SHEPWM in three-phase voltage source inverters by modified Newton–Raphson

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ABSTRACT
This paper describes a new strategy for optimizing the switching angles of a three-phase inverter in a photovoltaic system. It presents non-traditional solutions to the problem of selective harmonic elimination (SHE) in three-phase inverter (VSI)-fed induction motor drives. The aforementioned problem was solved independently by using hybrid genetic algorithms (HGAs) and a modified Newton–Raphson method. GAs can obtain the correct solution even if the first generation is arbitrary. The solution then converges rapidly. The modified Newton–Raphson method is used to solve transcendental equations of the SHE pulses width modulation (SHEPWM) technique, which is a unique method that produces all possible solutions without assuming the initial angles. This modified technique is not complex and ensures rapid convergence to the solution. A real-time experimental verification of the SHEPWM technique was performed in the OP5600 RT-Lab simulator. The results obtained show that the proposed SHEPWM algorithm controls the fundamental voltage and effectively eliminates the desired harmonics, and that the evolution of the signal quality increases according to the modulation index. For M=1.1 the SHE-PWM gives the best result: a current THD of 5% for a switching frequency of 1150 Hz.

Keywords: Hybrid genetic algorithms
Modified Newton–Raphson method
Selective harmonic elimination
SHEPWM
Three-phase inverter

1. INTRODUCTION
Selective harmonic elimination (SHE) pulse width modulation (SHEPWM) was first reported in 1964 [1, 2]. This modulation technique is very efficient and is used for controlling two-level inverters to improve the output voltage quality. The technique involves generating the inverter output wave in a succession of variable-width slots. This wave is characterized by the number of slots or pulses per alternation [3, 4]. The switching angles are determined such that the most troublesome harmonics, which are undesirable for the operation of loads, such as electric motors, are eliminated [5]. A set of nonlinear equations is simultaneously solved to determine the switching times of power switches. The solution of the aforementioned equation set is the major challenge involved in the SHEPWM technique. The equations of the SHEPWM technique are presented in [6], [7]. SHE techniques have been the subject of intensive research over the last two decades. Because of their complex implementation, only a few SHE techniques have gained...
acceptance in the industry [8], [9]. Difficulties are faced when using analytical methods to solve the SHE problem because SHE techniques have a complex implementation and require a massive computational load. Therefore, increasing research focus has been directed towards non-traditional optimization methods, particularly those inspired by natural biology, for solving the SHE problem. Genetic algorithms (GAs) [10], ant colony optimization [11], and particle swarm optimization (PSO) have been used effectively to solve the SHE problem. The artificial neural network algorithm [12], [13], colonial competition algorithm [14], and bee algorithm [15] have been used to generate the switching angles in real time. The search algorithm can be improved effectively and efficiently by using hybrid GAs (HGAs), which are a combination of GAs and local search (LS) algorithms [16], [17]. The performance of the Newton–Raphson method is considerably improved when the GA is used to determine the initial value of the solution [18]. The PSO algorithm is efficiently used to determine the optimal switching angles for three-phase pulse width modulation (PWM) inverters [19]–[21]. Al-Hitmi et al. [22] introduced the ‘any initial random assumption’ approach was used to obtain an analytical solution for solving SHE equations by using the Newton–Raphson method. The contribution of this study is shown in the elimination of the maximum harmonics of the inverter using a new modified method of Newton-Raphson and GA.

2. MATERIALS AND METHOD

2.1. Photovoltaic inverter topology

Figure 1 illustrates the studied system. Figure 1 is a PV system's configuration with the proposed inverter. The system comprises PV array, charge controller, battery, the proposed inverter, and AC load [23], [24]. The basic diagram of the three-phase two-step inverter is displayed in Figure 2. The SHEPWM technique was used for fundamental control and the elimination of preselected harmonics [25].

