Finite frequency H_{∞} control for wind turbine systems in T-S form

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Article Info	ABSTRACT		
Article history:	In this work, we study H_{∞} control wind turbine fuzzy model for finite frequency		
Received Sep 8, 2019 Revised Dec 10, 2019	(FF) interval. Less conservative results are obtained by using Finsler's lemma tech- nique, generalized Kalman Yakubovich Popov (gKYP), linear matrix inequality (LMI) approach and added several separate parameters, these conditions are given in terms		
Accepted Apr 6, 2020	of LMI which can be efficiently solved numerically for the problem that such fuzzy systems are admissible with H_{∞} disturbance attenuation level. The FF H_{∞} perfor-		
Keywords:	mance approach allows the state feedback command in a specific interval, the simula-		
Finite frequency	tion example is given to validate our results.		
H_{∞} control			
LMI Technique	<i>This is an open access article under the <u>CC BY-SA</u> license.</i>		
TS model			
Wind turbine system	BY SA		
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1. INTRODUCTION

In recent years, Takagi-Sugeno (TS) fuzzy models [1] described by a set of IF-THEN rules could approximate any smooth nonlinear function to any specified accuracy within any compact set. In other words, it formulates the complex nonlinear systems into a framework that interpolates some affine local models by a set of fuzzy membership functions. Based on this framework, a systematic analysis and design procedure for complex nonlinear systems can be possibly developed in view of the powerful control theories and techniques in linear systems. Thus, it is expected that the TS fuzzy systems can be used to represent a large class of nonlinear systems and many important results on the TS fuzzy systems have been reported in the literature see [2-12].

Furthermore, the interest in the above mentioned literature is that all performances are given in the full frequency interval. However, when the external disturbance belong to a certain frequency range which is known beforehand, it is not favorable to control the system in the full frequency domain, because this may introduce some conservatism and poor system performance. Recently, the control synthesis in a FF interval has been addressed, and there have appeared many results in this domain of fuzzy systems [13-18].

In this work, we present a new method for finding solution to problem H_{∞} state feedback wind turbine fuzzy model finite frequency specifications of TS model. Less conservative results are obtained by using the gKYP technique, Finslers lemma a to introduce, several separate parameters, and LMI approach, the sufficient conditions are given in terms of LMI which can be efficiently solved numerically for the problem that such fuzzy systems are admissible with H_{∞} disturbance attenuation level in a specific interval. Numerical example is given to illustrate the effectiveness the presented results.

2. PRELIMINARIES AND PROBLEM STATEMENT

2.1. Notations and lemma

In this part, We tell you a few symbols and Finslers lemme which will be hired in this article. Superscript " *" means matrix transposition. Notation Q > 0 means that the matrix Q > 0 is positive definite, symbol I represents the identity matrix where suitable dimension. sym(N) denotes $N + N^*$, $diag\{..\}$ means for block diagonal matrix.

[19] Let $\psi \in \mathbb{R}^n$, $\mathcal{Z} \in \mathbb{R}^{n \times n}$, $\mathcal{M} \in \mathbb{R}^{m \times n}$ (rank $(\mathcal{M}) = k < n$), $\mathcal{M}^{\perp} \in \mathbb{R}^{n \times (n-k)}$ be a classification matrix satisfactorily complete column $\mathcal{M}\mathcal{M}^{\perp} = 0$ such that the following conditions :

- $\psi^* \mathcal{Z} \psi < 0$: $\mathcal{M} \psi = 0$, $\forall \psi \neq 0$
- $\mathcal{M}^{\perp *} \mathcal{Z} \mathcal{M}^{\perp} < 0$
- $\exists \beta \in \mathbb{R} : \mathcal{Z} \beta \mathcal{M}^* \mathcal{M} < 0$
- $\exists \mathcal{Y} \in \mathbb{R}^{n \times m} : \mathcal{Z} + \mathcal{YM} + \mathcal{M}^* \mathcal{Y}^* < 0$

2.2. Problem statement

Consider the following linear continuous fuzzy system : **Rules** *l***:** IF ξ_1 is \tilde{N}_1^j ,... ξ_n is \tilde{N}_l^j THEN

$$\dot{X}(p) = A_{l}x(p) + B_{l}u(p) + B_{1l}w(p)
Z(p) = C_{l}x(p) + D_{1l}w(p)$$
(1)

where $(\tilde{N}_1^j, ..., \tilde{N}_l^j)$: fuzzy sets; j: number for **IF-THEN** rules (j = 1, 2, ..., n); ξ_j : premise variables. $A_l, B_l, B_{1l}, C_l, D_l$: real parameters where suitable dimension; $x(t) \in \mathbb{R}^{n_x}/u(t) \in \mathbb{R}^{n_u}$: state/input vector; $y(t) \in \mathbb{R}^{n_y}$: control output vector; $w(t) \in \mathbb{R}^{n_w}$: unknown noise input $(\ell_2\{[0, \infty), [0, \infty)\})$.

