Finite frequency $H_\infty$ control design for nonlinear systems

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ABSTRACT
The work deals finite frequency $H_\infty$ control design for continuous time nonlinear systems, we provide sufficient conditions, ensuring that the closed-loop model is stable. Simulations will be gifted to show level of attenuation that a $H_\infty$ lower can be by our method obtained developed where further comparison.

Keywords:
Finite frequency
LMIs
Nonlinear systems
T-S Model

Notations:
• $\ast$: Form symmetry
• $Q > 0$: Form positive
• $\text{sym}(M) > 0$: $M + M^*$
• $I$: form Identity
• $\text{diag}\{.\}$: Block diagonal form

1. INTRODUCTION
Fuzzy models [1] it generated widespread interest from engineers, mainly for renowned T-S systems my actually approach great category for non linear models. Then, the T-S systems is its universal approximation of a smooth non linear function by a family of IF and THEN non linear rules that represent the output/ input relationships of the models [2]-[11].

The interest of the literature mentioned above the $H_\infty$ control design in the FF range. whereas, in such cases, standard design methods of full frequency range can provide conservatism. Nevertheless, in an actual application, the design characteristics are generally given in selector Frequency domains (see, [12]-[21]).

In this work, we develop new our method concerning FF design of nonlinear continuous systems. Using the adequate conditions are developed, ensuring that the closed loop system is stable. Numerical examples are provides to prove the effectiveness of FF propose method.

2. T-S MODELS
Let’s the continuous model is given by

$$
\dot{x}(t) = \frac{\sum_{r=1}^{n} \sigma_r(t)(A_r x(t) + L_r u(t) + B_r v(t))}{\sum_{r=1}^{n} \sigma_r(t)},
g(t) = \frac{\sum_{r=1}^{n} \sigma_r(t)(C_r x(t) + E_r u(t) + D_r v(t))}{\sum_{r=1}^{n} \sigma_r(t)}
$$

(1)

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with
\[
\lambda_r(t) = \prod_{j=1}^{p} N_{rs}(\mu_s(t))
\]
and
\[
\sigma_r = \frac{\lambda_r(t)}{\sum_{r=1}^{n} \lambda_r(t)}; \quad 0 \leq \lambda_r \leq 1 \quad \text{and} \quad \sum_{r=1}^{n} \lambda_r = 1
\]
and \(\sigma = [\sigma_1, ..., \sigma_r]^*\), the T-S system can be rewritten as follows:
\[
\dot{x}(t) = A(\sigma)x(t) + L(\sigma)u(t) + B(\sigma)v(t) \\
y(t) = C(\sigma)x(t) + E(\sigma)u(t) + D(\sigma)v(t)
\]
where
\[
\{A(\sigma); B(\sigma); L(\sigma); B(\sigma); C(\sigma); E(\sigma); D(\sigma)\} = \sum_{r=1}^{n} \rho_r(t)\{A_r; L_r; B_r; C_r; E_r; D_r\}
\]

3. **PDC CONTROLLER SCHEME**

The fuzzy control as follows:
\[
u(t) = \sum_{s=1}^{n} \sigma_s K_s x(t)
\]
then, we have the closed loop model:
\[
\dot{x}(t) = A_c(\lambda)x(t) + B(\lambda)v(t) \\
y(t) = C_c(\lambda)x(t) + D(\lambda)v(t)
\]
with
\[
A_c(\sigma) = A_c(\sigma) + L(\sigma)K(\sigma); \quad c(\sigma) = C_c(\sigma) + E(\sigma)K(\sigma)
\]

4. **MAIN RESULTS**

4.1. **Useful lemma**

**Lemma 4.1** Tuan, H. D. *et al.*\[22\] If the following conditions are met:
\[
\Omega_{rr} < 0 \quad 1 \leq r \leq n \quad \frac{1}{n-1} \Omega_{rr} + \frac{1}{2} [\Omega_{rs} + \Omega_{sr}] < 0; \quad 1 \leq r \neq s \leq n
\]
and
\[
\sum_{r=1}^{n} \sum_{s=1}^{n} \lambda_r \lambda_s \Omega_{rs} < 0
\]

**Lemma 4.2** El-Amrani, A. *et al.*\[23\]. Let \(T \in \mathbb{R}^{n \times n}\) and \(M \in \mathbb{R}^{m \times n}\), so that the following conditions are equivalent:
1. $\mathcal{M}^\top T \mathcal{M}^\perp < 0$

