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Improved filtering H_∞ finite frequency of Takagi-Sugeno fuzzy systems

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ABSTRACT

The work treats the filter H_∞ finite frequency (FF) in Takagi-Sugeno (T-S) two-dimensional (2-D) systems described by Fornasini-Marchesini local state-space (FM LSS) models. The goal of this work is to find an FF H_∞ T-S fuzzy filter model design in such a way that the error system is stable and has a reduced FF H_∞ performance over FF areas with noise is established as a prerequisite. Via the use of the generalized Kalman Yakubovich Popov (gKYP) lemma, Lyapunov functions approach, Finsler's lemma, and parameterize slack matrices, new design conditions guaranteeing the FF H_∞ T-S fuzzy filter method of FM LSS models are developed by solving linear matrix inequalities (LMIs). At last, the simulation results are provided to show the effectiveness and the validity of the proposed FF T-S fuzzy of FM LSS models strategy by a practical application has been made.

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NOMENCLATURE

- $"T"$: Matrix transposition
- $" * "$: Matrix symmetry
- $M > 0$: matrix M is positive
- $sym(M)$: $M + M^T$
- $diag\{..\}$: Block diagonal matrix

1. INTRODUCTION

During the latter decennary, various searchers studied two-dimensional (2-D) systems inclusive discrete and continuous adjustments have a lot of practices in engineering as though process control, digital filter, and image processing [1]-[4]. Many important results based on LMI approach have already been reported. Among these results, such as [5]-[16]. The main existing sources on filtering problems and disturbances are based on the whole full frequency (EF) area, which will give several types of filtering design [15], [16]. However, most practical industrial applications work in a FF domain. So far, a few applications have been made [17]-[24]. Thus, for this we will present new approaches to solve these problems. The primary goal of our work is to define a fuzzy filter of discrete Fornasini-Marchesini models over FF ranges such a way that the error model is stable and have a reduced H_∞ FF index of a noise is established as a prerequisite. We have also presented an example of simulation in order to exemplify the efficiency of the suggested method.

2. PROBLEM STATEMENT

2.1. System formulation

Envisage the nonlinear FMLSS presented by:
Rule 1: if $\zeta_1(s)\ is\ \tilde{M}_1^l$, ... and $\zeta_\theta(s)\ is\ \tilde{M}_\theta^l$, then,

$$\begin{aligned} x_{n+1,k+1} &= A_{1l}x_{n,k+1} + A_{2l}x_{n+1,k} + B_{1l}u_{n,k+1} + B_{2l}u_{n+1,k} \\ y_{n,k} &= C_l x_{n,k} + D_l u_{n,k} \\ z_{n,k} &= E_l x_{n,k} \end{aligned} \tag{1}$$

where $(\tilde{M}_1^l, ..., \tilde{M}_\zeta^l)$ are fuzzy sets; $x \in \mathbb{R}^n$ is state vector; $y \in \mathbb{R}^{n_y}$ is measured output; $z \in \mathbb{R}_z^n$ is a signal to estimated; $u \in \mathbb{R}^p$ is supposed to appertain to a renowned rectangular domain Ω, where is the recognized noise signal and located in the following sets of frequencies:

$$\begin{aligned} \Omega \ \triangleq\ & \{(\mu_1, \mu_2) \in \mathbb{R} | \mu_1^a \le \mu_1 \le \mu_1^b;\ \mu_2^a \le \mu_2 \le \mu_2^b; \\ & \mu_1^a, \mu_2^a, \mu_1^b, \mu_2^b \in [-\pi, \pi]\} \end{aligned} \tag{2}$$

Where μ_1^a, μ_1^b, μ_2^a and μ_2^b are known scalars. We describe the nonlinear system (1) employ singleton fuzzifer, center-average and inference product by the following relation:

$$\begin{aligned} x_{n+1,k+1} &= A_1(q)x_{n,k+1} + A_2(q)x_{n+1,k} + B_1(q)u_{n,k+1} + B_2(q)u_{n+1,k} \\ y_{n,k} &= C(q)x_{n,k} + D(q)u_{n,k} \\ z_{n,k} &= E(q)x_{n,k} \end{aligned} \tag{3}$$

where

$$\begin{bmatrix} A_1(q) & B_1(q) \\ A_2(q) & B_2(q) \\ C(q) & D(q) \\ E(q) & 0 \end{bmatrix} = \sum_{l=1}^{r} q_l(\zeta(s)) \begin{bmatrix} A_{1l} & B_{1l} \\ A_{2l} & B_{2l} \\ C_l & D_l \\ E_l & 0 \end{bmatrix} \tag{4}$$

In this work, a fuzzy filter is designed ie being as:
Rule l: if $\zeta_1(s)\ is\ \tilde{M}_1^l$, ... and $\zeta_\theta(k)\ is\ \tilde{M}_\theta^l$, then,

$$\begin{aligned} \hat{x}_{n+1,k+1} &= \hat{A}_{1l}\hat{x}_{i,k+1} + \hat{A}_{2l}\hat{x}_{i+1,k} + \hat{B}_{1l}y_{n,k} + \hat{B}_{2l}y_{n,k} \\ \hat{z}_{n,k} &= \hat{C}_l\hat{x}_{n,k} \end{aligned} \tag{5}$$

where $\hat{x}_{n,k}$ is state filter vector; $\hat{z}_{n,k}$ estimation of $z_{n,k}$; $\breve{A}_{1l}$, $\breve{A}_{2l}$, $\breve{B}_{1l}$, $\breve{B}_{2l}$, $\breve{C}_l$ are parameters should be defined. We get defuzzified for system (5) is being as:

$$\begin{aligned} \hat{x}_{n+1,k+1} &= \hat{A}_1(q)\hat{x}_{n,k+1} + \hat{A}_2(q)x_{n+1,k} + \hat{B}_1(q)y_{n,k} + \hat{B}_2(q)y_{n,k} \\ \hat{z}_{n,k} &= \hat{C}(q)\hat{x}_{n,k} \end{aligned} \tag{6}$$

with

$$\begin{bmatrix} \hat{A}_1(q) & \hat{B}_1(q) \\ \hat{A}_2(q) & \hat{B}_2(q) \\ \hat{C}(q) & \end{bmatrix} = \sum_{l=1}^{r} h_l(\theta(k)) \begin{bmatrix} \hat{A}_{1l} & \hat{B}_{1l} \\ \hat{A}_{2l} & \hat{B}_{2l} \\ \hat{C}_l & \end{bmatrix} \tag{7}$$

