International Journal of Power Electronics and Drive Systems (IJPEDS)
Vol. 13, No. 2, June 2022, pp. 805~811
ISSN: 2088-8694, DOI: 10.11591/ijpeds.v13.i2.pp805-811

Modelling and control design of the electrostatic linear comb actuator

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Article Info

Article history:

Received Sep 16, 2021
Revised Mar 16, 2022
Accepted Apr 1, 2022

Keywords:

Displacement amplitude
Dynamic parameters
Electrostatic linear comb actuator
Micro-electro-mechanical systems
Nonlinear control

ABSTRACT

Control design plays a major part in electrostatic linear comb actuator (ELCA) systems. Most existing control methods of ECLA mainly focus on traditional PID method. However, proportional–integral–derivative (PID) controller is only appropriate for the reference to be known as constant. In this article, the problem of establishing the dynamic model and designing the controller is investigated for ECLA working in the time-varying reference. The tracking effectiveness is constructed for ECLA systems based on the consideration of roots of the equivalent differential equation in tracking error model. Moreover, the convergence velocity is also tuned by changing the parameters in the presented control scheme. Simulation results guarantee the performance of the nonlinear control scheme as well as dynamic model to be established.

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1. INTRODUCTION

Micro electro-mechanical system (MEMS) is a remarkable device and popularly considered in several directions such as robots, vehicles, aerospace, industrial electronics, medicine, etc. Micro actuators can be known as one of the MEMS devices as well as describe an efficient task creating a force to move the micro elements and micro systems. Electrostatic linear comb actuator (ELCA) is a remarkable instrument structure using the working description via electrostatic force for establishing linear motion. The ELCA systems have many advantages including high effectiveness, low power requirement, and simple structure. They are able to explain the fact of easy movement in MEMS elements, such as resonator systems [1], gyroscopes [2] and micro systems of the conveyer or transportation systems [3], micro motors or gripper [4], [5]. Several work of ELCA have been recently developed with many appropriate results including new structures, working effectiveness, manufacturing improvement, smart materials. Particularly, the fabrication effectiveness and the ELCA's movement stability guarantees the operating effectiveness of the elements. The characteristic of suspension beams with a crab-leg, a fold [6], [7] was developed to decrease the stiffness and change the deviation. A different direction on improving deviation was addressed [8], by decreasing the movement of out-of-plane from that generates the in-plane deviation. The work in [9] presented an actuator property after inserting more a module of a sensor as well as a sub-actuator for stability of control systems. The effect of side etching on deviation of ELCA with disability rent increase exemption DRIE method was pointed out in [10]. By computing the ELCA deviation in design objective [3], [4], [10]-[12], it is necessary to utilize the theoretical method to be developed by a static equilibrium equation and decreases a design time.

The fact is that these elements manipulate in statement of motion by using AC voltage such as traditional waves. Hence, under the effect of force and damping follows an efficient property in estimating the deviation of ELCA, which was considered in [13], [14] with the EAs to be developed for moving resonators, gyroscopes. Moreover, the motion dynamic model including an efficient mass and factor of conversion of air damping was not considered in these references. To achieve better behavior, the consideration of using control scheme is becoming a promising method [8]. However, the controller of ECLA have been considered with the traditional PID method. It should be noted that conventional PID controller does not satisfy the tracking effectiveness under the time-varying reference. Moreover, nonlinear controller and optimal control scheme have been usually proposed for different systems, such as robotic systems, power electronics [15]-[25]. Motivated by the above-mentioned discussions, this article studies the dynamic model of ECLA and the corresponding controller. To improve the tracking effectiveness under time-varying reference, the controller is carried out based on the consideration of roots of differential equations.

2. PROBLEM STATEMENT

Figure 1 describes the structure of ELCA including a moving element ① and fixed element (② and ③). The moving element with moving comb fingers is pressured by four EB systems driving in *y*-direction based on the electrostatic force (EF), one terminal of each beam is kept on ②. The configuration of movable and fixed comb fingers can be seen with the same shape. The work in [4] shows the ECLA geometrical parameters in Figure 1, where *b* is a beam width, *h* is the device layer thickness, and g_a is the gap.