2. $\exists \mathcal{N} \in \mathbb{R}^{n \times m} : T + \text{sym}[\mathcal{M}\mathcal{N}] < 0$

Lemma 4.3 Closed loop (6) is stable, if $R(\sigma) = R(\sigma)^* \in \mathbb{H}_n$, $0 < S = S^* \in \mathbb{H}_n$ such that

$$\left( \begin{array}{cc} A_c(\sigma) & B(\sigma) \\ I & 0 \end{array} \right)^* \Pi \left( \begin{array}{cc} A_c(\sigma) & B(\sigma) \\ I & 0 \end{array} \right) + \left( \begin{array}{cc} C^T(\sigma)C_c(\sigma) & 0 \\ D^T(\sigma)C_c(\sigma) & D^T(\sigma)D(\sigma) - \gamma^2 I \end{array} \right) < 0 \tag{11}$$

with $\Pi$ is given of Table 1.

4.2. Finite frequency analysis

Theorem 4.4 The fuzzy model (6) is stable, if $R(\sigma) \in \mathbb{H}_n$, $0 < S \in \mathbb{H}_n$, $0 < W(\sigma) \in \mathbb{H}_n$, $Z(\sigma) \in \mathbb{H}_n$. $H(\sigma) \in \mathbb{H}_n$ such that

$$\left[ \begin{array}{cc} -\text{sym}[Z(\sigma)] & W(\sigma) + Z(\sigma)A(\sigma) - H^*(\sigma) \\ * & \text{sym}[H(\sigma)A_c(\sigma)] \end{array} \right] < 0 \tag{12}$$

$$\left[ \begin{array}{ccc} \Psi_{11}(\sigma) & \Psi_{12}(\sigma) + Z(\sigma)A_c(\sigma) - H^*(\sigma) & Z(\sigma)B(\sigma) \\ * & \Psi_{22}(\sigma) + \text{sym}[H(\sigma)A_c(\sigma)] & H(\sigma)B(\sigma) \\ * & * & -\gamma^2 I \end{array} \right] < 0 \tag{13}$$

- Low frequency (LF) range:

$$\Psi_{11}(\sigma) = -S(\sigma) - Z(\sigma) - Z^*(\sigma); \quad \Psi_{12}(\sigma) = R(\sigma); \quad \Psi_{22}(\sigma) = \bar{\mu}_1^2 S(\sigma)$$

- Middle frequency range (MF) range:

$$\Psi_{11} = -S(\sigma) - Z(\sigma) - Z^*(\sigma); \quad \Psi_{12} = R(\sigma) + j\bar{\mu}_0 S(\sigma); \quad \Psi_{22} = -\bar{\mu}_1 \bar{\mu}_2 S(\sigma)$$

- High frequency (HF) range:

$$\Psi_{11}(\sigma) = S(\sigma) - Z(\sigma) - Z^*(\sigma); \quad \Psi_{12}(\sigma) = R(\sigma); \quad \Psi_{22}(\sigma) = -\bar{\mu}_2^2 S(\sigma)$$

Proof 4.5 Let $\bar{A}(\sigma), \bar{W}(\sigma) = W(\sigma)^* > 0$ such that

$$\left[ \begin{array}{c} A_c(\sigma) \\ I \end{array} \right]^* \left[ \begin{array}{cc} 0 & W(\sigma) \\ \bar{W}(\sigma) & 0 \end{array} \right] \left[ \begin{array}{c} A_c(\sigma) \\ I \end{array} \right] < 0 \tag{16}$$

define:

$$\mathcal{T} = \left[ \begin{array}{c} 0 \\ W(\sigma) \end{array} \right]; \quad \mathcal{N} = \left[ \begin{array}{c} Z(\sigma) \\ H(\sigma) \end{array} \right]; \quad \mathcal{M} = [-I, A_c(\sigma)]; \quad \mathcal{M}^\perp = \left[ \begin{array}{c} A_c(\sigma) \\ I \end{array} \right]$$