Let $\xi_{n,k} := [x_{n,q}^T\ \ \breve{x}_{n,q}^T]^T$, $\mathbf{e}_{n,k} = \mathbf{y}_{n,k} - \breve{y}_{n,k}$, Then, the error model as shown in:

$$\begin{aligned} \xi_{n+1,k+1} &= \bar{A}_1(q)\xi_{n,k+1} + \bar{B}_1(q)u_{n,k+1} + \bar{A}_2(q)\xi_{n+1,k} + \bar{B}_2(q)u_{n+1,k} \\ e_{n,k} &= \bar{C}(q)\xi_{n,k} \end{aligned} \tag{8}$$

with

$$\bar{A}_1(q) = \begin{pmatrix} A_1(q) & 0\hat{B}_1(q)C(q) \\ \hat{A}_1(q) & \end{pmatrix}; \bar{B}_1(q) = \begin{pmatrix} B_1(q) \\ \hat{B}_1(q)D(q) \end{pmatrix};\ \bar{B}_2(q) = \begin{pmatrix} B_2(q) \\ \hat{B}_2(q)D(q) \end{pmatrix};$$
$$\bar{\mathcal{A}}_2(q) = \begin{pmatrix} A_2(q) & 0 \\ \hat{B}_2(q)C(q) & \hat{A}_2(q) \end{pmatrix};\ \bar{C}(q) = \begin{pmatrix} E(q) & -\hat{C}(q) \end{pmatrix} \tag{9}$$

We express the question of this work by: we design an appropriate fuzzy filter (6) such that a error model is well-posed, stable satises the FF index:

$$\sup_{0\neq u_{n,k}\in l_2\{[0,\infty),[0,\infty)\}} ||e_{n,k}||_2 \leq \gamma^2||u_{n,k}||_2 \tag{10}$$

A equation is applicable of (8) knowing the following hold:

$$e^{j\mu_3^b}\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}[(e^{j\mu_1^b}\xi_{n,k+1} - \xi_{n+1,k+1})^T(\xi_{n+1,k+1} - e^{j\mu_1^a}\xi_{n,k+1})] \geq 0$$
$$e^{j\mu_4^b}\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}[(\xi_{n+1,k+1} - e^{j\mu_2^b}\xi_{n+1,k})^T(e^{j\mu_2^a}\xi_{n+1,k} - \xi_{n+1,k+1})] \geq 0 \tag{11}$$

2.2. Preliminaries

Lemma 2..1 in [25] from (12), we could get (13).

$$\begin{bmatrix} T + MU + U^TM^T & * \\ -M^T + GU & V - G - G^T \end{bmatrix} < 0 \tag{12}$$

$$T + U^TVU < 0 \tag{13}$$

Lemma 2..2 in [19] error system (8) is stable and FF in (10) is fulfilled, on condition that there are P_1, P_2, $0 < Q_1$, $0 < Q_2$, satisfying.

$$\begin{pmatrix} \mathcal{C}(q)^T\mathcal{C}(q) & \mathcal{C}(q)^T\mathcal{D}(q) \\ \mathcal{D}(q)^T\mathcal{C}(q) & -\gamma^2I + \mathcal{D}(q)^T\mathcal{D}(q) \end{pmatrix} + \begin{pmatrix} \mathcal{A}(q) & \mathcal{B}(q) \\ I & 0 \end{pmatrix}^T \begin{pmatrix} P & \Lambda^*Q \\ Q\Lambda & -\mathcal{R} \end{pmatrix} \begin{pmatrix} \mathcal{A}(q) & \mathcal{B}(q) \\ I & 0 \end{pmatrix} < 0 \tag{14}$$

with

$$\mathcal{R} = \begin{pmatrix} P_1 + 2cos(\mu_3^b)Q_1 & 0 \\ 0 & P_2 + 2cos(\mu_4^b)Q_2 \end{pmatrix}; \mathcal{D}(q) = \begin{pmatrix} \bar{D}(q) & 0 \\ 0 & \bar{D}(q) \end{pmatrix};\ \mathcal{A}(q) = \begin{pmatrix} \bar{A}_1(q) & \bar{A}_2(q) \end{pmatrix};$$
$$Q = \begin{pmatrix} Q_1 & Q_2 \end{pmatrix}; P = P_1 + P_2; \Lambda = \begin{pmatrix} e^{-j\mu_3^a}I_{2\bar{n}} & 0 \\ 0 & e^{-j\mu_4^a}I_{2\bar{n}} \end{pmatrix};\ \mathcal{B}(q) = \begin{pmatrix} \bar{B}_1(q) & \bar{B}_2(q) \end{pmatrix}; \tag{15}$$
$$\mathcal{C}(q) = \begin{pmatrix} \bar{C}(q) & 0 \\ 0 & \bar{C}(q) \end{pmatrix}; \mu_4^a = \frac{\mu_2^a + \mu_2^b}{2}; \mu_3^b = \frac{\mu_1^b - \mu_1^a}{2}; \mu_4^b = \frac{\mu_2^b - \mu_2^a}{2};\ \mu_3^a = \frac{\mu_1^a + \mu_1^b}{2};$$

3. FF PERFORMANCE ANALYSIS

3.1. Theorem 3..1

Error model (8) is stable, FF index (10) is fulfilled, on condition that there are P, Q, W_1, W_2, M_1, M_2, G_1, G_2, F_1, H satisfying $Q_1 > 0$, $Q_2 > 0$, $W_1 > 0$, $W_2 > 0$ and