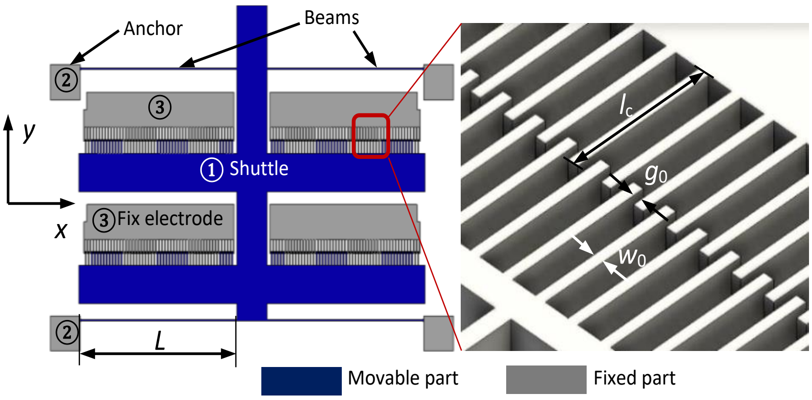

Figure 1. Structure of ELCA

Under the influence of a voltage V between ② and ③ of the ELCA, the total force in *y*-direction can be computed as (1).

$$F_e = \frac{nh\varepsilon\varepsilon_0}{g_0}V^2 \quad (1)$$

Where $\varepsilon_0 = 8.854 \times 10^{-6}\rho\text{F}/\mu\text{m}$ and ε = 1; *V* is the moving voltage. The stiffness in *y*-direction with four suspension beams system is described as (2) [4], [10].

$$K = \frac{4Ehb^3}{L^3} \quad (2)$$

Where *E* = 169GPa.

Due to the application of a *DC* voltage for ELCA, the deviation y_0 of ELCA (i.e. displacement of the shuttle) is directly computed from the equation of force balance:

$$F_e - F_{el} = 0\text{or:}\frac{nh\varepsilon\varepsilon_0}{g_0}V^2 = \frac{4Ehb^3}{L^3}.y_0$$

It implies that:

$$y_0 = \frac{n\varepsilon\varepsilon_0}{4Eg_0}\left(\frac{L}{b}\right)^3 V^2 \quad (3)$$

In (3) shows the quadratic relation between the deviation of the ELCA and the DC moving voltage. The objective of this article is to establish the dynamic model of ELCA for designing the controller with the reference of output $y(t)$ depending on time.

3. DYNAMIC MODEL OF ELCA AND CONTROL DESIGN

Under the AC moving voltage, it implies the movement of ELCA's shuttle to be similar with the movement to be described in Figure 2. Where K, C, M are the coefficients of conversion stiffness, conversion damping and conversion effective mass, respectively; $F_e(t)$ is an electrostatic moving force.

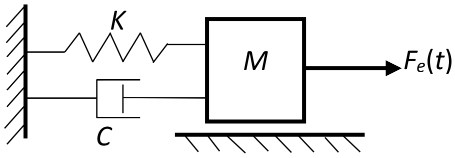

Figure 2. 1-DOF dynamic model of the ELCA

The dynamic model in *y*-direction (Figure 1) is represented as (4).

$$M\ddot{y} + C\dot{y} + Ky = F_e(t) \tag{4}$$

For the requirement of calculating exactly the deviation of the ELCA using dynamic model, the factors *K*, *M*, *C* are necessary to be accurately identified. Moreover, the factor *K* in *y*-direction is given in (2) and the appropriate mass of movable elements, including the elastic beams, is transformed into shuttle's motion with the principle of the kinetic energy of mass *M* being equal to all movable parts's kinetic energy. It follows that:

$$M = M_s + \frac{4M_b}{L}\int_0^L \left(\frac{\dot{y}(x)}{\dot{y}(L)}\right)^2 dx \tag{5}$$

moreover, the related equation between $\dot{y}(x)$ and $\dot{y}(L)$ is given through the relation of deviations:

$$y(x) = \frac{3Lx^2 - 2x^3}{L^3} y(L) \Rightarrow \dot{y}(x) = \frac{3Lx^2 - 2x^3}{L^3} \dot{y}(L) \tag{6}$$

according to (5) and (6), it leads:

