A modal analysis based on reactive power compensation on 6-bus Oman electrical grid

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ABSTRACT
This paper covers the modal analysis application (MATLAB 2019a) for improving the voltage profiles by optimum positioning of the capacitor banks for 6-bus Oman Electrical System because the Oman electricity Transmission Company (OETC) is suffering of drop voltage in these 6 buses especially during summer season as a peak period. The Newton-Raphson (N-R) method will help to determine the required reactive power for each load bus and as well the ideal position or point of capacitors. The process aims to maintain the Q-V relations of the electrical grid by correlating the lowest Eigen-values to related Eigen-vectors in obtained Jacobian matrix. It depends on the Eigen-values, if they are positive then the system’s voltage is stable otherwise it is not stable. In a stable system, the potential voltage collapse could be anticipated by checking the participation factors for a group of minimal positive Eigen-values. In general, the critical weak bus is associated with lower Eigen-values. Electrical system collapse is attributable to the weakest bus in the network and it could be avoided by determining the weak buses and providing capacitor banks at suitable locations which will lead to improve the voltage stability margin.

Keywords: Jacobian matrix, Load flow algorithm, Newton-Raphson, Reactive power compensation, Voltage stability

1. INTRODUCTION
In recent years, the electrical load demands have been increased and because of the system voltage security, the power transfer between villages, towns and industrial areas have emerged for the concern [1], [2]. A lot of disturbances in electrical power due to voltage collapse and instability as it happens with the selected 6-bus Oman grid. Therefore, in order to overcome these problems, the voltage stability needs to be analyzed and studied deeply. There are some reasons which lead to voltage instability such as inadequate of reactive power, high load on transmission line, lack of voltage sources and the distances between the source and utility [3]-[6]. Generally, voltage instability is related to imbalanced reactive power. In the power system, the load bus capability relies on reactive power support which it received from the electrical system. Voltage instability leads to voltage collapse where the system unable to recover again and usually it starts with rapid reduction of voltage [4]-[7]. There are two classifications of analysis methods of voltage stability issues which are dynamic and static. Static method involves [8]; i) modal analysis/Eigen vectors and values, ii) Q-V and P-V curves, and iii) power flow analysis.

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Flexible alternating current transmission system (FACTS) devices are widely used for solving the voltage instability. FACTS devices are considered one of the most effective solutions for reducing the power losses and improving the voltage profile in electrical grid because of their fast and flexibility of control [5]-[10]. In addition, they can provide high loading capacity and reducing or get rid of the blackouts. These devices can be used effectively by determining the weak bus in the electrical system where they should take a place to supply the required reactive power [11].

Optimal capacitor placement is considered one of the most effective solutions for reducing the power losses and improving the voltage profile in electrical grid [12]-[14]. By using the capacitors, the required reactive power for each load bus is supplied. There are issues in capacitors optimal placement for reducing the power losses in distribution network such as [15]; i) changeable behavior of the load buses, ii) intricacy of electrical grid, iii) different types of the loads network, iv) uncertainty in returning of the expenditure which used for capacitor placement.

In case the modal analysis of the Jacobean matrix is very close to the point of collapse [16], this will increase the ability of the operator to determine the weak bus and select the optimal placement of capacitor banks where the reactive power injection will support the system most. The information about the loads which are affecting the voltage collapse could be obtained from Eigen-vectors and their associated Eigenvalues [15]-[18]. The reduced Jacobean matrix for Newton-Raphson (N-R) can be established as per [19].