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \mathcal{A}^T(q)G_1^T - M_1 & \mathcal{A}^T(q)G_2^T & \mathcal{C}^T \\ * & \Phi_{22} & \mathcal{B}^T(q)G_1^T - M_2 & \mathcal{B}^T(q)G_2^T & \mathcal{D}^T \\ * & * & P - G_1 - G_1^T & \Lambda^*Q - G_2^T & 0 \\ * & * & * & -\mathcal{R} & 0 \\ * & * & * & * & -I \end{bmatrix} < 0$$
$$\Upsilon = \begin{bmatrix} -W_1 + sym(F_1\bar{A}_1(q)) & F_1\bar{A}_2(q) & -F_1 + \bar{A}_1^T(q)^TH^T \\ * & -W_2 & \bar{A}_2^T(q)^TH^T \\ * & * & W_1 + W_2 - sym(q) \end{bmatrix} < 0 \tag{16}$$

$$\Phi_{11} = M_1\mathcal{A}(q) + \mathcal{A}^T(q)M_1^T;\ \Phi_{12} = M_1\mathcal{B}(q) + \mathcal{A}^T(q)M_2^T;\ \Phi_{22} = -\gamma^2 I + M_2\mathcal{B}(q) + \mathcal{B}^T(q)M_2^T;$$

3.2. Proof 3..2

First, condition (14) maybe written as

$$T + N^T V N < 0 \tag{17}$$

Where

$$N = \begin{bmatrix} \mathcal{A}(q) & \mathcal{B}(q) \\ I & 0 \end{bmatrix};\ V = \begin{bmatrix} P & \Lambda^* Q \\ Q\Lambda & -\mathcal{R} \end{bmatrix};\ T = \begin{bmatrix} \mathcal{C}(q)^T\mathcal{C}(q) & \mathcal{C}(q)^T\mathcal{D}(q) \\ \mathcal{D}(q)^T\mathcal{C}(q) & -\gamma^2 I + \mathcal{D}(q)^T\mathcal{D}(q) \end{bmatrix} \tag{18}$$

By using Lemma 2, (17) is equivalent to

$$\begin{bmatrix} T + MU + U^T M^T & U^T G^T - M \\ -M^T + GU & V - G - G^T \end{bmatrix} < 0 \tag{19}$$

with

$$M = \begin{bmatrix} M_1 & 0 \\ M_2 & 0 \end{bmatrix};\ G = \begin{bmatrix} G_1 & 0 \\ G_2 & 0 \end{bmatrix} \tag{20}$$

which, using Schur complement, leads to given (16). Consider the Lyapunov equation, such that

$$\begin{pmatrix} W_1 & 0 \\ 0 & W_2 \end{pmatrix} - \begin{pmatrix} \bar{A}_1(q) & \bar{A}_2(q) \end{pmatrix}^T (W_1 + W_2) \begin{pmatrix} \bar{A}_1(q) & \bar{A}_2(q) \end{pmatrix} > 0 \tag{21}$$

Where

$$\hat{U} = \begin{pmatrix} \bar{A}_1(q) & \bar{A}_2(q) \end{pmatrix};\ \hat{T} = \begin{pmatrix} -W_1 & 0 \\ 0 & -W_2 \end{pmatrix};\ \hat{V} = W_1 + W_2 \tag{22}$$

We chose F is being as:

$$F = \begin{bmatrix} F_1 & 0 \end{bmatrix}^T \tag{23}$$

4. FF PERFORMANCE DESIGN

4.1. Theorem 4..1

Error model (8) is stable, FF index (10) is fulfilled, on condition that there are $\hat{A}_{1i}$, $\hat{B}_{1i}$, $\hat{A}_{2i}$, $\hat{B}_{2i}$, $\hat{C}_{1i}$, $\hat{D}_{1i}$, M_{1u}, G_{2u}, M_{2t}, G_{1t}, H_{1t}, F_{1t}, V, $u = 1, 2, 3, 4$, $t = 1, 2$, P_{1s}, $Q_{1s} > 0$, $W_{1s} > 0$, P_{2s}, $Q_{2s} > 0$, $W_{2s} > 0$, $s = 1, 2, 3$, satisfying.

$$\tilde{\triangle} = \left[\begin{array}{c|c} \tilde{\triangle}_1 & \tilde{\triangle}_2 \\ \hline * & \tilde{\triangle}_3 \end{array}\right] < 0; \tag{24}$$

$$\tilde{\Omega} = \begin{bmatrix} \tilde{\Omega}_{11} & \tilde{\Omega}_{12} & \tilde{\Omega}_{13} & E\hat{A}_{2i} & \tilde{\Omega}_{15} & \tilde{\Omega}_{16} \\ * & \tilde{\Omega}_{22} & \tilde{\Omega}_{23} & \hat{A}_{2i}^T & \tilde{\Omega}_{25} & \tilde{\Omega}_{26} \\ * & * & -W_{21} & -W_{22} & \tilde{\Omega}_{35} & A_{2j}^T H_{12}^T \\ * & * & * & -W_{23} & \tilde{\Omega}_{45} & \hat{A}_{2i}^T \\ * & * & * & * & \tilde{\Omega}_{55} & \tilde{\Omega}_{56} \\ * & * & * & * & * & \tilde{\Omega}_{66} \end{bmatrix} < 0 \tag{25}$$

with

$$\tilde{\triangle}_1 = \begin{bmatrix} \tilde{\triangle}_{1_{11}} & \tilde{\triangle}_{1_{12}} & \tilde{\triangle}_{1_{13}} & \tilde{\triangle}_{1_{14}} & \tilde{\triangle}_{1_{15}} & \tilde{\triangle}_{1_{16}} \\ * & \tilde{\triangle}_{1_{22}} & M_{12}A_{2j} & sym[\hat{A}_{2i}] & \tilde{\triangle}_{1_{25}} & \tilde{\triangle}_{1_{26}} \\ * & * & \tilde{\triangle}_{1_{33}} & A_{2j}^T M_{14}^T & \tilde{\triangle}_{1_{35}} & \tilde{\triangle}_{1_{36}} \\ * & * & * & \tilde{\triangle}_{1_{44}} & \tilde{\triangle}_{1_{45}} & \tilde{\triangle}_{1_{46}} \\ * & * & * & * & \tilde{\triangle}_{1_{55}} & \tilde{\triangle}_{1_{56}} \\ * & * & * & * & * & \tilde{\triangle}_{1_{66}} \end{bmatrix};$$