$$M = M_s + 4\frac{13M_b}{35} = \rho h(A_t + \frac{52bL}{35}) \tag{7}$$

where ρ = 2330 kg/m^3 is the silicon mass density; A_t is the total top side area of the shuttle. Based on [15], the coefficient *C* in (4) can be calculated:

$$C = C_1 + C_2 + C_3$$

where

$$C_1 = \mu\left(\frac{A_{bt}}{g_a} + \frac{2A_{sc}}{g_0}\right) \tag{8}$$

$$C_2 = \mu\frac{A_t + A_{ss}}{\delta} \tag{9}$$

$$C_3 \approx \frac{32}{3}\mu\frac{h}{2} \tag{10}$$

combination of in (8), (9) and (10), we have:

$$C = \mu\left(\frac{A_{bt}}{g_a} + \frac{2A_{sc}}{g_0} + \frac{A_t + A_{ss}}{\delta} + \frac{16h}{3}\right) \tag{11}$$

considering the driving voltage with the square wave (Figure 3).

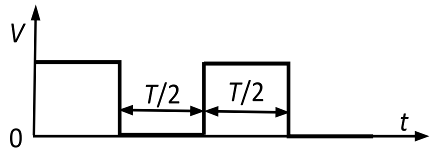

Figure 3. Square wave of driving voltage

The force function $F_e(t)$ is represented as:

$$F_e(t) = \begin{cases} \frac{nh\varepsilon\varepsilon_0}{g_0} V^2 & 0 \le t \le T/2 \\ 0 & T/2 < t \le T \end{cases} \quad (12)$$

$$y_1 = e^{-\beta t}(A_1 \cos(\omega t) + A_2 \sin(\omega t)) + \frac{F_e}{K} \quad (13)$$

Here: $\beta = \frac{C}{2M}$; $\omega = \sqrt{\omega_n^2 - \beta^2}$; $\omega_n = \sqrt{\frac{K}{M}}$. With the initial condition: $y(0) = 0$and $\dot{y}(0) = 0$, it follows: $A_1 = -\frac{F_e}{K}$; $A_2 = -\frac{\beta}{\omega}\frac{F_e}{K}$. The general displacement formula of the shuttle in this case can be expressed:

$$y_1 = \frac{n\varepsilon\varepsilon_0}{4Eg_0}\left(\frac{L}{b}\right)^3 V^2 \left[1 - \left(\cos(\omega t) + \frac{\beta}{\omega}\sin(\omega t)\right) e^{-\beta t}\right] \quad (14)$$

considering the driving voltage with the sine wave function (Figure 4).

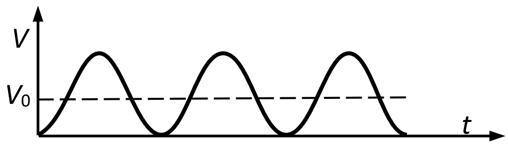

Figure 4. V = $V_0(1 + \sin(\Omega t))$

By setting: $F_0 = \frac{nh\varepsilon\varepsilon_0}{g_0} V_0^2$, the moving force is given:

$$F_{es} = F_0 \left[\frac{3}{2} + 2\sin(\Omega t) - \frac{1}{2}\cos(2\Omega t)\right] \quad (15)$$

in this case, the deviation of dynamic model (4) is represented:

$$\begin{aligned} y_{1s} = {} & \frac{3F_0}{2K} + \frac{2F_0}{(K - M\Omega^2)^2 + C^2\Omega^2}\left[(K - M\Omega^2)\sin(\Omega t) - C\Omega\cos(\Omega t)\right] \\ & - \frac{F_0}{2(K - 4M\Omega^2)^2 + 8C^2\Omega^2}\left[(K - 4M\Omega^2)\cos(2\Omega t) + 2C\Omega\sin(2\Omega t)\right] \\ & + e^{-\beta t}\left[B_1\cos(\omega t) + B_2\sin(\omega t)\right] \end{aligned} \quad (16)$$

according to initial condition: $y_{1s}(0) = 0$and$\dot{y}_{1s}(0) = 0$, we have:

$$B_1 = F_0\left\{\frac{2C\Omega}{(K - M\Omega^2)^2 + C^2\Omega^2} + \frac{K - 4M\Omega^2}{2(K - 4M\Omega^2)^2 + 8C^2\Omega^2} - \frac{3}{2K}\right\}$$

$$B_2 = \frac{F_0}{\omega}\left\{\frac{2C\Omega^2}{(K - 4M\Omega^2)^2 + 4C^2\Omega^2} - \frac{2\Omega(K - M\Omega^2)}{(K - M\Omega^2)^2 + C^2\Omega^2}\right\} + \frac{\beta}{\omega}B_1$$

for the mathematical model (4) established above$M\ddot{y} + C\dot{y} + Ky = F_e(t)$. The control objective is to achieve the input signal$F_e(t)$ such that $y(t)$ tracks $y^{ref}(t)$. Due to the difficulties in finding the precise parameters

M, C, K ; it is assumed that the parameters $M, C, K$are unknown and bounded by known constantsM_0, C_0, K_0, respectively. Let's define that $e(t) = \{y^{ref}(t) - y(t)\}$, according to (4):

$$M\frac{d^2e}{dt^2} + C\frac{de}{dt} + Ke = \left(M\frac{d^2y^{ref}}{dt^2} + C\frac{dy^{ref}}{dt} + Ky^{ref} - F_e(t)\right)$$

that:

$$M\frac{d^2e}{dt^2} + C\frac{de}{dt} + Ke = u(t) \tag{17}$$

therefore, the proposed controller is chosen as $F_e(t) = \left(M\frac{d^2y^{ref}}{dt^2} + C\frac{dy^{ref}}{dt} + Ky^{ref}\right) - u(t)$, where:

$$u(t) = D\frac{de}{dt} \tag{18}$$

with D to be chosen as:

From (17) and (18), it implies that: $M\frac{d^2e}{dt^2} + (C - D)\frac{de}{dt} + Ke = 0$ (19)

hence, we only choose the parameter D such that the corresponding polynomial $Mx^2 + (C - D)x + K = 0$ have all solutions being on the left side of complex plane.

4. THE SIMULATION AND EVALUATION OF RESEARCH RESULTS

The following simulations are performed to validate the tracking problem of output with the reference being the sinusoidal signal. The parameters of the proposed controller (18) are given as $M = 1, C = 10, D = 9, K = 1$. Two cases of sinusoidal reference signals are discussed as $y_{1,\text{ref}}(t) = 2\,sin\left(t + \frac{\pi}{2}\right)$, $y_{2,\text{ref}}(t) = 2\,sin\left(t + \frac{\pi}{2}\right)$. The simulations will focus on the command tracking effect of the proposed controller (18). It is clearly seen that, although the references are depended on time but the tracking effectiveness is guaranteed (Figures 5 and 6). Moreover, the performance of tracking problem can be adjusted by choosing the parameters in (8) based on the roots of corresponding polynomial $Mx^2 + (C - D)x + K = 0$.