The use of a central average defuzzification, a product deduction and a singleton fuzzifier, gives the global fuzzy refined system.

$$\dot{X}(p) = \sum_{l=1}^{n} \alpha_{l}(\mu) \{ A_{l}x(p) + B_{l}u(p) + B_{1l}w(p) \}$$

$$Z(p) = \sum_{l=1}^{n} \alpha_{l}(\mu) \{ C_{l}x(p) + D_{l}w(p) \}$$
(2)

where

$$\alpha_{l}(\mu(p)) = \frac{\theta_{l}(\mu(p))}{\sum_{j=1}^{n} \theta_{j}(\mu(p))}; \quad \theta_{j}(\mu(p)) = \prod_{j=1}^{n} \tilde{N}_{lj}(\mu(p)); \quad \mu(p) = [\mu_{1}(p), \mu_{2}(p), ..., \mu_{n}(p)]^{T}$$

 $\tilde{N}_{lj}(\mu_j(p))$ is the member of grade $\mu_j(p)$ for \tilde{N}_{lj} ; where it is proposed that

$$\sum_{l=1}^{n} \theta_l(\mu(p)) > 0; \quad \theta_l(\mu(p)) \ge 0; \quad l = 1, 2, ..., n$$
(3)

for all t. Then we can get the following conditions:

$$\sum_{j=1}^{n} \alpha_j(\mu(p)) > 0; \quad \alpha_j(\mu(p)) \ge 0; \quad l = 1, 2, ..., n$$
(4)

then we may have rewritten the fuzzy models chooses as :

$$\dot{X}(p) = A(\alpha)x(p) + B(\alpha)u(p) + B_1(\alpha)w(p)$$

$$Z(p) = C(\alpha)x(p) + D(\alpha)w(p)$$
(5)

where

$$A(\alpha) = \sum_{l=1}^{n} \alpha_{l}(\xi(p))A_{l}; \quad B(\alpha) = \sum_{l=1}^{n} \alpha_{l}(\xi(p))B_{l}; \quad B_{l}(\alpha) = \sum_{l=1}^{n} \alpha_{l}(\xi(p))B_{1l};$$

$$C(\alpha) = \sum_{l=1}^{n} \alpha_{l}(\xi(p))C_{l}; \quad D(\alpha) = \sum_{l=1}^{n} \alpha_{l}(\xi(p))D_{l}$$

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We propose the fuzzy logic controller chosen as:

$$u(p) = \sum_{j=1}^{n} \alpha_j(\xi(p)) K_j x(p) 6$$
(6)

where K_j are gain matrices with appropriate dimension.

By substituting (6) in (5) we obtain the following augmented model:

$$\dot{X}(p) = A_{cl}(\alpha)x(p) + B_1(\alpha)w(p)$$

$$Z(p) = C(\alpha)x(p) + D(\alpha)w(p)$$
(7)

where

$$A_{cl}(\alpha) = A(\alpha) + B(\alpha)K(\alpha).$$
(8)

Let $\gamma > 0$, augmented fuzzy systems in (7) is said may be in H_{∞} performance, the following index holds:

$$\int_0^\infty z^T(p)Z(p)dt \le \gamma^2 \int_0^\infty w^T(p)w(p)dt \tag{9}$$

From Parsevals theorems in [20, 21] we have the following index holds:

$$\int_{-\infty}^{+\infty} \tilde{Z}^T(\tau) \tilde{Z}(\tau) d\tau \le \gamma^2 \int_{-\infty}^{+\infty} \tilde{W}^T(\tau) \tilde{W}(\tau) d\omega$$
(10)

with $\tilde{W}(\tau)$, $\tilde{Z}(\tau)$ the Fourier transform of w(p) and Z(p).