(17)

let lemma 4.1., (16) and (17) are equivalent to:

$$\left[ \begin{array}{cc} 0 & W(\sigma) \\ \bar{W}(\sigma) & 0 \end{array} \right] + \left[ \begin{array}{c} Z(\sigma) \\ H(\sigma) \end{array} \right] \left[ \begin{array}{c} -I \\ A_c(\sigma) \end{array} \right] + \left[ \begin{array}{c} -I \\ A_c(\sigma) \end{array} \right]^* \left[ \begin{array}{c} Z(\sigma) \\ H(\sigma) \end{array} \right]^* < 0$$

$$\left( \begin{array}{cc} -S & \bar{\mu}_0^2 S + C_c^*(\sigma)C_c(\sigma) \\ * & -\gamma^2 I + D^*(\sigma)D(\sigma) \end{array} \right) \tag{18}$$

which is nothing but (12), let LF case:

$$\mathcal{T} = \left[ \begin{array}{cc} -S & R(\sigma) \\ * & \bar{\mu}_0^2 S + C_c^*(\sigma)C_c(\sigma) \\ * & -\gamma^2 I + D^*(\sigma)D(\sigma) \end{array} \right]; \quad \mathcal{M}^\perp = \left[ \begin{array}{cc} A_c(\sigma) & B(\sigma) \\ I & 0 \end{array} \right]; \quad \mathcal{M} = [-I, A_c(\sigma), B(\sigma)]; \quad \mathcal{N} = \left[ \begin{array}{c} Z(\sigma)^T \\ H(\sigma)^T \end{array} \right]^T \tag{19}$$

we have

$$\mathcal{T} + \text{sym}[\mathcal{N}\mathcal{M}] < 0 \tag{20}$$

using Lemma 4.1., we obtain (11).
4.3. Finite frequency design

Theorem 4.6 The fuzzy model (6) is stable, if $\bar{R}(\sigma) \in \mathbb{H}_n$, $0 < \bar{S} \in \mathbb{H}_n$, $0 < \bar{W}(\sigma) \in \mathbb{H}_n$, $G(\sigma)$, $\bar{Z}(\sigma)$ such that:

$$
\begin{bmatrix}
-Z^*(\sigma) - \bar{Z}(\sigma) & \bar{W}(\sigma) + A(\sigma)\bar{Z}^*(\sigma) + B(\sigma)G^*(\sigma) - \beta\bar{Z}(\sigma) \\
\ast & \text{sym} [\beta A(\sigma)\bar{Z}^*(\sigma) + B(\sigma)G^*(\sigma)]
\end{bmatrix} < 0
$$

(21)

$$
\begin{bmatrix}
\Psi_{11}(\sigma) & \Psi_{12}(\sigma) & B(\sigma) & 0 \\
\ast & \Psi_{22}(\sigma) & \beta B(\sigma) & \bar{Z}(\sigma)C^*(\sigma) + G(\sigma)E^*(\sigma) \\
\ast & \ast & -\gamma^2 I & D^*(\sigma) \\
\ast & \ast & \ast & -I
\end{bmatrix}
< 0
$$

(22)

$\Psi_{11}(\sigma) = -\bar{S}(\sigma) - Z^*(\sigma) - \bar{Z}(\sigma)$; 
$\Psi_{22}(\sigma) = \mu_1^2\bar{S}(\sigma) + \text{sym} [\beta A(\sigma)\bar{Z}^*(\sigma) + B(\sigma)G^*(\sigma)]$; 
$\Psi_{12}(\sigma) = \bar{R}(\sigma) - \beta\bar{Z}(\sigma) + A(\sigma)\bar{Z}^*(\sigma) + B(\sigma)G^*(\sigma)$.

$\Psi_{11}(\sigma) = -\bar{S}(\sigma) - Z^*(\sigma) - \bar{Z}(\sigma)$; 
$\Psi_{22}(\sigma) = -\mu_1\mu_2\bar{S}(\sigma) + \text{sym} [\beta A(\sigma)\bar{Z}^*(\sigma) + B(\sigma)G^*(\sigma)]$; 
$\Psi_{12}(\sigma) = \bar{R}(\sigma) + j\mu_0\bar{S}(\sigma) - \beta\bar{U}(\sigma) + A(\sigma)\bar{Z}^*(\sigma) + B(\sigma)G^*(\sigma)$

$\Psi_{11}(\sigma) = \bar{S}(\sigma) - Z^*(\sigma) - \bar{Z}(\sigma)$; 
$\Psi_{22}(\sigma) = -\mu_1^2\bar{S}(\sigma) + \text{sym} [\beta A(\sigma)\bar{Z}^*(\sigma) + B(\sigma)G^*(\sigma)]$.