![Figure 1. Configuration of the proposed inverter in a PV system application](image1)

![Figure 2. Three-phase two-level voltage source inverter](image2)

2.2. Calculation method

SHEPWM control involves calculating in advance the inverter switching angles, which are stored in a memory to control the semiconductors. Odd harmonics that are multiples of three deserve particular attention. Harmonics 5, 7, 11... (and not harmonics 3, 9, 15...), are often eliminated because harmonics of rank h=3k in a three-phase system coincide in phase and time. Therefore, for delta coupling in a balanced regime, the compound voltages V12(t), V23(t), and V31(t) do not contain odd harmonics that are multiples of three even if the single voltages contain them.
The calculated modulation is characterized by $k$ electrical angles, which are denoted as $\alpha_1, \alpha_2, ... \alpha_k$. These angles allow (1) the cancellation of $k$ harmonics or (2) the cancellation of $k-1$ harmonics and the setting of the fundamental voltage amplitude [22], [26]. Usually, a wave that is symmetrical with respect to the quarter period is used, and the other angles are deduced according to the symmetry. Figure 3 illustrates the existence of odd-order harmonics after the decomposition of a Fourier series of a PWM signal that is symmetrical with respect to the quarter period and antisymmetric with respect to the half period [27]–[29].

The switching angles $\alpha_1, \alpha_2, ... \alpha_{11}$, and $\alpha_{11}$ allow us to eliminate the harmonics and control the fundamental [30], [31]. The line-to-neutral output voltage is shown in Figure 3. As shown in Figure 3, eleven and seven notch angles are created on voltage waveform and it has also quarter symmetry and odd symmetry. The voltage waveform shown in Figure 3 can be stated with regards to the Fourier series coefficient $b_n$ and eleven and seven notch angles as follows. The Fourier transform of a waveform that is periodic and symmetrical with respect to an odd quarter-wave is given as [14]:

$$v(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos(n \omega t) + b_n \sin(n \omega t)]$$

(1)

the coefficients $a_n$ and $b_n$ are given by (2) and (3) [14]:

$$a_n = \frac{1}{T} \int_{0}^{T} u(t) \cos(n \omega t) dt$$

(2)

$$b_n = \frac{1}{T} \int_{0}^{T} u(t) \sin(n \omega t) dt$$

(3)

for a periodic signal with quarter-period symmetry and half-period antisymmetric, the (4) is obtained:

$$a_0 = 0, a_n = 0$$

$$b_n = \frac{4}{\pi} \delta_{n/2} \int_{0}^{T/2} u(t) \sin(n \omega t) dt$$

(4)

we set $N=11$, and the SHEPWM technique was used to control the fundamental and eliminate 10 harmonics (the $5^{th}$, $7^{th}$, $11^{th}$, $13^{th}$, $17^{th}$, $19^{th}$, $23^{rd}$, $25^{th}$, $29^{th}$, and $31^{st}$ order harmonics). The line-neutral output voltage in Figure 3 can be expressed in terms of the series Fourier coefficient $b_n$ and 11 switching angles [14], [32].

$$b_n = \frac{4}{n \pi} \left[ -1 - 2 \sum_{k=1}^{k} (-1)^k \cos(n \alpha_k) \right]$$

(5)

The (6) and (7) are used to determine $\alpha_1, \alpha_2, ... \alpha_{11}$ with the elimination of the $5^{th}$, $7^{th}$, $11^{th}$, $13^{th}$, $17^{th}$, $19^{th}$, $23^{rd}$, $25^{th}$, $29^{th}$, and $31^{st}$ order harmonics:

$$b(1) = \frac{4}{\pi} \left[ 1 - 2 \cos(\alpha 1) + 2 \cos(\alpha 2) - ... - 2 \cos(\alpha_k) \right] - M$$

$$b(5) = \frac{4}{5 \pi} \left[ 1 - 2 \cos(5 \alpha 1) + 2 \cos(5 \alpha 2) - ... - 2 \cos(5 \alpha_k) \right]$$

$$b(7) = \frac{4}{7 \pi} \left[ 1 - 2 \cos(7 \alpha 1) + 2 \cos(7 \alpha 2) - ... - 2 \cos(7 \alpha_k) \right]$$

$$b(11) = \frac{4}{11 \pi} \left[ 1 - 2 \cos(11 \alpha 1) + 2 \cos(11 \alpha 2) - ... - 2 \cos(11 \alpha_k) \right]$$

$$b(13) = -\frac{4}{13 \pi} \left[ 1 - 2 \cos(13 \alpha 1) + 2 \cos(13 \alpha 2) - ... - 2 \cos(13 \alpha_k) \right]$$

$$b(31) = \frac{4}{31 \pi} \left[ 1 - 2 \cos(31 \alpha 1) + 2 \cos(31 \alpha 2) - ... - 2 \cos(31 \alpha_k) \right]$$