The problem proposed in this work reads chosen as: The goal is to design a controller in (6) of model (5) such that :

- System (7) is asymptotically stable.
- FF index holds:

$$\int_{\tau \in \Delta} Z^T(\tau) Z(\tau) d\tau \le \gamma^2 \int_{\tau \in \Delta} W^T(\tau) W(\tau) d\tau$$
(11)

where \triangle is defined in Table 1;

Table 1. Different frequency ranges				
_	low-frequency	middle-frequency	high-frequency	
∇	$ au \leq ar{ au}_l$	$\bar{\tau}_1 \le \tau \le \bar{\tau}_2$	$ \tau \ge \bar{\tau}_h$	

with $\bar{\tau}_l$, $\bar{\tau}_1$, $\bar{\tau}_2$, $\bar{\tau}_h$ are known scalars. For $\triangle = (-\infty, +\infty)$, (11) is shortened to (10) (full frequency range (EFR)).

3. FINITE FREQUENCY H_{∞} CONTROLLER ANALYSIS

Let $\gamma > 0$. For the system (7) is asymptotically stable satisfied FF index in (11), if there exists Hermitian parameters $0 < Q = Q^T \in \mathbb{H}_n$, $P = P^T \in \mathbb{H}_n$ in such a way that

$$\begin{pmatrix} A_{cl}(\alpha) & B_1(\alpha) \\ I & 0 \end{pmatrix}^T \Xi \begin{pmatrix} A_{cl}(\alpha) & B_1(\alpha) \\ I & 0 \end{pmatrix} + \begin{pmatrix} C^T(\alpha)C(\alpha) & C^T(\alpha)D(\alpha) \\ D^T(\alpha)C(\alpha) & -\gamma^2 I + D^T(\alpha)D(\alpha) \end{pmatrix} < 0$$
(12)

• Low-frequency range (LFR) : $|\tau| \leq \bar{\tau}_l$

$$\Xi = \begin{pmatrix} -Q & P \\ P & \bar{\tau}_l^2 Q \end{pmatrix}$$
(13)

• Middle-frequency range (MFR) : $\bar{\tau}_1 \leq \tau \leq \bar{\tau}_2$, $\bar{\tau}_0 = \frac{\bar{\tau}_1 + \bar{\tau}_2}{2}$

$$\Xi = \begin{pmatrix} -Q & P + j\bar{\tau}_0 Q \\ P - j\bar{\tau}_0 Q & -\bar{\tau}_1 \bar{\tau}_2 Q \end{pmatrix}$$
(14)

• High-frequency range (HFR) : $|\tau| \ge \bar{\tau}_h$

$$\Xi = \begin{pmatrix} Q & P \\ P & -\bar{\tau}_h^2 Q \end{pmatrix}$$
(15)

If only if all the parameters of the theorem 3. are non-party of membership functions, then the systems are a linears, and theorem 3. is shrunken to lemme in [22] which has proven to be an efficient being to treat the FF method for linear time-invariant models. Let $\gamma > 0$, system (7) is asymptotically stable, if there exists parameters $0 < Q = Q^T \in \mathbb{H}_n$, $0 < W = W^T \in \mathbb{H}_n$, $P \in \mathbb{H}_n$, $G \in \mathbb{H}_n$ such that:

$$\Upsilon(\xi(p)) = \begin{pmatrix} -G - G^T & W + GA_c(\alpha) - G^T \\ * & sym[GA_c(\alpha) \end{pmatrix} < 0$$
(16)

$$\Psi(\xi(p)) = \begin{pmatrix} \Psi_{11}(\xi(p)) & \Psi_{12}(\xi(p)) & GB_1(\alpha) & 0 \\ * & \Psi_{22}(\xi(p)) & GB_1(\alpha) & C^T(\alpha) \\ * & * & -\gamma^2 I & D^T(\alpha) \\ * & * & * & -I \end{pmatrix} < 0$$
(17)

where

• LFR:
$$|\tau| \leq \bar{\tau}_l$$

 $\Psi_{11}(\xi(p)) = -Q - G - G^T; \quad \Psi_{12}(\xi(p)) = P + GA_c(\alpha) - G^T; \quad \Psi_{22}(\xi(p)) = \bar{\tau}_l^2 Q + sym[GA_c(\alpha)]$