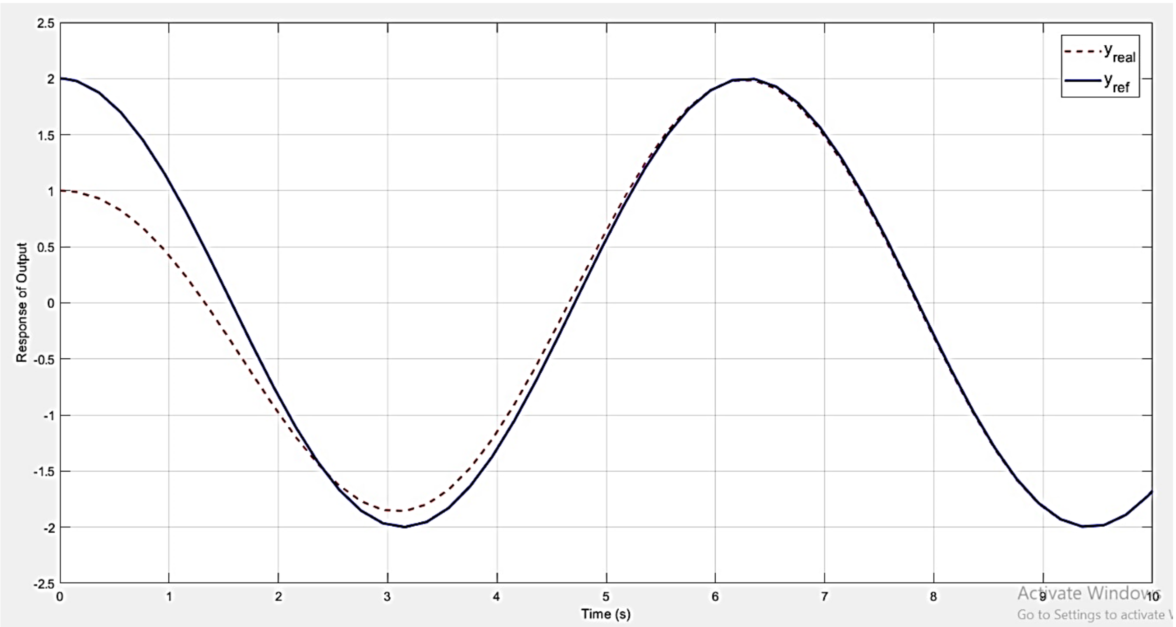

Figure 5. The result of the output with the reference being $y_{\text{ref}}(t) = 2\,sin\left(t + \frac{\pi}{2}\right)$

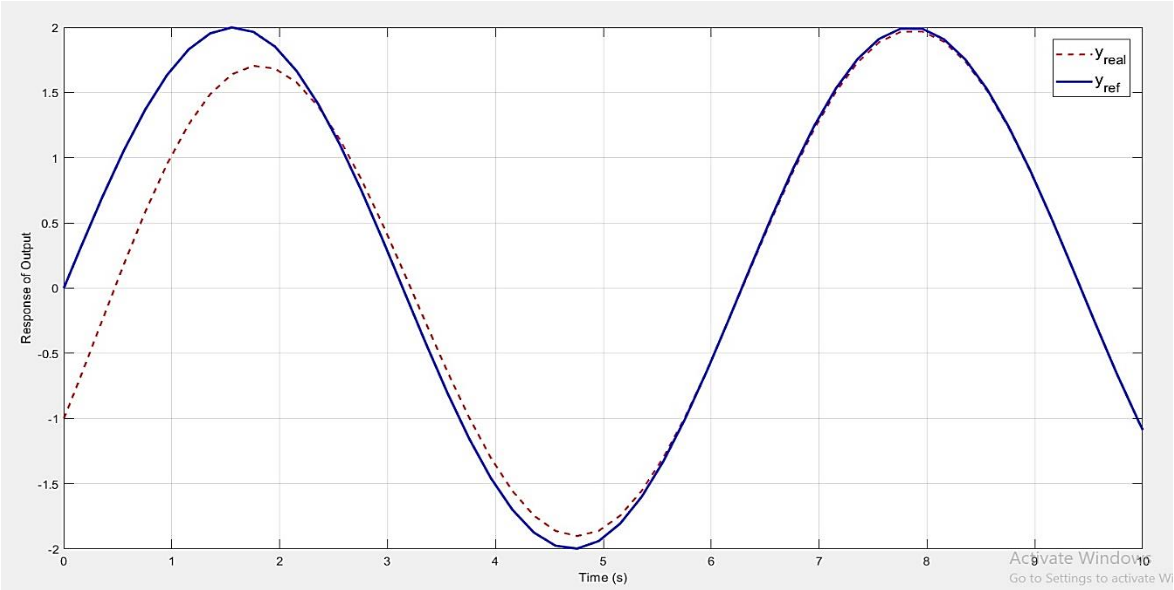

Figure 6. The result of the output with the reference being $y_{\text{ref}}(t) = 2\,sin(t)$

5. CONCLUSION

The article studies the dynamic model and control design of ELCA systems. Through the method of solving differential equation, the control scheme is implemented for ECLA in the presence of time varying reference. The tracking problem is analyzed through considering the solution of differential equation. By carrying out two simulation cases, it is concluded that the tracking problem is absolutely guaranteed with convergence rate to be depended on controller parameters. For future work, the problem of practical ECLA systems can be developed.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the Thai Nguyen University of Technology for supporting this work.

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