

# NARMAX approach for micro positioning stage piezoelectric actuator hysteresis identification

Ounissi Amor<sup>1</sup>, Azeddine Kaddouri<sup>2</sup>, Rachid Abdessemed<sup>1</sup>

<sup>1</sup>Department of Electrical Engineering, Faculty of Technology, University Batna 2, Batna, Algeria

<sup>2</sup>Department of Electrical Engineering, Faculty of Engineering, University of Moncton, Moncton, Canada

---

## Article Info

### Article history:

Received Oct 7, 2022

Revised Dec 15, 2022

Accepted Dec 28, 2022

---

### Keywords:

Hysteresis

Identification

Mechanism PEA

NARMAX approach

Piezo-positioning

---

## ABSTRACT

The structure of the model may either be inferred via an experimental study or just by looking at the input and output data. A novel nonlinear autoregressive with exogenous inputs (NARMAX) method for identifying PEA piezoelectric positioning mechanisms is put forward in this study. The developed model enables accurate prediction of the hysteresis of the PEAs. The accuracy of the model built from the input and output data will be assessed by comparison with a LuGre model. The results of the identification show that the recommended approach is successful and that it has a high degree of identification precision within an absolute error range of one micron. The findings demonstrated the potential of the suggested method for classifying nonlinear PEAs.

*This is an open access article under the [CC BY-SA](https://creativecommons.org/licenses/by-sa/4.0/) license.*



---

## Corresponding Author:

Ounissi Amor

Department of Electrical Engineering, Faculty of Technology, University of Batna 2

Route de Constantine. Fésdis, Batna 05078, Algeria

Email: [ounissi\\_omar@yahoo.fr](mailto:ounissi_omar@yahoo.fr)

---

## 1. INTRODUCTION

Piezoelectric actuators (PEA) powered micro/nano manipulators are useful elements in ultra-high precision positioning applications where nanoscale resolution in displacement, high stiffness, and quick reaction are required. However, the fundamental disadvantage of these actuators is the nonlinearity of the hysteresis between the excitement signal and the structure's reaction; additional reasons of loss of positioning accuracy are drift owing to creep effects and temperature effects [1]–[3]. The phenomena of friction constitute the operating principal of PEA, these models of friction must integrate as well as possible these phenomena (stiction, rising, viscous friction, elastic friction, effects of Strobeck, fractional memory, non-derivative) [4]–[6]. The number and nature of the parameters that compose them and that must therefore be identified, the precision of description of the real phenomena of friction, the qualities in simulation are as much criteria of differentiating and which must be taken into account during the selection [7], [8].

Thus, the identification of certain parameters proves to be very complex, either the method used is difficult to implement or is very sensitive to measurement errors for example, the test procedure does not make it possible to demonstrate the phenomenon to be modeled [9]–[11]. The speed of PEA varied according to the applied load and the friction of the device. The control is recommended for applications requiring exact position and precise speed [12], [13]. However, due to the complex structure and the factors mentioned, it is impossible to establish an accurate physical model. The experimental modeling giving rise to a model of representation is very interesting in the measure or it makes it possible to obtain a representation of the system from the input/output data [14].

In this case, the problem of modeling becomes essentially an identification problem and the objective is to find the structure and the values of the parameters of a model capable of describing as precisely as possible the external behavior of the system without any consideration of the phenomenon internal [15]. In recent years, much effort has been devoted to modeling the mechanism positioning piezo actuator (PEA) using bilinear models [16], [17]. Many of these models have been evaluated by actual plant tests and are well established. However, the identification approach used by the maximum likelihood method and the prediction error require appropriately specified initial values of unknown parameters and system states [18], [19]. An appropriate choice leads to problems of convergence and singularity, which are very difficult to solve in real applications. There are several types of literature such as the Volterra model, the Weiner-Hammerstein cascade models [20], [21]. In the publications [22], [23], describes the input/output parametric models for nonlinear systems have developed a NARMAX model, which has been compared to that of Volterra and affine in the state. Among the existing nonlinear system identification approach the NARMAX model is widely applied in the modeling of many engineering chemical, [24], [25] biological, medical [26], [27], geographic [28], [29], and economic systems [30], [31].

The NARMAX models are well suited to model design for nonlinear process control, where a relatively simple structure system is often required, in order to be able to perform many calculations, and adjust model parameters, if necessary, by the experimental data sets new. This article describes the dynamic behavior so the interrelations between the controlled variables translates into the difficulty of defining correct values of the parameters. To reduce (avoid) complexity, one solution is to limit the number of terms, it is often necessary to identify the input-output model.