$$\tilde{\triangle}_2 = \begin{bmatrix} \tilde{\triangle}_{11} & \tilde{\triangle}_{12} & \tilde{\triangle}_{13} & \tilde{\triangle}_{14} & \tilde{\triangle}_{15} & \tilde{\triangle}_{16} & \tilde{\triangle}_{17} & 0 \\ \tilde{\triangle}_{21} & \tilde{\triangle}_{22} & \tilde{\triangle}_{23} & \tilde{\triangle}_{24} & 0 & \tilde{\triangle}_{26} & 0 & \tilde{\triangle}_{88} \\ \tilde{\triangle}_{31} & \tilde{\triangle}_{32} & \tilde{\triangle}_{33} & \tilde{\triangle}_{34} & \tilde{\triangle}_{35} & \tilde{\triangle}_{36} & \tilde{\triangle}_{37} & 0 \\ \tilde{\triangle}_{41} & \tilde{\triangle}_{42} & \tilde{\triangle}_{43} & \tilde{\triangle}_{44} & 0 & \tilde{\triangle}_{46} & 0 & -\hat{C}_i^T \\ \tilde{\triangle}_{51} & \tilde{\triangle}_{52} & \tilde{\triangle}_{53} & \tilde{\triangle}_{54} & \tilde{\triangle}_{55} & \tilde{\triangle}_{56} & \tilde{\triangle}_{57} & 0 \\ \tilde{\triangle}_{61} & \tilde{\triangle}_{62} & \tilde{\triangle}_{63} & \tilde{\triangle}_{64} & \tilde{\triangle}_{65} & \tilde{\triangle}_{66} & 0 & \tilde{\triangle}_{88} \end{bmatrix};$$

$$\tilde{\triangle}_3 = \begin{bmatrix} \tilde{\triangle}_{3_{11}} & \tilde{\triangle}_{3_{12}} & \tilde{\triangle}_{3_{13}} & \tilde{\triangle}_{3_{14}} & \tilde{\triangle}_{3_{15}} & \tilde{\triangle}_{3_{16}} & 0 & 0 \\ * & \tilde{\triangle}_{3_{22}} & \tilde{\triangle}_{3_{23}} & \tilde{\triangle}_{3_{24}} & \tilde{\triangle}_{3_{25}} & \tilde{\triangle}_{3_{26}} & 0 & 0 \\ * & * & \tilde{\triangle}_{3_{33}} & \tilde{\triangle}_{3_{34}} & 0 & 0 & 0 & 0 \\ * & * & * & \tilde{\triangle}_{3_{44}} & 0 & 0 & 0 & 0 \\ * & * & * & * & \tilde{\triangle}_{3_{55}} & \tilde{\triangle}_{3_{56}} & 0 & 0 \\ * & * & * & * & * & \tilde{\triangle}_{3_{66}} & 0 & 0 \\ * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & -I \end{bmatrix}$$