Therefore:

$$K(\sigma) = (Z^{-1}(\sigma)G(\sigma))^*$$

(23)

Proof 4.7 First, for matrix variable $H(\sigma)$ in theorem 4.2., let $H(\sigma) = \beta Z(\sigma)$, after that by replacing (7) into (12) and (13), respectively, moreover, let,

$$\bar{Z}(\sigma) = Z^{-1}(\sigma); \ G(\sigma) = \bar{Z}(\sigma)K^*; \ \bar{S}(\sigma) = Z^{-1}(\sigma)S(\sigma)Z^{-*}(\sigma);$$

$$\bar{R}(\sigma) = Z^{-1}(\sigma)R(\sigma)Z^{-*}(\sigma); \ \bar{W}(\sigma) = Z^{-1}(\sigma)W(\sigma)Z^{-*}(\sigma)$$

Multiplying (12) by $\text{diag}[Z^{-1}(\sigma), \ Z^{-1}(\sigma)]$, and (13) by $\text{diag}[Z^{-1}(\sigma), \ Z^{-1}(\sigma), \ I, \ I]$, we have (12) and (13) are equivalent (21) and (22), respectively. Theorem 4.8 The fuzzy model (6) is stable. If $R_r \in \mathbb{H}_n, 0 < S \in \mathbb{H}_n, 0 < W_r \in \mathbb{H}_n, Z_s, G_s$ such that

$$\bar{\Psi}_{rr} < 0; \ \bar{\Upsilon}_{rr} < 0; \quad 1 \leq r \leq n$$

(24)

$$\frac{1}{r-1} \bar{\Psi}_{rr} + \frac{1}{2} |\bar{\Psi}_{rs} + \bar{\Psi}_{sr}| < 0; \quad 1 \leq r \neq s \leq n$$

(25)

$$\frac{1}{r-1} \bar{\Upsilon}_{rr} + \frac{1}{2} |\bar{\Upsilon}_{rs} + \bar{\Upsilon}_{sr}| < 0; \quad 1 \leq r \neq s \leq n$$

(26)

where

$$
\tilde{\Psi}_{rs} = \begin{bmatrix}
\tilde{\Psi}_{11rs} & \tilde{\Psi}_{12rs} & B_r & 0 \\
* & \tilde{\Psi}_{22rs} & \beta B_t & \bar{F}_sC_r^* + G_sE_r^* \\
* & * & -\gamma^2 I & D_r^* \\
* & * & * & -I
\end{bmatrix};
$$
\[ \Psi_{rs} = \begin{bmatrix} -\hat{Z}_s^* - \hat{Z}_s & W_r + A_r \hat{Z}_s^* + B_r \hat{G}_s^* - \beta \hat{Z}_s \\ \text{sym} & \beta(A_r \hat{Z}_s^* + B_r \hat{G}_s^*) \end{bmatrix} \]

- \( |\mu| \leq \bar{\mu} \)
  \[ \tilde{\Psi}_{11rs} = -\hat{S}_s - \hat{Z}_s^* - \hat{Z}_s; \]
  \[ \tilde{\Psi}_{12rs} = \tilde{R}_r - \beta \hat{Z}_s + A_r \hat{Z}_s^* + B_r \hat{G}_s^*; \]
  \[ \tilde{\Psi}_{22rs} = \bar{\mu}_1^2 \hat{S}_s + \text{sym}[\beta(A_r \hat{Z}_s^* + B_r \hat{G}_s^*)]. \]

- \( \bar{\mu}_1 \leq \mu \leq \bar{\mu}_2 \)
  \[ \tilde{\Psi}_{11rs} = -\bar{S}_s - \hat{Z}_s^* - \hat{Z}_s; \]
  \[ \tilde{\Psi}_{12rs} = \bar{R}_r + j \bar{\mu}_0 \bar{S}_s - \beta \bar{U}_s + A_r \hat{Z}_s^* + B_r \hat{G}_s^*; \]
  \[ \tilde{\Psi}_{22rs} = -\bar{\mu}_1 \bar{\mu}_2 \hat{S}_s + \text{sym}[\beta(A_r \hat{Z}_s^* + B_r \hat{G}_s^*)]. \]