2. MODAL ANALYSIS TO IMPROVE VOLTAGE PROFILES

A system is voltage steady at given operating circumstance if for each bus in the system, the bus voltage magnitude will increase as reactive energy injection at that bus is increased. In other words, a system is voltage secure if V-Q sensitivity is positive for each bus and unstable if it is negative for at least one bus. The Jacobian of N-R load flow algorithm is considered in terms of (1) and (2). Where, \( \Delta P \) is the incremental change in bus real power, \( \Delta Q \) is the incremental change in bus reactive power injection, \( \Delta \delta \) is the incremental change in bus voltage angle, \( \Delta V \) is the incremental change in bus voltage magnitude and the sub matrices \( J_1 \) to \( J_4 \) are the partitioned matrices of the Jacobian Matrix \( J \).

\[
\begin{bmatrix}
\Delta P \\
\Delta Q \\
\end{bmatrix} =
\begin{bmatrix}
\frac{\partial P}{\partial \delta} & \frac{\partial P}{\partial V} \\
\frac{\partial Q}{\partial \delta} & \frac{\partial Q}{\partial V} \\
\end{bmatrix}
\begin{bmatrix}
\Delta \delta \\
\Delta V \\
\end{bmatrix}
\tag{1}
\]

\[
\begin{bmatrix}
\Delta P \\
\Delta Q \\
\end{bmatrix} =
\begin{bmatrix}
J_1 & J_2 \\
J_3 & J_4 \\
\end{bmatrix}
\begin{bmatrix}
\Delta \delta \\
\Delta V \\
\end{bmatrix}
\tag{2}
\]

This conventional load flow is used for reactive power compensation studies and it also helps in voltage stability studies. The power flow equations are solved using the N-R Method. Real and reactive power will affect the voltage stability. However, at each operating point, P is constant and stability is evaluated by considering the incremental relation between Q and V, although incremental changes in (P) are neglected in the formulation. The expression of the incremental changes in each bus reactive power injection. \( \Delta Q \) is obtained based on the (1) and (2) [20].

\[
\Delta Q = [-J_3J_1^{-1}J_2 + J_4] \Delta [V]
\tag{3}
\]

It is obvious that the modal analysis by using the Jacobian matrix has more computational difficulty than of the reduced Jacobian matrix, thus the reduced Jacobian approach (J_R) is followed here as (4) for 6-bus Oman grid [20], [21]. It represents a linear relation between the change in bus voltage (\( \Delta V \)) and the injection of reactive power at each bus (\( \Delta Q \)) at unchangeable active power.

\[
\Delta Q = J_R \Delta [V]
\tag{4}
\]

2.1. Modes and voltage stability

The reduced Jacobian matrix has right and left Eigen-vectors as (4) which is resulting in participation factors that can be defined as [21], [22].

\[
J_R = \xi \lambda \eta
\tag{5}
\]
where $\xi$ is the matrix of right eigenvectors, $\eta$ is the matrix of left eigenvectors and $\lambda$ is the matrix of diagonal eigenvalues of matrix $J_R$. The smaller the eigenvalues, the more the bus’s participation factor indicates the point closer to voltage instability: i) $\lambda_i = 0$, voltage collapses, ii) $\lambda_i > 0$, stable voltage-modal voltage and reactive power changes are proportional to each, and iv) $\lambda_i < 0$, unstable voltage-modal voltage and reactive power changes are in the reverse direction.

Thus, by checking the status of the eigenvalues, machine stability can be evaluated. It should be remembered that the entire individual values of a voltage stable system would be positive. If all of the eigenvalues are negative, the voltage is unstable. The reduced Jacobian matrix’s zero value means that the system is on the verge of voltage instability. The smaller the magnitude of eigenvalues, the closer the voltage is unstable to the corresponding modal voltage.

Eigen value analysis is considered as one of the tools to determine voltage collapse in power system grid [11]. It has computational process of the eigenvalues for reduced Jacobian matrix which obtained from Newton-Raphson load flow solution. It gives a relative measure to voltage instability because Eigen values show the relation between the voltage and reactive power differences which will help to find out the weakest bus in the network by effective use of participation factor [16], [19], [23]. The magnitude of the Eigen-values can evaluate the instability of the electrical system, whereas the Eigen-vectors give the data for loss of the voltage stability [24], [25]. Generally, the modal analysis relies on the matrix of diagonal eigen-values of matrix $J_R$ ($\lambda$). Thus, the reduced Jacobian matrix determines the condition of the given electrical system with respect to voltage stability [22].