## 2. HYSTERESIS MODELING

The LuGre model is the most widely used to model systems with hysteresis. In this model, the structure is obtained from physical laws. the LuGre model is a model based on the microscopic approach of the modeling of the contact surfaces by the blades. The dynamic model of the entire micro-positioning system with hysteresis includes velocity-dependent friction phenomena such as variable breaking force and friction lag can be established as (1) [32].

$$M\ddot{x} + kx + c.\dot{x} + F(x, \dot{x}) = F_e(x, \dot{x}) \quad (1)$$

Where  $M$  denotes the equivalent mass of the controlled piezo-positioning mechanism;  $x$  is the displacement of the mechanism;  $\ddot{x}$  denotes the second-order derivative of the state with respect where  $k$  and  $c$  coefficient of stiffness and viscous damping respectively. The hysteresis force given by the (2) essentially causing the hysteresis effect in the motion dynamics of the PEA positioning mechanism is given by (2).

$$F(x, \dot{x}) = \sigma_0 Z + \sigma_1 \frac{dz}{dt} + \sigma_2 \dot{x} \quad (2)$$

Where  $\sigma_0$  the bristle stiffness parameter,  $\sigma_2$  is the viscous friction coefficient,  $\sigma_1$  is the bristle damping and  $\frac{dz}{dt}$  the average bristle deflexion is given by (3).

$$\frac{dz}{dt} = x - \frac{\sigma_0}{g(\dot{x})} Z |\dot{x}| \quad (3)$$

The stibek function,  $g(v)$  is a decreasing function for inceasing velocity bounded by upper limit equal to statc function force and a lower limit equals to the coulomb friction force, and  $\dot{x}$ . The stribeck velocity:

$$g(\dot{x}) = F_c + (F_s - F_c)e^{-\left(\frac{x}{\dot{x}}\right)^2} \quad (4)$$

$F_e(x, \dot{x})$ : represents the external force excitation.

## 3. NARMAX MODEL FOR HYSTERESIS

The parametric identification problem consists in determining the numerical values of the parameters of a model of the system under consideration. The choice of the input/output variables of a system is a preliminary phase that makes it possible to reveal the cause and effect relationships between these variables. The identification is an experimental approach, it is used to look for a mathematical model. This model aims

to correctly predict the dynamic behavior of the real system. The identification of a system is then an iterative procedure that includes several stages, Figure 1, or the choice must be questioned during a failure of the validation of the identified model. In this work, we have been particularly interested in input-output models. Indeed, Leontaritis and Billings have shown that possible to represent a large class of non linear noisy systems in time by the NARMAX model [33].

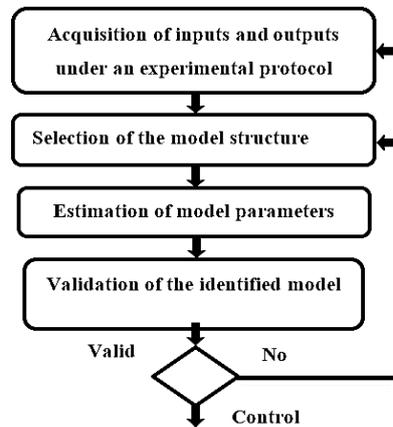


Figure 1. Flowchart of the nonlinear identification algorithm

Its most general expression is (5).

$$x(k) = a_0 + \sum_{i=1}^{N_1} a_i x(k - i) + \sum_{i=1}^{N_2} b_i u(k - i) + \sum_{i=1}^{N_3} c_i e(k - i) + e(k) \tag{5}$$

Where  $x(k)$  and  $u(k)$  are respectively the output and input signals;  $e(k)$  accounts for uncertainties and possible noise;  $a_i x(k - i)$  is the variable auto-regressive (AR);  $b_i u(k - i)$  is the variable input (exogenous);  $c_i e(k - i)$  represents the variable moving average (MA) supposed white noise model variable,  $i$  is the dimension of the vector,  $N_1, N_2, N_3$  represent the dimension of the AR (system output) variable of the  $i^{\text{th}}$  input, respectively, the size of the input vector and the parameters of the model to be estimated. The resilience force used to identify the mechanism positioning piezo actuator PEA is described by the following [34]:

$$F_r(x, \dot{x}) = M\ddot{x} + kx + c.\dot{x} + F(x, \dot{x}) = \alpha_1 x(k + x_0) + \alpha_3 x(k + x_0)^3 + \alpha_5 x(k + x_0)^5 + \lambda_1 \dot{x}(k) + \lambda_3 \dot{x}^3(k) + \lambda_9 \dot{x}^9(k) + z(t) \tag{6}$$