- \( |\mu| \geq \bar{\mu}_h \)
  \[ \bar{\Psi}_{11rs} = \bar{S}_s - \bar{Z}_s^* - \bar{Z}_s; \]
  \[ \bar{\Psi}_{12rs} = \bar{R}_r - \beta \bar{Z}_s + A_r \hat{Z}_s^* + B_r \hat{G}_s^*; \]
  \[ \bar{\Psi}_{22rs} = -\bar{\mu}_h^2 \hat{S}_s + \text{sym}[\beta(A_r \hat{Z}_s^* + B_r \hat{G}_s^*)]. \]

The matrices gains are obtained by:
\[ K_s = (\hat{Z}_s^{-1}G_s)^*, \quad 1 \leq s \leq n \]  \hspace{1cm} (27)

**Proof 4.9** by applying the Lemma 4.1., we have Theorem 4.3.

5. **SIMULATIONS**

5.1. **Example 1**

Consider fuzzy system (3) with two rules [24]:
\[
A_1 = \begin{bmatrix} 0 & 17.2941 \\ 1 & 0 \end{bmatrix}; \quad A_2 = \begin{bmatrix} 0 & 1 \\ 12.6305 & 0 \end{bmatrix};
\]
\[
E_1 = \begin{bmatrix} 0 \\ -0.1765 \end{bmatrix}; \quad L_1 = L_2 = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix};
\]
\[
E_2 = \begin{bmatrix} 0 \\ -0.0779 \end{bmatrix}; \quad B_1 = B_2 = 0.1;
\]
\[
C_1 = C_2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}; \quad D_1 = D_2 = 0 \]  \hspace{1cm} (28)

and
\[
N_2(x_1) = \frac{1}{1 + \exp(-7(x_1 - \frac{\pi}{4}))} \frac{1}{1 + \exp(-7(x_1 + \frac{\pi}{4}))};
\]
\[ N_1(x_1) = 1 - N_2(x_1). \]  \hspace{1cm} (29)

let
\[
v(t) = \begin{cases} 2 & 2 \leq t \leq 3 \\ 2 & 5 \leq t \leq 6 \\ 0 & \text{others} \end{cases} \]  \hspace{1cm} (30)

We propose in table 2 shows the values of \( \gamma \) obtained in different frequency ranges. By Theorem 4.3., the controller gains are given by:

- **Low frequency (LF) range** (with \( \beta_1 = 0.0502 \) and \( \gamma = 0.2507 \)):
  \[ K_1 = \begin{bmatrix} 168.2205 & 22.9439 \end{bmatrix}; \quad K_2 = \begin{bmatrix} 353.5002 & 115.2592 \end{bmatrix} \]  \hspace{1cm} (31)
• Middle frequency (MF) range (with $\beta_1 = 0.5025$ and $\gamma = 0.8355$):

$$K_1 = \left[ \begin{array}{c}
186.8988 \\
36.1978 \\
\end{array} \right] ; \hspace{0.5cm} K_2 = \left[ \begin{array}{c}
378.9459 \\
109.7698 \\
\end{array} \right]$$ (32)

• High frequency (HF) range (with $\beta_1 = 0.2478$ and $\gamma = 0.5702$):

$$K_1 = \left[ \begin{array}{c}
203.4092 \\
43.1392 \\
\end{array} \right] ; \hspace{0.5cm} K_2 = \left[ \begin{array}{c}
356.0428 \\
101.8833 \\
\end{array} \right]$$ (33)