- MFR: $\bar{\tau}_1 \leq \tau \leq \bar{\tau}_2$; $\bar{\tau}_0 = \frac{\bar{\tau}_1 + \bar{\tau}_2}{2}$ $\Psi_{11}(\xi(p)) = -Q - G - G^T$; $\Psi_{12}(\xi(p)) = P + j\bar{\tau}_0 Q + GA_c(\alpha) - G^T$; $\Psi_{22}(\xi(p)) = -\bar{\tau}_1 \bar{\tau}_2 Q + sym[GA_c(\alpha)]$
- HFR : $|\tau| \ge \bar{\tau}_h$ $\Psi_{11}(\xi(p)) = Q - G - G^T; \quad \Psi_{12}(\xi(p)) = P + GA_c(\alpha) - G^T; \quad \Psi_{22}(\xi(p)) = -\bar{\tau}_{\alpha}^2 Q + sym[GA_c(\alpha)]$

First, $\bar{A}(\mu(p))$ is stable, si $S = S^T > 0$ in such a way that

$$\begin{pmatrix} A_{cl}(\alpha) \\ I \end{pmatrix}^T \begin{pmatrix} 0 & S \\ S & 0 \end{pmatrix} \begin{pmatrix} A_{cl}(\alpha) \\ I \end{pmatrix} < 0$$
(18)

Let

$$\mathcal{Z} = \begin{pmatrix} 0 & S \\ S & 0 \end{pmatrix}; \ \mu = \begin{pmatrix} \dot{X}(p) \\ x(p) \end{pmatrix}; \ \mathcal{Y} = \begin{pmatrix} G \\ G \end{pmatrix}; \ \mathcal{M} = \begin{pmatrix} -I & A_{cl}(\alpha) \end{pmatrix}; \ \mathcal{M}^{\perp} = \begin{pmatrix} A_{cl}(\alpha) \\ I \end{pmatrix}$$
(19)

By applying the lemma 2.1. from (18) and (19), we obtain the inequality :

$$\begin{bmatrix} 0 & W \\ W & 0 \end{bmatrix} + \begin{bmatrix} G \\ G \end{bmatrix} \begin{bmatrix} -I & A_c(h) \end{bmatrix} + \begin{bmatrix} -I & A_c(h) \end{bmatrix}^T \begin{bmatrix} G \\ G \end{bmatrix}^T < 0$$
(20)

who is nothing (16).

Moreover, we consider the middle-frequency case. Applying lemma 3., the equation (12) are given by:

$$\mathcal{Z} = \begin{pmatrix}
-Q & P + j\bar{\tau}_0 Q & 0 \\
* & -\bar{\tau}_1\bar{\tau}_2 Q + C^T(\alpha)C(\alpha) & C^T(\alpha)D(\alpha) \\
* & * & -\gamma^2 I + D^T(\alpha)D(\alpha)
\end{pmatrix}; \quad \tau = \begin{pmatrix}
\dot{X}(p) \\
x(p) \\
w(p)
\end{pmatrix}; \quad \mathcal{Y} = \begin{pmatrix}
G \\
G \\
0
\end{pmatrix};$$

$$\mathcal{M} = \begin{pmatrix}
-I & A_{cl}(\alpha) & B_1(\alpha)
\end{pmatrix}.$$
(21)

By Schur complement, the following inequality

$$\mathcal{Z} + \mathcal{YM} + \mathcal{M}^T \mathcal{Y}^T < 0 \tag{22}$$

with

$$\mathcal{M}^{\perp} = \begin{pmatrix} A_{cl}(\alpha) & B_1(\alpha) \\ I & 0 \\ 0 & I \end{pmatrix}$$

Applying the terms (2) and some easy manipulation we obtain exactly the inequalities (12), (13) and (14).

4. FINITE FREQUENCY H_{∞} CONTROLLER DESIGN

Let $\gamma > 0$, system (7) is asymptotically stable, if there exists parameters $0 < Q = Q^T \in \mathbb{H}_n$, $0 < S = S^T \in \mathbb{H}_n$, $P \in \mathbb{H}_n$, Y(h), G such that the LMI (23) (24) feasible :

$$\bar{\Upsilon}(\alpha) = \begin{pmatrix} -\bar{G}^T - \bar{G} & \tilde{W} + A(\alpha)\bar{G}^T + B_1(\alpha)Y^T(\alpha) - \bar{G} \\ * & sym[A(\alpha)\bar{G}^T + B_1(\alpha)G^T] \end{pmatrix} < 0$$
(23)

$$\bar{\Psi}(\alpha) = \begin{pmatrix} \bar{\Psi}_{11}(\alpha) & \bar{\Psi}_{12}(\alpha) & B_1(\alpha) & 0\\ * & \bar{\Psi}_{22}(\alpha) & B_1(\alpha) & \bar{G}C^T(\alpha)\\ * & * & -\gamma^2 I & D^T(\alpha)\\ * & * & * & -I \end{pmatrix} < 0$$
(24)