The expression of the hysteretic variable  $z$  is expressed by (7).

$$\dot{z}(t) = \frac{\alpha_s}{2} [1 + \text{sgn}(z_s - |z(t)|)] \dot{x}(t) \tag{7}$$

Where  $\alpha_s = \frac{z_s}{x_s}$ .  $\alpha_s$  is the mechanical stiffness,  $z_s$  is not measurable, it expresses the restoring force memorized during sliding between threads, when sliding between wires happens, is the memorized restoring force, and the elastic deformation limit is denoted by  $x_s$ , and the  $\text{sgn}$  function is defined as (8).

$$\text{sgn}(\cdot) = \begin{cases} 1 & \text{if } x > 0 \\ -1 & \text{if } x \leq 0 \end{cases} \tag{8}$$

In the static displacement produced by the preload  $x_0$ ,  $F_r(x, \dot{x})$  is the resilience force. In the NARMAX approach, the data of  $(x(k), \dot{x}(k), F_{ex})$  and are used, such that the output  $y(k)$  is the hysteresis resilience force and their derivative and the excitation force resilience signal. The model (6) has a clear physical meaning, or the first three elements are the spring forces and the last element is the damping force: there are six indeterminate parameters to identify. The system is constructed only from input and output data and it is not difficult to identify all the parameters with great precision.

#### 4. DISCUSSION RESULTS

We identify the hysteresis property of the piezoelectric actuator using the algorithms of (6) and the parameters of the suggested technique. To illustrate the performance of the identification technique, the feasibility and efficacy of this research on a piezoelectric stage are offered. These findings, however, are obtained by using a sinusoidal input signal. Based on the relationship between the excitation of the input force and the displacement of the output displacement. Figure 2 compares the response of the force restoring and displacement in the NARMAX method to that of the LuGre model. Figure 3 shows a comparison of the displacement caused by the recognized model and the displacement caused by the LuGre model in pursuit. According to the results, the proposed identification method has the ability to minimize absolute error to the order of 458.6 nanometers.

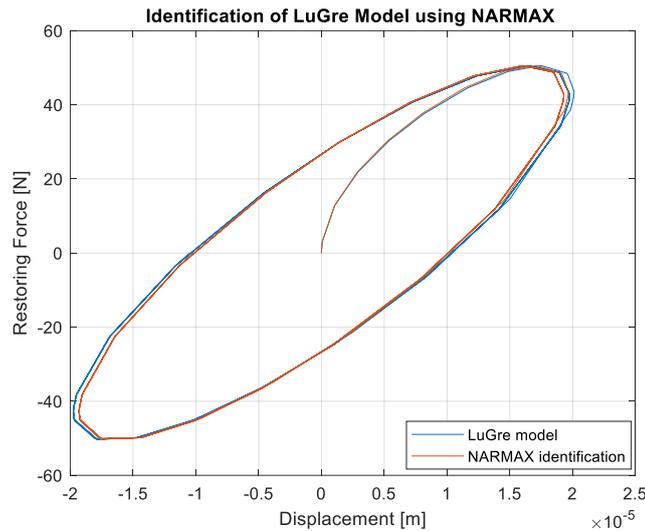


Figure 2. Hysteresis cycle obtained by NARMAX and LuGre model

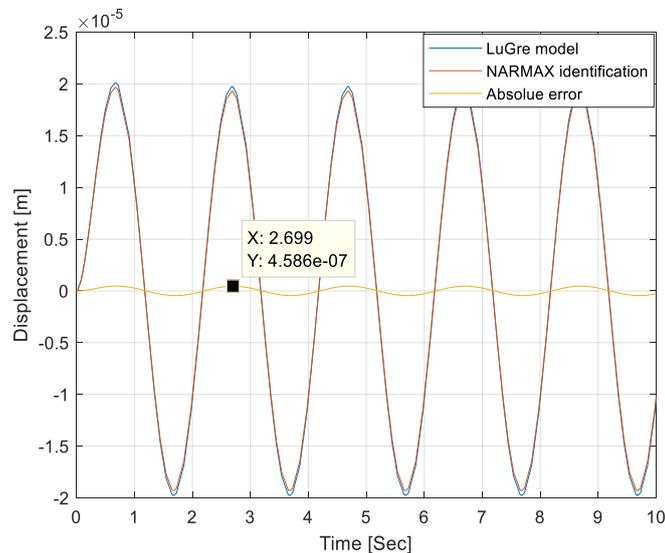


Figure 3. Response displacement with NARMAX and LuGre model

#### 5. CONCLUSION

The application of a NARMAX technique for identifying the mechanism positioning piezo actuator stage PEA will be examined in this research. The process of identifying a model's parameters may be phrased as an optimization problem with the goal of minimizing the prediction error between the observed PEA outputs

and the model outputs. The analysis and control of these actuators are made easier by the behavior of the findings, which demonstrated the behavior of restoring force-displacement hysteresis in the PEA.

