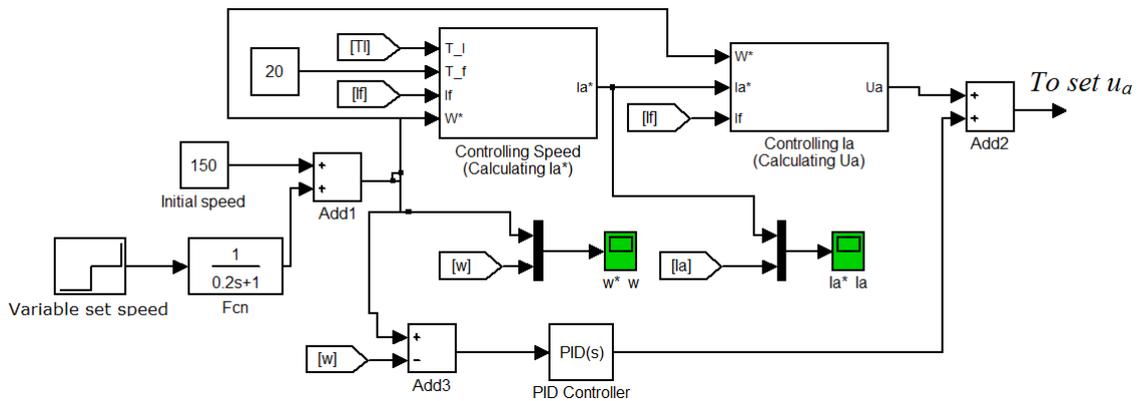
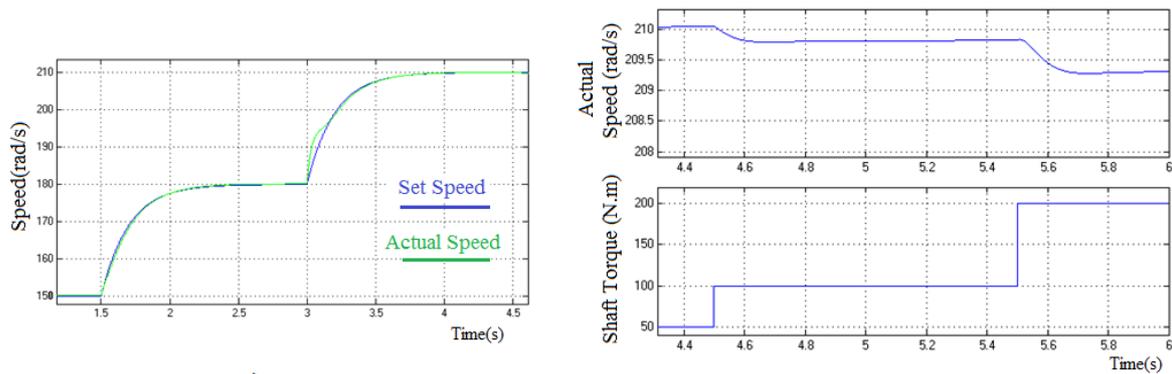


Figure 9. The simulation diagram of the control system with the additional PID controller

Running the model with the additional PID controller with  $K_p=3$ ,  $K_i=0.5$ , the simulation results are shown in figure 10. The results show that when changing the set speed, the actual speed of DC Motor is very close to the set speed, and when the torque of the shaft changes, the speed of DC Motor is not affected much. So the flatness control system for DC Motor with the support of the PID controller has worked very well.



a) b)  
Figure 10. The simulated results with the additional PID controller

#### 4. CONCLUSION

In this paper, the author has successfully built the control system for DC Motor based on a flatness control. The simulation results demonstrate that the system with the flatness controller works very well if we know the parameters of DC Motor exactly. However, in fact, the parameters of the DC Motor have not known accurately, so there is the error between set speed and actual speed when only the flatness control is applied. A PID controller has been put into operation in parallel with the flat controller to eliminate their error. The simulation results show that the quality of the whole system is very good and work effectively.

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