- LFM : $|\tau| \leq \bar{\tau}_l$

$$\begin{split} \bar{\Psi}_{11}(\alpha) &= -\tilde{Q} - sym[\bar{G}]; \ \bar{\Psi}_{12}(\alpha) = \tilde{P} - \bar{G} + A(\alpha)\bar{G}^T + B_1(\alpha)Y^T(\alpha); \\ \bar{\Psi}_{22}(\alpha) &= \bar{\tau}_l^2\tilde{Q} + sym[A(\alpha)\bar{G}^T + B_1(\alpha)Y^T(\alpha)] \end{split}$$

- MFR :
$$\bar{\tau}_1 \leq \tau \leq \bar{\tau}_2$$
; $\bar{\tau}_0 = \frac{\bar{\tau}_1 + \bar{\tau}_2}{2}$
 $\bar{\Psi}_{11}(\alpha) = -\tilde{Q} - \bar{G}^T - \bar{G}; \quad \bar{\Psi}_{12}(\alpha) = \tilde{P} + j\bar{\tau}_0\tilde{Q} - \bar{G} + A(\alpha)\bar{G}^T + B_1(\alpha)Y^T(\alpha);$
 $\bar{\Psi}_{22}(\alpha) = -\bar{\tau}_1\bar{\tau}_2\tilde{Q} + sym[A(\alpha)\bar{G}^T + B_1(\alpha)Y^T(\alpha)]$

- HFR : $|\tau| \geq \bar{\tau}_h$

$$\bar{\Psi}_{11}(\alpha) = \tilde{Q} - \bar{G}^T - \bar{G}; \quad \bar{\Psi}_{12}(\alpha) = \tilde{P} - \bar{G} + A(\alpha)\bar{G}^T + B_1(\alpha)Y^T(\alpha); \\ \bar{\Psi}_{22}(\alpha) = -\bar{\tau}_{\alpha}^2\tilde{Q} + sym[A(\alpha)\bar{G}^T + B_1(\alpha)Y^T(\alpha)]$$

The matrices gains are obtained by

$$K(\alpha) = (\bar{G}^{-1}Y(\alpha))^T \tag{25}$$

Let $\overline{G} = G^{-1}$, $\widetilde{P} = G^{-1}PG^{-T}$, $Y(\alpha) = \overline{G}K(\alpha)^T$, $\widetilde{Q} = G^{-1}QG^{-T}$, $\widetilde{S} = G^{-1}SG^{-T}$. Pre/postmultiplying (16) by invertible parameters $\widehat{\Xi} = diag\{G^{-1}; G^{-1}\}$ and its transpose from the left and right

we get that (16) is equal to (23). Somewhere else, pre/post-multiplying (17) by invertible parameters $\Xi = diag\{G^{-1}, G^{-1}, I, I\}$ and its transpose from the left and right we get that (17) is equal to (24).

Then, theorem 4. is resolved the FF H_{∞} performance for fuzzy continuous systems. Let $\gamma > 0$, system (7) is asymptotically stable, if there exists parameters $0 < Q = Q^T \in \mathbb{H}_n$, $0 < W = W^T \in \mathbb{H}_n$, $P \in \mathbb{H}_n$, $G \in \mathbb{H}_n$ such that:

$$\tilde{\mathbf{f}}_{lj} = \begin{pmatrix} -\tilde{G}^T - \tilde{G} & \tilde{W} + A_l \tilde{G}^T + B_{1l} Y_j^T - \tilde{G} \\ * \quad sym[A_l \tilde{G}^T + B_{1l} G^T] \end{pmatrix} < 0$$
(26)

$$\tilde{\Psi}_{lj} = \begin{pmatrix} \tilde{\Psi}_{11lj} & \tilde{\Psi}_{12lj} & B_{1l} & 0\\ * & \tilde{\Psi}_{22lj} & B_{1l} & \tilde{G}C_l^T\\ * & * & -\gamma^2 I & D_l^T\\ * & * & * & -I \end{pmatrix} < 0$$
(27)

where

- LFR : $|\tau| \leq \tilde{\tau}_l$

$$\tilde{\Psi}_{11lj} = -\tilde{Q} - \tilde{G}^T - \tilde{G}; \quad \tilde{\Psi}_{12lj} = \tilde{P} - \tilde{G} + A_l \tilde{G}^T + B_{1l} Y_j^T; \quad \tilde{\Psi}_{22lj} = \tilde{\tau}_l^2 \tilde{Q} + sym[A_l \tilde{G}^T + B_{1l} Y_j^T]$$