$$\begin{aligned}
\bar{\Omega}_{13} &= F_{11}A_{2j};\ \bar{\Omega}_{23} = F_{12}A_{2j};\ \bar{\Omega}_{35} = A_{2j}^T H_{11}^T;\ \bar{\Omega}_{45} = \hat{A}_{2i}^T E^T;\ \tilde{\triangle}_{1_{22}} = sym[\hat{A}_{1i}];\ \tilde{\triangle}_{1_{33}} = sym[E\hat{A}_{2j}];\\
\bar{\triangle}_{2_{16}} &= A_{1j}^T G_{24}^T;\ \tilde{\triangle}_{2_{88}} = D_j^T - \hat{D}_i^T;\ \tilde{\triangle}_{2_{17}} = C_j^T;\ \tilde{\triangle}_{2_{88}} = -\hat{C}_i^T;\ \tilde{\triangle}_{1_{33}} = sym[M_{13}A_{2j}];\ \tilde{\triangle}_{2_{37}} = C_j^T;\\
\tilde{\triangle}_{1_{11}} &= sym[M_{11}A_{1j}];\ \tilde{\triangle}_{1_{12}} = A_{1j}^T M_{12}^T + E\hat{A}_{1i};\ \tilde{\triangle}_{1_{13}} = M_{11}A_{2j} + A_{1j}^T M_{13}^T;\ \tilde{\triangle}_{1_{14}} = A_{2j}^T M_{14}^T + E\hat{A}_{2i};\\
\tilde{\triangle}_{1_{15}} &= M_{11}B_{1j} + A_{1j}^T M_{21}^T + E\hat{B}_{1i};\ \tilde{\triangle}_{1_{16}} = M_{11}B_{2j} + A_{1j}^T M_{22}^T + E\hat{B}_{2i};\ \tilde{\triangle}_{1_{25}} = M_{12}B_{1j} + \hat{B}_{1i};\\
\tilde{\triangle}_{1_{26}} &= M_{12}B_{2j} + \hat{B}_{2i};\ \tilde{\triangle}_{1_{35}} = M_{13}B_{1j} + A_{2j}^T M_{21}^T;\ \tilde{\triangle}_{1_{56}} = M_{21}B_{2j} + B_{1j}^T M_{22}^T;\\
\tilde{\triangle}_{1_{36}} &= M_{13}B_{2j} + A_{2j}^T M_{22}^T;\ \tilde{\triangle}_{1_{45}} = M_{14}B_{1j} + E\tilde{B}_{1i};\ \tilde{\triangle}_{1_{46}} = M_{14}B_{2j};\ \tilde{\triangle}_{1_{55}} = -\gamma^2 I + sym[M_{21}B_{1j}];\\
\tilde{\triangle}_{1_{66}} &= -\gamma^2 I + sym[M_{22}B_{2j}];\ \tilde{\triangle}_{11} = A_{1j}^T G_{11}^T - M_{11};\ \tilde{\triangle}_{12} = A_{1j}^T G_{12}^T - EV;\ \tilde{\triangle}_{13} = A_{1j}^T G_{21}^T;\\
\tilde{\triangle}_{14} &= A_{1j}^T G_{22}^T;\ \tilde{\triangle}_{15} = A_{1j}^T G_{23}^T;\ \tilde{\triangle}_{16} = A_{1j}^T G_{24}^T;\ \tilde{\triangle}_{21} = \hat{A}_{1i}^T E^T - M_{12};\ \tilde{\triangle}_{22} = \hat{A}_{1i}^T - V;\\
\tilde{\triangle}_{23} &= \hat{A}_{1i}^T E^T;\ \tilde{\triangle}_{24} = \hat{A}_{1i}^T;\ \tilde{\triangle}_{26} = \hat{A}_{1i}^T E^T;\ \tilde{\triangle}_{31} = A_{2j}^T G_{11}^T - M_{13};\ \tilde{\triangle}_{32} = A_{2j}^T G_{12}^T;\\
\tilde{\triangle}_{33} &= A_{2j}^T G_{21}^T;\ \tilde{\triangle}_{34} = A_{2j}^T G_{22}^T;\ \tilde{\triangle}_{35} = A_{2j}^T G_{23}^T;\ \tilde{\triangle}_{36} = A_{2j}^T G_{24}^T;\ \tilde{\triangle}_{41} = \hat{A}_{2i}^T E^T - M_{14};\\
\tilde{\triangle}_{22} &= \hat{A}_{2i}^T - V;\ \tilde{\triangle}_{23} = \hat{A}_{2i}^T E^T;\ \tilde{\triangle}_{24} = \hat{A}_{2i}^T;\ \tilde{\triangle}_{26} = \hat{A}_{2i}^T E^T;\\
\tilde{\triangle}_{51} &= B_{1j}^T G_{11}^T + \hat{B}_{1i}^T E^T - M_{21};\ \tilde{\triangle}_{52} = B_{1j}^T G_{12}^T + \hat{B}_{1i}^T;\ \tilde{\triangle}_{53} = B_{1j}^T G_{21}^T + \hat{B}_{1i}^T E^T;\ \tilde{\triangle}_{54} = B_{1j}^T G_{22}^T + \hat{B}_{1i}^T;\\
\tilde{\triangle}_{55} &= B_{1j}^T G_{23}^T;\ \tilde{\triangle}_{56} = B_{1j}^T G_{24}^T + \hat{B}_{1i}^T E^T;\ \tilde{\triangle}_{61} = B_{2j}^T G_{11}^T - M_{22};\ \tilde{\triangle}_{62} = B_{2j}^T G_{12}^T + \hat{B}_{2i}^T;\\
\tilde{\triangle}_{63} &= B_{2j}^T G_{21}^T + \hat{B}_{2i}^T E^T;\ \tilde{\triangle}_{64} = B_{2j}^T G_{22}^T + \hat{B}_{2i}^T;\ \tilde{\triangle}_{65} = B_{2j}^T G_{23}^T;\ \tilde{\triangle}_{66} = B_{2j}^T G_{24}^T + \hat{B}_{2i}^T E^T;\\
\tilde{\triangle}_{3_{11}} &= P_{11} + P_{21} - G_{11} - G_{11}^T;\ \tilde{\triangle}_{3_{13}} = e^{-j\mu_3^a} Q_{11} - G_{21}^T;\ \tilde{\triangle}_{3_{12}} = P_{12} + P_{22} - EV - G_{21}^T;\\
\tilde{\triangle}_{3_{14}} &= e^{-j\mu_3^a} Q_{12} - G_{22}^T;\ \tilde{\triangle}_{3_{15}} = e^{-j\mu_3^a} Q_{21} - G_{23}^T;\ \tilde{\triangle}_{3_{16}} = e^{-j\mu_3^a} Q_{22} - G_{24}^T;\\
\tilde{\triangle}_{3_{22}} &= P_{13} + P_{23} - V - V^T;\ \tilde{\triangle}_{3_{23}} = e^{-j\mu_4^a} Q_{21}^T - V^T E^T;\ \tilde{\triangle}_{3_{24}} = e^{-j\mu_4^a} Q_{13} - V^T;\ \tilde{\triangle}_{3_{25}} = e^{-j\mu_4^a} Q_{22}^T;\\
\tilde{\triangle}_{3_{26}} &= e^{-j\mu_4^a} Q_{23} - V^T E^T;\ \tilde{\triangle}_{3_{33}} = -P_{11} - 2cos(\mu_3^b) Q_{11};\ \tilde{\triangle}_{3_{34}} = -P_{12} - 2cos(\mu_3^b) Q_{12};\\
\tilde{\triangle}_{3_{44}} &= P_{13} - 2cos(\mu_3^b) Q_{13};\ \tilde{\triangle}_{3_{55}} = -P_{21} - 2cos(\mu_3^b) Q_{21};\ \tilde{\triangle}_{3_{56}} = -P_{22} - 2cos(\mu_3^b) Q_{22};\\
\tilde{\triangle}_{3_{66}} &= -P_{23} - 2cos(\mu_3^b) Q_{23}; \bar{\Omega}_{12} = -W_{12} + A_{1j}^T F_{12}^T + E\hat{A}_{1i};\ \bar{\Omega}_{11} = -W_{11} + sym(F_{11}A_{1j});\\
\bar{\Omega}_{22} &= -W_{13} + sym(\hat{A}_{1i});\ \bar{\Omega}_{15} = -F_{11} + A_{1j}^T H_{11}^T;\ \bar{\Omega}_{16} = -EV + A_{1j}^T H_{12}^T;\ \bar{\Omega}_{25} = -F_{12} + \hat{A}_{1i}^T E^T;\\
\bar{\Omega}_{26} &= \hat{A}_{1i}^T - V;\ \bar{\Omega}_{55} = W_{11} + W_{21} - sym(H_{11});\ \bar{\Omega}_{56} = W_{12} + W_{22} - EV - H_{12}^T;\\
\bar{\Omega}_{66} &= W_{13} + W_{23} - sym(V);\ E = \begin{bmatrix} I & 0 \end{bmatrix}^T
\end{aligned}$$