2.2. Bus participations factors

The use of right and left eigenvectors of the reduced Jacobian matrix $J_R$ of (5) leads to the notion of participation factors. The participation factor of bus-k to mode-i is defined as [13].

$$P_{ki} = \xi_{ki}^T \eta_{ki}$$

where $P_{ki}$ indicates the contribution of eigenvalues corresponding to load bus i to the V-Q sensitivity at bus k. The higher values of $P_{ki}$ result in larger contribution of $\lambda_i$ in determining V-Q sensitivity at bus k. For all the small eigenvalues, bus participation factors determine the area closer to voltage instability.

2.3. Proposed algorithm

The stable electrical system could move towards the instability and this situation can be determined by participation factor for minimum positive Eigen-values. The flowchart shows the steps how to reach the weakest bus is in Figure 1. The proposed algorithm consists of calculation of the required $V_{AR}$, reactive voltage and calculation of eigen-values of matrix $J_R$. Finally, the weakest bus in the grid is determined only after the identification of least positive Eigen-values and the determination of the corresponding participation factor, $P_{ki}$.

![Flowchart of steps using to determinate the weakest bus](image-url)
3. RESULTS AND DISCUSSION

A computer program implementing in MATLAB2019a in order to evaluate the efficiency of the chosen Oman electrical system. In this paper, a single line diagram of 6-bus Oman electrical grid as shown in Figure 2 has been used to apply the modal analysis. The Oman 6 bus Electrical System is used to to show the practicalability of proposed algorithm. Bus 1 is the reference bus; bus 2 is a PV bus, while bus 4 to 6 are load buses. The line data and bus data are shown in Tables 1 and 2 respectively.

Figure 2. 6-bus Oman electrical grid

Table 1. Line data for a 6-bus Oman electrical grid

<table>
<thead>
<tr>
<th>Line No.</th>
<th>Bus Number</th>
<th>Length(km)</th>
<th>Total Impedances Ω/Length (km)</th>
<th>Total Impedances p.u/ Length(km)</th>
<th>Half line charging Admittance p.u</th>
<th>Tap ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>From</td>
<td>To</td>
<td></td>
<td>R</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>3</td>
<td>20</td>
<td>0.4283</td>
<td>2.82100</td>
<td>0.002458</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>4</td>
<td>15.5</td>
<td>0.3319</td>
<td>2.18620</td>
<td>0.001905</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>5</td>
<td>16.5</td>
<td>0.3533</td>
<td>2.32730</td>
<td>0.002027</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>6</td>
<td>60.9</td>
<td>1.30417</td>
<td>8.58994</td>
<td>0.007484</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>2</td>
<td>34.5</td>
<td>0.738817</td>
<td>4.86622</td>
<td>0.004240</td>
</tr>
</tbody>
</table>

Table 2. Bus data for a 6-bus Oman electrical grid

<table>
<thead>
<tr>
<th>Bus No.</th>
<th>V (p.u.)</th>
<th>O (deg)</th>
<th>P (MW)</th>
<th>Q (MVAR)</th>
<th>P (MW)</th>
<th>Q (MVAR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.00</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>1.00</td>
<td>0</td>
<td>70</td>
<td>0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>1.0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>115</td>
<td>38</td>
</tr>
<tr>
<td>4</td>
<td>1.0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>1.0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>50</td>
<td>17</td>
</tr>
<tr>
<td>6</td>
<td>1.0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>102</td>
<td>34</td>
</tr>
</tbody>
</table>

The load flow is performed on the 6-bus Oman electrical grid and the base-case results are obtained. Since it is required to improve the bus-voltage profile, the modal analysis-based approach is preferred. Figure 3 is required active power required to maintain voltage at 1PU. The load flow solution had been implemented on 6-bus Oman electrical system in order to improve the voltage profiles and its results as shown in Table 5. Then, the Eigen-values were calculated for reduced Jacobian matrix and the modal analysis was applied. As seen in Table 3, the Eigen-values are all positive which indicates that the 6-bus Oman system is stable and the smallest value is λ=19.9833 which corresponding to bus no 6. Therefore, the participation factor could be calculated by (6) corresponding to the lowest value as shown in the Table 3. Now, the required reactive power values to be injected are calculated (3) as per the proposed techniques and the values so determined are tabulated in Table 4.