(6)

where $M$ is the modulation index and the variables $\alpha_1$–$\alpha_{11}$ are the normalized amplitudes of the harmonics to be eliminated under a constraint.

$$0 < \alpha_1 < \alpha_2 < \alpha_3 < \alpha_4 < ... < \alpha_{11} < \frac{\pi}{2}$$

(7)

This aforementioned is repeated for various modulus indices $(M)$ ranging from 0.01 to 1.20.

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2.3. Optimization techniques

2.3.1. Hybrid genetic algorithms HGAs

Genetic algorithms are part of ‘Nonlinear Adaptive Networks’. These algorithms are inspired by the mechanisms of natural selection (Darwin) and the genetics of evolution. They allow the search for a global extremum. Because of their high parallel processing capacity, robustness, and global search capability, they are used to solve the problem of nonlinear functions [33].

A genetic algorithm evolves a set of solutions \((a_1, a_2, a_3, ..., a_k)\) called the population, which is randomly initialized, to efficiently guide the search towards good solutions in the research space [13]. GAs is characterized by the following parameters:

- Maximum number of generations (generation max)
- Population size \((n)\)
- Number of elites \((n_{elites})\)
- Type of selection (roulette or tournament selection)
- Type of crossover (simple, arithmetic, or heuristic)
- Type of mutation (e.g., uniform, nonuniform, or boundary)
- Probability of selection \((p_{selection})\)
- Probability of crossover \((pc)\)
- Probabilities of mutation \((pm_1\) and \(pm_2)\)
- Transfer rate \((\tau_{m1}\) and \(\tau_{m2})\)
- Function to be minimized

Minimization function

The fitness function makes it possible to minimize the low-order harmonics of a three-phase inverter \((5^{th}, 7^{th}, 11^{th}, ..., 31^{th}\) harmonics) for obtaining an optimal solution to eliminate prespecified harmonics [34]. The fitness function for five switching angles is given as follows. Up to the 19th-order harmonic should be eliminated for seven switching angles, and up to the 31st-order harmonic should be eliminated for 11 switching angles \([35]–[37]\).

HGAs are combinations of GAs and LS algorithms. They are used to eliminate the problem of developing LS in the GA [15]. HGAs can be used to determine 7 and 11 switching angles. The \((6)\) is optimized with the constraint of \((7)\) for different values of \(M\) by using the MATLAB GA-Toolbox. The optimization options, fitness function, number of variables, lower- and upper-bound constraints, and nonlinear stress function of the MATLAB GA-Toolbox are displayed in Figure 4.

The hybrid function is used after the GA to enhance the value of the fitness function. The final point determined by the GA is used by the hybrid function as the starting point. The \textit{fmincon} hybrid technique, which is based on the sequential quadratic programming algorithm, is used to determine a minimum of multivariate functions with nonlinear constraints.

A total of 7 and 11 switching angles can be determined with an error tolerance of \(10^{-9}\) by using the MATLAB GA-Toolbox. These angles are obtained for the following condition: \(0 =< Ma =< 1.15\). The Figure 5(a) and Figure 5(b) showing the switching angles for different modulation indices, 7 and 11 switching angles.
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Figure 4. MATLAB GA-Toolbox

Figure 5. Switching angles for various modulation indices (a) 7 switching angles and (b) 11 switching angles

Greatest Error
2.3.2. Modified Newton–Raphson method

The difference between the Newton-Raphson and the Modified Newton-Raphson method is the point at which the stiffness matrix is evaluated. The modified Newton-Raphson method generally requires more iterations, but each iteration is faster than the Newton-Raphson method. The modified Newton-Raphson process can sometimes still converge in cases where Newton-Raphson no longer converges. Small variations of both processes are possible by making the first prediction using the linear or previous stiffness and constructing the current stiffness matrix after the first prediction. In the case of unloading, it may be advantageous to restore a linear stiffness.