The following parameters as.

$$\hat{A}_{1i} = V^{-1}\breve{A}_{1i};\ \hat{A}_{2i} = V^{-1}\breve{A}_{2i};\ \hat{B}_{1i} = V^{-1}\breve{B}_{1i};\ \hat{B}_{2i} = V^{-1}\breve{B}_{2i};\ \hat{C}_i = \breve{C}_i. \tag{26}$$

4.2. Proof 4..2

Parameterise slack matrices M_1, M_2, G_1, G_2, F_1 and H in Theorem 3..1 as.

$$M_1 = \begin{bmatrix} M_{11} & V \\ M_{12} & V \\ M_{13} & 0 \\ M_{14} & V \end{bmatrix};\ G_2 = \begin{bmatrix} G_{21} & V \\ G_{22} & V \\ G_{23} & 0 \\ G_{24} & V \end{bmatrix};\ M_2 = \begin{bmatrix} M_{21} & 0 \\ M_{22} & 0 \end{bmatrix};\ G_1 = \begin{bmatrix} G_{11} & V \\ G_{12} & V \end{bmatrix};$$
$$F = \begin{bmatrix} F_{11} & V \\ F_{12} & V \end{bmatrix},\ H = \begin{bmatrix} H_{11} & V \\ H_{12} & V \end{bmatrix} \tag{27}$$

4.3. Remark 4..3

If we take $Q_m = diag\{Q_{mh}, Q_{mv}\} = 0$, $m = 1, ..., 3$, we can employ theorem 3 to settle the H_∞ filter for FMLSS nonlinear 2-D systems in EF range.

5. NUMERICAL EXAMPLE

Consider a 2D discrete-time model, given by [19].

Rule 1: if $\zeta_1(s)\ is\ \tilde{M}_1^1$, $\zeta_2(s)\ is\ \tilde{M}_2^1$, then,

$$\begin{aligned} x_{n+1,k+1} &= A_{11}x_{n,k+1} + A_{21}x_{n+1,k} + B_{11}u_{n,k+1} + B_{21}u_{n+1,k} \\ y_{n,k} &= C_1x_{n,k} + D_1u_{n,k}; \\ z_{n,k} &= E_1x_{n,k} \end{aligned} \tag{28}$$

Rule 2: if $\zeta_1(s)\ is\ \tilde{M}_1^2$, $\zeta_2(s)\ is\ \tilde{M}_2^2$, then,

$$\begin{aligned} x_{n+1,k+1} &= A_{12}x_{n,k+1} + A_{22}x_{n+1,k} + B_{12}u_{n,k+1} + B_{22}u_{n+1,k} \\ y_{n,k} &= C_2x_{n,k} + D_2u_{n,k}; \\ z_{n,k} &= E_1x_{n,k} \end{aligned} \tag{29}$$

with

$$\begin{aligned} A_{11} &= \begin{bmatrix} 0.1 & -0.2 \\ -0.2 & 0.1 \end{bmatrix} A_{21} = \begin{bmatrix} 0.25 & 0.1 \\ -0.05 & 0.3 \end{bmatrix};\ A_{12} = \begin{bmatrix} 0.3 & -0.2 \\ -0.2 & 0.3 \end{bmatrix} A_{22} = \begin{bmatrix} 0.25 & 0.1 \\ -0.05 & 0.5 \end{bmatrix}; \\ B_{11} &= \begin{bmatrix} 0.1 \\ 0.25 \end{bmatrix};\ B_{12} = \begin{bmatrix} 0.1 \\ 0.4 \end{bmatrix};\ D_1 = D_2 = 0.1; B_{21} = \begin{bmatrix} 0 \\ 0.28 \end{bmatrix};\ B_{22} = \begin{bmatrix} 0 \\ 0.2 \end{bmatrix};\ E_1 = E_2 = 0.1; \\ C_1 &= C_2 = \begin{bmatrix} 0 & 1 \end{bmatrix} \end{aligned} \tag{30}$$

The normalized membership function:

$$q_1(\zeta_{n,k}) = 1 - \frac{1}{1 + \exp(-2(\zeta_{n,k} - 3))};\ q_2(\zeta_{n,k}) = \frac{1}{1 + \exp(-2(\zeta_{n,k} - 3))} \tag{31}$$

Suppose that the FF domain of disturbance input signal is $[\frac{\pi}{8}, \frac{\pi}{4}] \times [\frac{\pi}{8}, \frac{\pi}{4}]$. Via using Theorem 4, the obtained matrix parameters of FF H_∞ filter are the following:

$$\begin{aligned} \hat{A}_{11} &= \begin{bmatrix} 0.1454 & -0.1092 \\ 0.0638 & -0.2298 \end{bmatrix}; \breve{B}_{11} = \begin{bmatrix} -0.3219 \\ 0.0002 \end{bmatrix}; \breve{A}_{12} = \begin{bmatrix} 0.2454 & -0.0738 \\ 0.0010 & -0.0974 \end{bmatrix}; \breve{B}_{21} = \begin{bmatrix} -0.0923 \\ -0.1105 \end{bmatrix}; \\ \breve{A}_{21} &= \begin{bmatrix} -0.1527 & 0.7145 \\ 0.0305 & -0.2105 \end{bmatrix}; \breve{B}_{12} = \begin{bmatrix} -0.1504 \\ 0.0204 \end{bmatrix};\ \breve{A}_{22} = \begin{bmatrix} -0.0101 & 0.8745 \\ 0.0174 & -0.0202 \end{bmatrix}; \breve{B}_{22} = \begin{bmatrix} -0.0017 \\ 0.0201 \end{bmatrix}; \\ \breve{C}_1 &= \begin{bmatrix} -1.9560 & 0.4749 \end{bmatrix};\ \breve{C}_2 = \begin{bmatrix} -1.9560 & 0.4749 \end{bmatrix}; \end{aligned} \tag{32}$$