Figure 4 shows the participation factor for least eigen values i.e., λ=19.9833. As shown in the Table 3, column-4 of the participation factors for λ=19.9833 is conforming to bus no 4 returned a maximum
participation factors of 0.8310, which indicates that bus no 4 is the weakest bus contributing to maximum voltage collapse, hence the evaluation of the participation factor could help to determine the weakest bus. After the total required reactive ($Q_T=120.51$ MVAR) is injected in bus no 4, the magnitude of load bus voltage improves, the improvement in voltage as Table 5 and as depicted in Figure 5 shows the comparison of voltage before compensation and after compensation.

### Table 3. Participation factors 6-bus Oman electrical grid

<table>
<thead>
<tr>
<th>Load Buses</th>
<th>Eigen-values</th>
<th>Participation factors for ( \lambda = 19.98 ) (for load buses 3-6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>465.88</td>
<td>0.4032 -0.5912 -0.6532 0.2440</td>
</tr>
<tr>
<td>4</td>
<td>225.68</td>
<td>-0.1573 0.4841 -0.2227 0.8310</td>
</tr>
<tr>
<td>5</td>
<td>89.28</td>
<td>-0.5924 -0.6375 0.3447 0.3508</td>
</tr>
<tr>
<td>6</td>
<td>19.98</td>
<td>-0.6805 0.0937 -0.6364 -0.3558</td>
</tr>
</tbody>
</table>

### Table 4. Additional VAr's required to maintaining voltage at 1 p.u. (Proposed method)

<table>
<thead>
<tr>
<th>Load Bus No.</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Additional VAr's Required p.u</td>
<td>0.5783</td>
<td>0.0057</td>
<td>0.0370</td>
<td>0.5841</td>
</tr>
<tr>
<td>Total Reactive Power, ($Q_T$) = 120.51 MVAR</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 5. Comparison of Results before and after capacitor placement at bus 4

<table>
<thead>
<tr>
<th>Bus Number</th>
<th>Eigen-Values-( \lambda )</th>
<th>Participation factor (( \lambda = 19.98 ))</th>
<th>Magnitude of Voltages (p.u.)</th>
<th>Base Case (Before)</th>
<th>( Q_T ) at bus-4 (After)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-</td>
<td>-</td>
<td>1.000</td>
<td>1.000</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>465.88</td>
<td>0.2440</td>
<td>0.989</td>
<td>1.001</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>225.68</td>
<td>0.8310</td>
<td>0.987</td>
<td>1.008</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>89.28</td>
<td>0.3508</td>
<td>0.986</td>
<td>1.001</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>19.98</td>
<td>-0.3558</td>
<td>0.985</td>
<td>0.998</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real power loss (MW)</td>
<td>1.355</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 3. Additional reactive power required to maintain voltage 1PU

Figure 4. Participation factor for least Eigen values \( \lambda = 19.98 \)
4. CONCLUSION

In this paper, the modal analysis for Eigen-values was used to determine the weakest bus or the optimal location for 6-bus Oman electrical system where the capacitor bank could be installed in order to maintain the magnitude of voltages. The weakest bus is considering as the higher contributing factor for system voltage instability and hence the voltage collapse. In addition, the condition of the system could be checked by using the Eigen-values for reduced Jacobian matrix $J_R$ with respect to voltage stability.

ACKNOWLEDGMENT

The author would like to thank financial support from the international Islamic University Malaysia under research grant IRF19-026-002: Highly Efficient Lithium-Ion Battery Recycling with Capacity-Sorted Optimization for Secondary Energy Storage System.

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