The modified Newton–Raphson method is initialized by determining the initial angles in the range of 0 to π/2, and the solution generally converges to zero for a given system of nonlinear equations. If a solution exists, it generally works for a large number of iterations. The Newton–Raphson algorithm is executed in a discrete number of points, usually with small steps of modulation index variation. The multiplicity of solutions in a particular range of the modulation index is due to its convergence to different sets of solutions in the vicinity of M. The step size can be further reduced to search for other solutions in a certain range of M.

The algorithm displayed in Figure 6 is executed repeatedly over the entire range of the modulation index (i.e., from 0 to 1.20) to find multiple solutions for PWM with guaranteed convergence. The algorithm automatically generates the initial estimated value until convergence. This value is then used to estimate the next initial value of m. The solutions for m and m + 0.001 are expected to be close to each other.

A total of 7 and 11 switching angles are determined with an error tolerance of 10⁻¹⁵. These angles are obtained for the following condition: 0 ≤ M ≤ 1.15. The Figure 7(a) and Figure 7(b) showing the switching angles for different modulation indices, 7 and 11 switching angles.

![Figure 6. Flow chart of modified NR for solving SHE problem](image-url)
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The block diagram of the experimental assembly is shown in the Figure 8. The SHEPWM algorithm based on an HGA and a modified Newton–Raphson method was implemented in the Opal-RT Technologies OP5600 digital signal controller. The SEMITEACH B6U+E1CIF+B6CI low-loss intelligent molded IGBT module was used in the inverter circuit. The value of the dc-link capacitor was 233 uF/450 V. A three-phase induction motor was used as load for the inverter ($R=60 \ \Omega$ and $L=300 \ \text{mH}$). The output voltage of the inverter and the charging current were measured with an energy meter (FLUKE 435ii) [38]–[42].

Figure 9 displays the experimental test bench used for validating the developed control method. The design methodology implements MATLAB/Simulink and MATLAB/SimPowerSystem as well as an RT-LAB interface to allow access to control system variables. The experimental results obtained for seven switching angles are displayed in Figures 10-13 and those obtained for 11 switching angles are displayed in Figures 14-18. The operating frequency was kept constant at 50 Hz during the validation test. The results for the inverter load voltage and current are displayed in Figure 10(a) to Figure 13(a) and Figure 15(a) to Figure 18(a), respectively. The output current waveform was analyzed by using the fast Fourier transform at the 49th harmonic, as displayed in Figure 10(b) to Figure 13(b) and Figure 15(b) to Figure 18(b). The voltage harmonic spectra obtained for 11 and 7 switching angles are displayed in Figure 15(c) to Figure 18(c) and Figure 10(c) to Figure 13(c), respectively. The 5th–19th-order voltage harmonics were eliminated from the harmonic spectrum for seven switching angles, whereas the 5th–31st voltage harmonics were eliminated for
11 switching angles. Triple harmonics were eliminated due to the connection of the three-phase system. The Figure 14 and Figure 19 showing the comparative study between voltage and current distortion for 7 and 11 switching angles respectively (simulation and experimental).

a. Use of seven angles

For each modulation index $M$, we measure the harmonic distortion rate (current and voltage) using an energy meter (FLUKE 435ii). The evolution of the voltage and current was visualized using the oscilloscope. A comparative study by simulation and experimentally between the distortion of the voltage and the current was made for each modulation index.