The comparison result with the technique proposed in Theorem 4..1 Illustrate in Table 1, that indicate the little conservation of the method suggest in the paper. Figures 1-3 indicate the path of states filters vectors $\hat{x}_1$, $\hat{x}_2$ and filtering error system of $e_{n,k}$, respectively. From Figures 1-3, also, we could notice whether the error model is stable, knowing that the initial Condition are null, we can work out $\frac{||e_{n,k}||_2}{||u_{n,k}||_2} = 0.2205$. So, the condition (10) is fulfilled, that signify that the error system get a specific H_∞ index ($\gamma = 0.2357$).

Table 1. H_∞ performance apply from various approaches

Frequency domain	Methods	γ
$[0;\pi] \times [0;\pi]$	Theorem 6 in [26]	1.7302
$[0;\pi] \times [0;\pi]$	Theorem 3.4 (Q=0) in [19]	*Inf*
$[0;\pi] \times [0;\pi]$	*Theorem* 4..1 (Q=0)	0.6321
$[\frac{\pi}{8};\frac{\pi}{4}] \times [\frac{\pi}{8};\frac{\pi}{4}]$	Theorem 3.4 in [19]	0.6000
$[\frac{\pi}{8};\frac{\pi}{4}] \times [\frac{\pi}{8};\frac{\pi}{4}]$	Theorem 4..1	0.2357

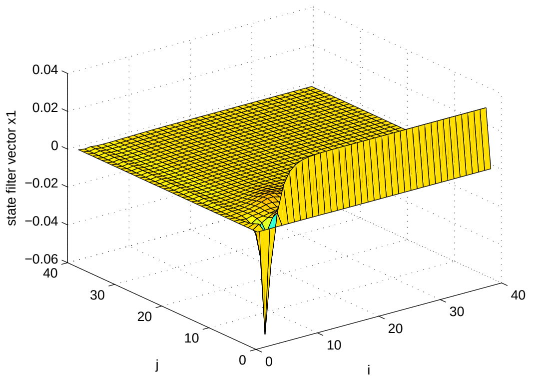

Figure 1. Trajectory of state filter vectors $\hat{x}_1$

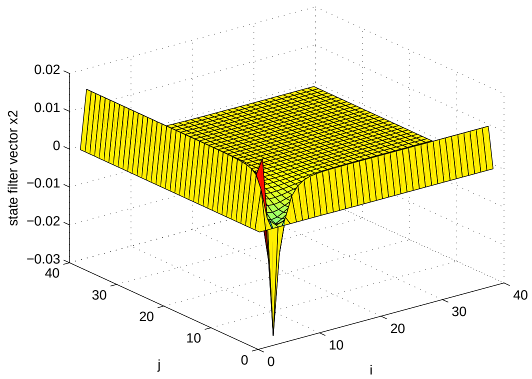

Figure 2. Trajectory of state filter vectors $\hat{x}_2$

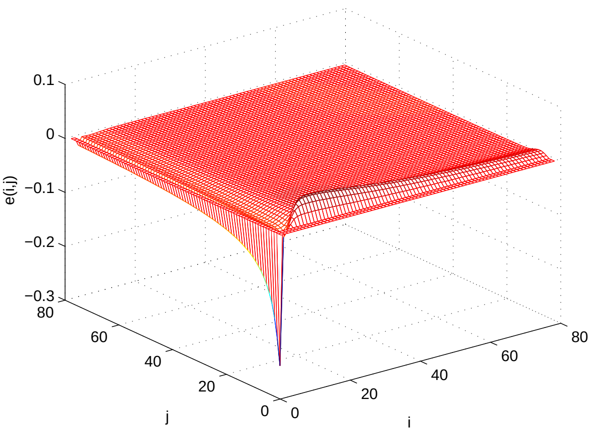

Figure 3. Error $e_{n,k}$

6. CONCLUSION

This work, we dealt with problematic for the FF design of FMLSS nonlinear 2-D systems over FF ranges. We have suggested a filter process in order to minimize the conservatism design using the frequency information of the disturbances and we have assumed that the disturbances are known in a recognized FF domain. Also, systematic techniques have been suggested for the generation of a filtering which ensures asymptotic stability and FF H_∞ index, at the basis of a more general linearization procedure.

REFERENCES

[1] M. S. Branicky, V. S. Borkar, and S. K. Mitter, "A unified framework for hybrid control: Model and optimal control theory," in *IEEE Transactions on Automatic Controls*, vol. 43, no. 1, pp. 31-45, January 1998, doi: 10.1109/9.654885.

[2] S. Xu and P. V. Dooren, "Robust H_∞ filtering for a class of non-linear systems with state delay and parameter uncertainty," *International Journal of Control*, vol. 75, no. 10, pp. 766-774, 2002, doi: 10.1080/00207170210141815.

[3] T. Kaczorek, "Two-Dimensional Linear Systems," *Lecture Notes in Control and Information Science*, vol. 68, pp. 283-284, 1999, doi: /10.1007/978-1-4471-0853-$5_1$0.

[4] C. Du and L. Xie, "H_∞ Control and Filtering of two-dimensional Systems," *Springer Verlag*, vol. 278, 2002.

[5] X. Li, J. Lam, H. Gao, and Y. Gu, "A frequency-partitioning approach to stability analysis of 2-D discrete systems," *Multidimensional System and Signal Processing*, vol 26, no. 1, pp. 67-93, 2015, doi: 10.1007/s11045-013-0237-4.