## REFERENCES

- [1] M. S. Rana, H. R. Pota, and I. R. Petersen, "Creep, hysteresis, and vibration effects attenuation in an AFM PTS," *2014 European Control Conference, ECC 2014*, pp. 2022–2027, 2014, doi: 10.1109/ECC.2014.6862417.
- [2] E. Ayvali and J. P. Desai, "Pulse width modulation-based temperature tracking for feedback control of a shape memory alloy actuator," *Journal of Intelligent Material Systems and Structures*, vol. 25, no. 6, pp. 720–730, 2014, doi: 10.1177/1045389X13502576.
- [3] D. V. Sabarianand, P. Karthikeyan, and T. Muthuramalingam, "A review on control strategies for compensation of hysteresis and creep on piezoelectric actuators based micro systems," *Mechanical Systems and Signal Processing*, vol. 140, 2020, doi: 10.1016/j.ymsp.2020.106634.
- [4] F. Marques, P. Flores, J. C. P. Claro, and H. M. Lankarani, "Modeling and analysis of friction including rolling effects in multibody dynamics: a review," *Multibody System Dynamics*, vol. 45, no. 2, pp. 223–244, 2019, doi: 10.1007/s11044-018-09640-6.
- [5] M. Wojtyra, "Modeling of static friction in closed-loop kinematic chains—Uniqueness and parametric sensitivity problems," *Multibody System Dynamics*, vol. 39, no. 4, pp. 337–361, 2017, doi: 10.1007/s11044-016-9535-6.
- [6] Y. F. Liu *et al.*, "Modeling and control of piezoelectric inertia-friction actuators: Review and future research directions," *Mechanical Sciences*, vol. 6, no. 2, pp. 95–107, 2015, doi: 10.5194/ms-6-95-2015.
- [7] Z. M. Zhang, Q. An, J. W. Li, and W. J. Zhang, "Piezoelectric friction–inertia actuator—a critical review and future perspective," *The International Journal of Advanced Manufacturing Technology*, vol. 62, no. 5–8, pp. 669–685, Sep. 2012, doi: 10.1007/s00170-011-3827-z.
- [8] M. Hunstig, "Piezoelectric inertia motors—a critical review of history, concepts, design, applications, and perspectives," *Actuators*, vol. 6, no. 1, 2017, doi: 10.3390/act6010007.
- [9] J. M. Rodríguez-Fortun, J. Orus, F. Buil, and J. A. Castellanos, "General Bond Graph model for piezoelectric actuators and methodology for experimental identification," *Mechatronics*, vol. 20, no. 2, pp. 303–314, 2010, doi: 10.1016/j.mechatronics.2010.01.004.
- [10] G. Y. Gu, L. M. Zhu, C. Y. Su, H. Ding, and S. Fatikow, "Modeling and control of piezo-actuated nanopositioning stages: A survey," *IEEE Transactions on Automation Science and Engineering*, vol. 13, no. 1, pp. 313–332, 2016, doi: 10.1109/TASE.2014.2352364.
- [11] R. K. Jain, S. Majumder, B. Ghosh, and S. Saha, "Design and manufacturing of mobile micro manipulation system with a compliant piezoelectric actuator based micro gripper," *Journal of Manufacturing Systems*, vol. 35, pp. 76–91, 2015, doi: 10.1016/j.jmsy.2014.12.001.
- [12] P. R. Ouyang, R. C. Tjioptoprodo, W. J. Zhang, and G. S. Yang, "Micro-motion devices technology: The state of arts review," *International Journal of Advanced Manufacturing Technology*, vol. 38, no. 5–6, pp. 463–478, 2008, doi: 10.1007/s00170-007-1109-6.
- [13] S. Mohith, A. R. Upadhya, K. P. Navin, S. M. Kulkarni, and M. Rao, "Recent trends in piezoelectric actuators for precision motion and their applications: a review," *Smart Materials and Structures*, vol. 30, no. 1, 2021, doi: 10.1088/1361-665X/abc6b9.
- [14] L. Wang, Y. Ji, H. Yang, and L. Xu, "Decomposition-based multi-innovation gradient identification algorithms for a special bilinear system based on its input-output representation," *International Journal of Robust and Nonlinear Control*, vol. 30, no. 9, pp. 3607–3623, 2020, doi: 10.1002/rnc.4959.