- MFR : $\tilde{\tau}_1 \leq \tau \leq \tilde{\tau}_2$; $\tilde{\tau}_0 = \frac{\tilde{\tau}_1 + \tilde{\tau}_2}{2}$

$$\begin{split} \tilde{\Psi}_{11lj} &= -\tilde{Q} - \tilde{G}^T - \tilde{G}; \quad \tilde{\Psi}_{12lj} = \tilde{P} + j\tilde{\tau}_0 \tilde{Q} - \tilde{G} + A_l \tilde{G}^T + B_{1l} Y_j^T; \\ \tilde{\Psi}_{22lj} &= -\tilde{\tau}_1 \tilde{\tau}_2 \tilde{Q} + sym[A_l \tilde{G}^T + B_{1l} Y_i^T] \end{split}$$

- HFR : $|\tau| \geq \tilde{\tau}_h$

$$\tilde{\Psi}_{11lj} = \tilde{Q} - \tilde{G}^T - \tilde{G}; \quad \tilde{\Psi}_{12lj} = \tilde{P} - \tilde{G} + A_i \tilde{G}^T + B_{1l} Y_j^T; \quad \tilde{\Psi}_{22lj} = -\tilde{\tau}_h^2 \tilde{Q} + sym[A_l \tilde{G}^T + B_{1l} Y_j^T]$$

The matrices gains are obtained by

$$K_j = (\bar{G}^{-1}Y_j)^T, \quad 1 \le j \le n$$

$$(28)$$

The proposed formulas following are:

$$\sum_{i=1}^r \sum_{j=1}^r h_i h_j \tilde{\Upsilon}_{ij}, \ \sum_{i=1}^r \sum_{j=1}^r h_i h_j \bar{\Psi}_{ij}$$

so we gave theorem 4.. : We propose that the linear parameter equations (29) to non-real defined variables. by virtue of [23], the LMIs in non-real parameters can be transformd to an LMIs for greatmeasure in real parameters. While the equations $\Omega_1 + j\Omega_2 < 0$ is equivalent to $\begin{bmatrix} \Omega_1 & \Omega_2 \\ -\Omega_2 & \Omega_1 \end{bmatrix} < 0$, which involved the LMIs in (29) can be taken into account.

5. EXAMPLE

To demonstrate the effectiveness of FF proposed methods in this work. we provide a problem in the generator of the wind turbine. The variables in the wind turbine are assumed varying in the operating range: $\phi_1 \leq \phi \leq \phi_2$ and $\nabla_1 \leq \nabla \leq \nabla_2$, Consequently the nonlinear system (1) can be represented by the following four IF-THEN rules [24] with the numerical values given in Table 2 are proposed under a variable wind speed

	Table 2.	Numerical value	es of a three-b	plade wind turbine	
T		0		N7 · 1 1	

Parameters	Description	Numericalvalue
g_j	Inertia of the generator	$5.9 Kgm^2$
g_r	Inertia of rotor	$830000 Kgm^{2}$
ω	Air mass thickness	$1.225 Kg/m^3$
ω	Length of rotor blades	30m
t	Delay time	500m.s
k_g	the stiffness of the transmission	$1.556 \times 10^{6} N/m$
∇_s	sinking of transmission	$3029.5 Nm.s.rad^{-1}$
$ abla_g$	sinking of generator	$15.993 Nm.s.rad^{-1}$

Therefore, the wind turbine system is given by the following approximated fuzzy model T-S : **Rule 1:** IF ∇ is $\tilde{N}_1(p)$) and ϕ is $\tilde{M}_1(p)$) THEN

$$X(p) = A_1 x(p) + B_1 u(p) + B_{11} w(p)$$

$$Z(p) = C_1 x(p) + D_{11} w(p)$$
(29)