b. Use of 11 angles

For each modulation index $M$, we measure the harmonic distortion rate (current and voltage) using an energy meter (FLUKE 435ii). A comparative study by simulation and experimentally between the distortion of the voltage and the current was visualized using the oscilloscope. The experimental results indicated that the number of eliminated harmonics increased with the number of angles. Moreover, the signal quality increased as a function of the modulation index. A superior SHEPWM result was obtained when $M=1.1$ where the current THD considerably decreased. The switching frequency ($f_s$) was 1150 Hz for SHEPWM 11 angles in 50 Hz. The THD of the current wave was 5.0% when $M=1.1$, and $f_s=750$ Hz for seven angles in 50 Hz, the THD of the current is increased according to the low frequency to 5.2%. The advantage of this technique is that the switching frequency is reduced and therefore the switching losses in the semiconductor components decrease.
Figure 10. Experimental results for $Ma=0.7$, $f=50$ Hz, (a) output voltage and load current, (b) harmonic spectrum of voltage, and (c) harmonic spectrum of current

Figure 11. Experimental results for $Ma=0.85$, $f=50$ Hz, (a) output voltage and load current, (b) harmonic spectrum of voltage, and (c) harmonic spectrum of current
Figure 12. Experimental results for $Ma=0.9$, $f=50$ Hz, (a) output voltage and load current, (b) harmonic spectrum of voltage, and (c) harmonic spectrum of current.

Figure 13. Experimental results for $Ma=1.1$, $f=50$ Hz, (a) output voltage and load current, (b) harmonic spectrum of voltage, (c) harmonic spectrum of current.
Two techniques are used to calculate the switching angles (modified Newton Raphson and hybrid genetic algorithm). The first one allows effectively guiding the search towards the best solutions, in the search space, for different values of modulation index. On the other hand, the second allows minimizing the harmonics expressed by an objective function, in order to obtain an optimal solution after a few iterations. This technique is characterized by a high capacity of parallel processing, great robustness and a global research capacity. These algorithms have the advantage of finding the best solution, even if we take the first generation at random. The calculation will then quickly converge on the right solution. The modified Newton-Raphson approach is used without assuming an initial switching angle, to obtain an analytical solution of SHE equations and ensure rapid convergence. Despite the short computation time for the HGA method, the modified Newton-Raphson method is still the best, due to its computation precision "very low error", The error values are 1e-15 and 1.5e-7 in case of modified Newton-Raphson and hybrid genetic algorithm respectively.
Figure 16. Experimental results for $M_a=0.7$, $f=50$ Hz, (a) output voltage and load current, (b) harmonic spectrum of voltage, (c) harmonic spectrum of current.

Figure 17. Experimental results for $M_a=0.9$, $f=50$ Hz, (a) output voltage and load current, (b) harmonic spectrum of voltage, (c) harmonic spectrum of current.
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3. CONCLUSION

In this paper, two approaches are proposed to obtain multiple sets of solutions for a wide range of modulation indices. HGAs were used to evaluate and adjust the switching angles. During the optimization process, the possible solutions were evaluated through a function involving four performance criteria based on the response of the global system. The modified Newton–Raphson approach was used without assuming an initial switching angle to obtain an analytical solution for solving the SHE equations and ensuring rapid convergence. In addition, multiple solutions were obtained while maintaining a very small increase in the modulation index. Such results have not been obtained in previously used calculation methods for
modulation indices. The experimental results confirm that the THD is low at high modulation indices but is very high at very low modulation indices.

Finally, the proposed method offers remarkable improvements on the inverter voltage output energy quality by calculating commutation angles in a range of 0 to π/2 and the simulated results are highly applicable in practice. The computation time of the Modified Newton Raphson method is low compared to the Hybrid Genetic Algorithm because this method automatically generates the estimated initial value until convergence occurs. Once found, this value serves as the initial estimated value for the next value of m. Indeed, we postulate that for the solutions m and m+0.01 which must not be very far apart. Accuracy is 1e-15.

REFERENCES


**Biographies of Authors**

Imen Souhila Bousmaha was born in Brahami, obtained her state engineer diploma in electrical networks in 2009, then her master's degree in 2011 at the Djillali Liabes University of Sidi Bel Abbès Algeria. She has held a doctorate electrical engineering since 2018. She worked as an engineer at Sonelgaz, an electricity and gas distribution company from 2009 to 2018. Since 2018, she has been a lecturer at the Higher School in Applied Sciences of Temcen. She can be contacted at email: imensousmaha@yahoo.fr.
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