Table 2. Obtained $\gamma$ by different domains

<table>
<thead>
<tr>
<th>Frequency ranges</th>
<th>methods</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>EF ($0 \leq \mu \leq \infty$)</td>
<td>Theorem 4.3. ($\hat{S}_k = 0$)</td>
<td>1.1789</td>
</tr>
<tr>
<td>EF ($0 \leq \mu \leq \infty$)</td>
<td>Theorem 2 in [16]</td>
<td>1.3598</td>
</tr>
<tr>
<td>LF ($</td>
<td>\mu</td>
<td>\leq 0.7$)</td>
</tr>
<tr>
<td>LF ($</td>
<td>\mu</td>
<td>\leq 0.7$)</td>
</tr>
<tr>
<td>MF ($1 \leq \mu \leq 5$)</td>
<td>Theorem 4.3.</td>
<td>0.2102</td>
</tr>
<tr>
<td>HF ($</td>
<td>\mu</td>
<td>\geq 6$)</td>
</tr>
<tr>
<td>HF ($</td>
<td>\mu</td>
<td>\geq 6$)</td>
</tr>
<tr>
<td>MF ($628 \leq \mu \leq 6283$)</td>
<td>Theorem 4.3.</td>
<td>0.8355</td>
</tr>
<tr>
<td>MF ($628 \leq \mu \leq 6283$)</td>
<td>Corollary 1 in [16]</td>
<td>1.5786</td>
</tr>
<tr>
<td>HF ($</td>
<td>\mu</td>
<td>\geq 6283$)</td>
</tr>
<tr>
<td>HF ($</td>
<td>\mu</td>
<td>\geq 6283$)</td>
</tr>
</tbody>
</table>

Figure 1. Trajectories of $x_i(t), i = 1, 2, u(t)$ and $y(t)$ for LF $|\mu| \leq 0.7$ range, (a) state $x_1(t)$, (b) state $x_2(t)$, (c) estimation controlled output $y(t)$, (d) estimation controlled output $u(t)$
5.2. Example 2

Let the fuzzy system (3) [25], where:

\[ A_1 = \begin{bmatrix} -1 & -1.155 \\ 1 & 0 \end{bmatrix}; A_2 = \begin{bmatrix} -1 & -1.155 \\ 1 & 0 \end{bmatrix}; \]

\[ L_1 = \begin{bmatrix} 1.4387 \\ 0 \end{bmatrix}; L_2 = \begin{bmatrix} 0.5613 \\ 0 \end{bmatrix}; B_1 = B_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}; \]

\[ C_1 = C_2 = \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}; E_1 = E_2 = \begin{bmatrix} 0 \\ 2 \end{bmatrix}; D_1 = D_2 = \begin{bmatrix} 0 \\ 2 \end{bmatrix} \]

(34)

and

\[ N_2(x_1(t)) = 0.5 - \frac{x_1^3(t)}{6.75}; \]

\[ N_1(x_1(t)) = 1 - N_2(x_1(t)); \quad x_1(t) \in (1.5, 1.5) \]  

(35)

Figure 2. Trajectories of \( x_i(t), i = 1, 2, u(t) \) and \( y(t) \) for MF \( 1 \leq |\mu| \leq 5 \) range, (a) state \( x_1(t) \), (b) state \( x_2(t) \), (c) estimation controlled output \( y(t) \), (d) estimation controlled output \( u(t) \)
Table 3. $H_\infty$ performance bounds $\gamma$ by different domains

<table>
<thead>
<tr>
<th>Frequency ranges</th>
<th>Methods</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 \leq \mu \leq \infty$</td>
<td>[16]</td>
<td>Infeasible</td>
</tr>
<tr>
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<td>Theorem 4.3. ($\tilde{S}_\mu = 0$)</td>
<td>1.4517</td>
</tr>
<tr>
<td>$</td>
<td>\mu</td>
<td>\leq 628$</td>
</tr>
<tr>
<td>$</td>
<td>\mu</td>
<td>\leq 628$</td>
</tr>
<tr>
<td>$628 \leq \mu \leq 628$</td>
<td>[16]</td>
<td>1.025</td>
</tr>
<tr>
<td>$628 \leq \mu \leq 628$</td>
<td>Theorem 4.3.</td>
<td>0.5274</td>
</tr>
<tr>
<td>$</td>
<td>\mu</td>
<td>\geq 628$</td>
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<tr>
<td>$</td>
<td>\mu</td>
<td>\geq 628$</td>
</tr>
</tbody>
</table>

The FF case of $u(t)$ is assumed to satisfy 100 Hz; [100 1000] Hz and 1000 Hz, i.e., $|\mu| \leq 628$ rad/s; $628 \leq \mu \leq 628$ rad/s; and $|\mu| \geq 628$ rad/s, respectively for $u(t)$ and $\beta = 1$.

6. CONCLUSION

We sent the FF state feedback design. To reduce the closed-loop system and establish less conservative results, we have considered two practical examples has been provides to show the feasibility of tuning FF $H_\infty$ fuzzy control design method.

REFERENCES