Rule 2: IF ∇ is $\tilde{N}_1(p)$) and ϕ is $\tilde{M}_2(p)$) THEN

$$\dot{X}(p) = A_2 x(p) + B_2 u(p) + B_{12} w(p)
Z(p) = C_2 x(p) + D_{12} w(p)$$
(30)

Rule 3: IF ∇ is $\tilde{N}_2(p)$) and ϕ is $\tilde{M}_1(p)$) THEN

$$\dot{X}(p) = A_3 x(p) + B_3 u(p) + B_{13} w(p)
Z(p) = C_3 x(p) + D_{13} w(p)$$
(31)

Rule 4: IF ∇ is $\tilde{N}_2(p)$) and ϕ is $\tilde{M}_1(p)$) THEN

$$X(p) = A_4 x(p) + B_4 u(p) + B_{14} w(p)$$

$$Z(p) = C_4 x(p) + D_{14} w(p)$$
(32)

with

$$A_{1} = A_{2} = \begin{pmatrix} 0 & 1 & -1 & 0 \\ \frac{-k_{g}}{g_{r}} & \frac{-b_{s}}{g_{r}} & \frac{b_{s}}{g_{r}} & \frac{-\nu_{b}\nabla_{1}}{g_{r}} \\ \frac{-k_{g}}{g_{j}} & \frac{-(b_{s}+b_{g})}{g_{j}} & \frac{b_{s}}{g_{j}} & 0 \\ 0 & 0 & 0 & \frac{-1}{t} \end{pmatrix}; A_{3} = A_{4} = \begin{pmatrix} 0 & 1 & -1 & 0 \\ \frac{-k_{g}}{g_{r}} & \frac{-b_{s}}{g_{r}} & \frac{b_{s}}{g_{r}} & \frac{-Y_{b}\nabla_{3}}{g_{r}} \\ \frac{-k_{g}}{g_{j}} & \frac{-(b_{s}+b_{g})}{g_{j}} & \frac{b_{s}}{g_{j}} & 0 \\ 0 & 0 & 0 & \frac{-1}{t} \end{pmatrix};;$$
$$B_{1} = B_{2} = B_{3} = B_{4} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & \frac{b_{g}}{g_{j}} \\ \frac{1}{t} & 0 \end{pmatrix}; B_{11} = B_{12} = \begin{pmatrix} 0 \\ \frac{Y_{b\phi_{1}}}{g_{r}} \\ 0 \\ 0 \end{pmatrix}; B_{13} = B_{14} = \begin{pmatrix} 0 \\ \frac{Y_{b\phi_{2}}}{g_{r}} \\ 0 \\ 0 \end{pmatrix};$$
$$C_{1} = C_{2} = C_{3} = C_{4} = \begin{pmatrix} 0 & 0 & 1 & 0 \end{pmatrix}; D_{1} = D_{2} = D_{3} = D_{4} = 0$$
(33)

Numerical value:

$$Y_{b\phi_1} = 106440; \quad Y_{b\phi_2} = 85370; \quad Y_{b\nabla_1} = 723980; \quad Y_{b\nabla_2} = 376070$$

When the membership parameters are given by:

$$\alpha_1 = \tilde{M}_1(\nabla)\tilde{N}_1(\phi); \quad \alpha_2 = \tilde{M}_1(\nabla)\tilde{N}_2(\phi); \quad \alpha_3 = \tilde{M}_2(\nabla)\tilde{N}_1(\phi); \quad \alpha_4 = \tilde{M}_2(\nabla)\tilde{N}_2(\phi)$$

with

$$\begin{split} \tilde{N}_1(\nabla) &= \frac{\nabla - \nabla_1}{\nabla_2 - \nabla_1}; \quad \tilde{M}_2(\nabla) = \frac{\nabla_2 - \nabla}{\nabla_2 - \nabla_1}; \\ \tilde{N}_1(\phi) &= \frac{\phi - \phi_1}{\phi_2 - \phi_1}; \quad \tilde{M}_2(\phi) = \frac{\phi_2 - \phi}{\phi_2 - \phi_1} \end{split}$$

To illustrate the advantage of our method, we show in Table 3 the state feedback H_{∞} performance, which shows the conservativeness of our method in this work.