[6] C. Du, L. Xie, and C. Zhang, "H_∞ control and robust stabilization of two-dimensional systems in Roesser models," *Automatica*, vol. 37, no. 2, pp. 205-211, February 2001, doi: 10.1016/S0005-1098(00)00155-2.

[7] H. Gao, X. Meng, and T. Chen, "New Design of Robust H_∞ Filters for 2-D Systems," *Signal Processing Letters*, vol. 15, pp. 217-220, 2008, doi: 10.1109/LSP.2007.913136.

[8] H. Gao, J. Lam, C. Wang, and S. Xu, "H_∞ model reduction for uncertain two-dimensional discrete systems," *Optimal Control Application and Method*, vol. 26, no. 4, pp. 199-227, July 2005, doi: 10.1002/oca.760.

[9] Z. Duan and Z. Xiang, "State feedback H_∞ control for discrete 2-D switched systems," *Journal of the Franklin Institute*, vol. 350, no. 6, pp. 1512-1530, August 2013, doi: 10.1016/j.jfranklin.2013.04.001.

[10] C. Lin, Q. Wang, T. H. Lee and B. Chen, "H_∞ filter design for non-linear systems with time-delay through TS fuzzy model approach," *IEEE Transactions on Fuzzy Systems*, vol. 16, no. 3, pp. 739-746, June 2008, doi: 10.1109/TFUZZ.2007.905915.

[11] E. Tian and D. Yue, "Reliable H_∞ filter design of T-S fuzzy model based networked control systems with random sensor failure,", *Int. J. of Robust and Nonlinear Control*, vol. 23, no. 1, pp. 15-32, January 2013, doi: 10.1002/rnc.1811.

[12] Y. Luo, Z. Wang, J. Liang, G. Wei, and F. E. Alsaadi, "H_∞ control for 2-D fuzzy systems with interval time-varying delay and missing measurements," in *IEEE Transactions on Cybernetics*, vol. 47, no. 2, pp. 365-377, Feb. 2017, doi: 10.1109/TCYB.2016.2514846.

[13] H. Nobahari, S. M. Zandavi, and H. Mohammadkarimi, "A novel heuristic filter for nonlinear systems state estimation," *Applied Soft Computing*, vol. 49, pp 474-484, August 2016, doi: 10.1016/j.asoc.2016.08.008.

[14] J. Woods and V. Ingle, "Kalman filtering in two dimensions: Further results," in *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 29, no. 2, pp. 188-197, April 1981, doi: 10.1109/TASSP.1981.1163533.

[15] H. D. Tuan, P. Apkarian, T. Q. Nguyen, and T. Narikiyo, "Robust mixed H_2/H_∞ filtering of 2-D system," *Transactions on Signal Processing*, vol. 50, no. 7, pp. 1759-1771, July 2002.

[16] L. Li, W. Wang, and X. Li, "New approach to H_∞ filtering of 2D TS fuzzy system," *International Journal of Robust and Nonlinear Control*, vol. 23, no. 17, pp 1900-2012, July 2012, doi: 10.1002/rnc.2866.

[17] X. Li and H. Gao, "Robust finite frequency image filtering for uncertain 2-D system: The FM model case," *Automatica*, vol. 29, no. 8, pp. 2446-2452, August 2013, doi: 10.1016/j.automatica.2013.04.014.

[18] A. El-Amrani, B, Boukili, A. Hmamed, A. El Hajjaji, and I. Boumhidi, "Robust H_∞ filtering for 2D continuous system where finite frequency specification," *International Journal of System Science*, vol. 49, no. 1, pp. 43-58, October 2017, doi: 10.1080/00207721.2017.1391960.

[19] Z. Duan, J. Zhou, and J. Shen, "Filter design for discrete 2D TS fuzzy systems with finite frequency specification," *Int. J. of Sys. Sci.*, vol. 50, no. 3, pp. 599-613, 2019, doi: 10.1080/00207721.2018.1564086.

[20] A. El-Amrani, A. E. Hajjaji, B. Boukili, and A. Hmamed, "H_∞ Model Reduction for Two-Dimensional Discrete Systems in Finite Frequency Ranges," *2018 26th Mediterranean Conference on Control and Automation (MED)*, 2018, pp. 1-9, doi: 10.1109/MED.2018.8442969.

[21] A. El-Amrani, A. Hmamed, B. Boukili, and A. El Hajjaji, "H_∞ filtering of TS systems in Finite Frequency domain," *2016 5th International Conference on System and Control (ICSC)*, 2016, pp. 306-312, doi: 10.1109/ICoSC.2016.7507038.

[22] S. Aboulem, A. El-Amrani, and I. Boumhidi, "Finite frequency H_∞ control for wind turbine system in TS form," *International Journal of Power Electronics and Drive Systems (IJPEDS)*, vol. 11, no. 3, pp. 1313-1323, September 2020, doi: 10.11591/ijpeds.v11.i3.pp1313-1322.

[23] Z. Lahlou, A. El-Amrani, and I. Boumhidi, "Finite frequency H_∞ control design for nonlinear system," *International Journal of Power Electronics and Drive Systems (IJPEDS)*, vol. 12, no. 1, pp. 567-577, March 2021, doi: 10.11591/ijpeds.v12.i1.pp567-575.

[24] A. El-Amrani, B. Boukili, A. El Hajjaji, I. Boumhidi, and A. Hmamed, "H_∞ model reduction design in finite frequency ranges of discrete TS systems," *Int. J. of System Engineering*, vol. 11, no. 2, pp. 89-104, 2021.

[25] D. Peaucelle, "Unified Formulation for Robust Analysis and Synthesis where Parameters Dependent Lyapunov Function," *Thesis of Toulouse University*, France, 2000.

[26] L. Li, W. Wang, and X. Li, "New approach to filtering of 2D TS system," *International Journal of Robust and Nonlinear Control*, vol. 23, pp. 1989-2012, 2013, doi: 10.1002/RNC.2866.