∞ I \sim \sim \sim	1	· · · · · · · · · · · · · · · · · · ·
Frequency	Approaches	γ
EFR ($0 \le \tau \le +\infty$)	Th 2 in [11]	2.3214
LFR ($\tau \leq 2$)	Th 4 .	0.7815
MFR ($2 \le \tau \le 6$)	Th 4 .	1.1102
HFR ($\tau \ge 6$)	Th 4 .	0.2145

Table 3. H_{∞} performance levels γ obtained in different approaches

Resolution of Theorem 4. based the Toolbox LMI optimization algorithm [25], the gain state feedback controller matrices are obtained as follows:

• LFR :

$$K_{1} = 10^{3} \times \begin{bmatrix} 1.0382 & 3.0212 & 1.2487 & 1.1052 \\ -95.1382 & 1.4425 & -0.2487 & -0.4052 \end{bmatrix};$$

$$K_{2} = 10^{3} \times \begin{pmatrix} 1.0214 & 3.1485 & 1.2458 & 1.1125 \\ -95.1452 & 1.4512 & -0.2215 & -0.4725 \end{pmatrix};$$

$$K_{3} = 10^{3} \times \begin{pmatrix} 1.0175 & 3.1425 & 1.2714 & 1.1154 \\ -95.1214 & 1.4325 & -0.2514 & -0.3015 \end{pmatrix};$$

$$K_{4} = 10^{3} \times \begin{pmatrix} 10.0147 & 3.4515 & 1.2198 & 1.0714 \\ -94.5874 & 1.4425 & -0.2524 & -0.3817 \end{pmatrix}.$$
(34)

• MFR :

$$K_{1} = 10^{3} \times \begin{pmatrix} 0.9914 & 2.9541 & 1.1124 & 1.3245 \\ -95.2458 & 1.1214 & -0.2784 & -0.5111 \end{pmatrix};$$

$$K_{2} = 10^{3} \times \begin{pmatrix} 0.9847 & 2.9478 & 1.5478 & 1.0524 \\ -95.1825 & 1.2741 & -0.2325 & -0.5014 \end{pmatrix};$$

$$K_{3} = 10^{3} \times \begin{pmatrix} 0.9812 & 3.1478 & 1.3248 & 1.0741 \\ -94.8715 & 1.7185 & -0.7548 & -0.9548 \end{pmatrix};$$

$$K_{4} = 10^{3} \times \begin{pmatrix} 0.9578 & 3.2174 & 1.2945 & 1.3325 \\ -94.1748 & 2.0014 & -0.8471 & -0.3948 \end{pmatrix}.$$
(35)

• **HFR** :

$$K_{1} = 10^{3} \times \begin{pmatrix} 1.0102 & 2.9518 & 1.1502 & 1.3208 \\ -94.8417 & 1.2018 & 0.2525 & -0.2908 \end{pmatrix};$$

$$K_{2} = 10^{3} \times \begin{pmatrix} 1.0984 & 3.2546 & 1.0578 & 1.0174 \\ -96.0364 & 1.3206 & -0.1465 & -0.1108 \end{pmatrix};$$

$$K_{3} = 10^{3} \times \begin{pmatrix} 1.1187 & 3.0847 & 1.1974 & 1.2176 \\ -96.0147 & 1.6605 & -0.5847 & -0.5943 \end{pmatrix};$$

$$K_{4} = 10^{3} \times \begin{pmatrix} 1.0487 & 3.1425 & 1.2845 & 1.0987 \\ -95.1211 & 1.3387 & -0.2528 & -0.4125 \end{pmatrix}.$$
(36)

We suppose that $(2 \le \omega \le 6)$, let the disturbance be $w(p) = (2 + p^{1.3})^{-1}$, and the initial conditions $(x(0) = [-0.1 - 0.1 \ 0.1 \ 0.1]^T)$. The trajectories of Z(p), u(p), $x_1(p)$, $x_2(p)$, $x_3(p)$ and $x_4(p)$ are represented in Figures 1, 2 and 3. It is clear that indeed, the closed loop fuzzy model is converges towards zerois. Then, asymptotically stable.



Figure 3. Estimation output/input Z(p) and u(p).

6. CONCLUSION

In this work, an effective finite frequency approach fuzzy systems has been studied and applied for the state feedback problem in disturbed wind turbine. founded on gKYP lemma and lyapunov function for stability with the states feedback control, a sufficient stability conditions proposed to deal with problem of control in specific domain. Based on this, new conditions have been given to guarantee the standard H_{∞} performance has been revealed which has been illustrated by numerical examples.

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