- [15] L. Tian, Z. Xiong, J. Wu, and H. Ding, "A new rate-dependent model for high-frequency tracking performance enhancement of piezoactuator system," *Smart Materials and Structures*, vol. 26, no. 5, 2017, doi: 10.1088/1361-665X/aa672a.
- [16] H. Ji and Y. Wen, "Study on bilinear interpolation Preisach model of piezoelectric actuator," *Advanced Materials Research*, vol. 443–444, pp. 437–441, 2012, doi: 10.4028/www.scientific.net/AMR.443-444.437.
- [17] Q. Xu and K. K. Tan, "Advanced Control of Piezoelectric Micro-/Nano-Positioning Systems," *Advanced Control of Piezoelectric Micro-/Nano-Positioning Systems*, pp. 1–257, 2015, doi: 10.1007/978-3-319-21623-2.
- [18] S. N. Wood, N. Pya, and B. Säfken, "Smoothing Parameter and Model Selection for General Smooth Models," *Journal of the American Statistical Association*, vol. 111, no. 516, pp. 1548–1563, Oct. 2016, doi: 10.1080/01621459.2016.1180986.
- [19] A. Pajor, "Estimating the marginal likelihood using the arithmetic mean identity," *Bayesian Analysis*, vol. 12, no. 1, pp. 261–287, 2017, doi: 10.1214/16-ba1001.
- [20] M. Ławryńczuk, "Nonlinear predictive control of dynamic systems represented by Wiener–Hammerstein models," *Nonlinear Dynamics*, vol. 86, no. 2, pp. 1193–1214, 2016, doi: 10.1007/s11071-016-2957-0.
- [21] L. Janjanam, S. Kumar Saha, R. Kar, and D. Mandal, "Optimal design of cascaded Wiener-Hammerstein system using a heuristically supervised discrete Kalman filter with application on benchmark problems," *Expert Systems with Applications*, vol. 200, 2022, doi: 10.1016/j.eswa.2022.117065.
- [22] P. M. S. Burt and J. H. De Moraes Goulart, "Efficient Computation of Bilinear Approximations and Volterra Models of Nonlinear Systems," *IEEE Transactions on Signal Processing*, vol. 66, no. 3, pp. 804–816, 2018, doi: 10.1109/TSP.2017.2777391.
- [23] L. Aggoun and Y. Chetouani, "Fault detection strategy combining NARMAX model and Bhattacharyya distance for process monitoring," *Journal of the Franklin Institute*, vol. 358, no. 3, pp. 2212–2228, 2021, doi: 10.1016/j.jfranklin.2021.01.001.
- [24] C. M. Cheng, Z. K. Peng, W. M. Zhang, and G. Meng, "Volterra-series-based nonlinear system modeling and its engineering applications: A state-of-the-art review," *Mechanical Systems and Signal Processing*, vol. 87, pp. 340–364, 2017, doi: 10.1016/j.ymsp.2016.10.029.
- [25] X. C. Guan, D. Y. Zhao, and Q. M. Zhu, "NARMAX modelling and U-model control design for continuous stirred tank reactor (CSTR)," *Chinese Control Conference, CCC*, vol. 2016–August, pp. 1964–1969, 2016, doi: 10.1109/ChiCC.2016.7553655.
- [26] C. Tao, Y. Gong, H. Xu, and Z. Zhao, "Introduction: The International Conference on Intelligent Biology and Medicine (ICIBM) 2016: Special focus on medical informatics and big data," *BMC Medical Informatics and Decision Making*, vol. 17, 2017, doi: 10.1186/s12911-017-0462-0.
- [27] G. N. Beligiannis, L. V. Skarlas, S. D. Likothanassis, and K. G. Perdikouri, "Nonlinear model structure identification of complex biomedical data using a genetic-programming-based technique," *IEEE Transactions on Instrumentation and Measurement*, vol. 54, no. 6, pp. 2184–2190, 2005, doi: 10.1109/TIM.2005.858573.
- [28] R. J. Boynton, M. A. Balikhin, S. A. Billings, A. S. Sharma, and O. A. Amariutei, "Data derived narmax dst model," *Annales Geophysicae*, vol. 29, no. 6, pp. 965–971, 2011, doi: 10.5194/angeo-29-965-2011.

- [29] W. Lacerda, L. da Andrade, S. Oliveira, and S. Martins, "SysIdentPy: A Python package for System Identification using NARMAX models," *Journal of Open Source Software*, vol. 5, no. 54, p. 2384, Oct. 2020, doi: 10.21105/joss.02384.
- [30] L. A. Aguirre and A. Aguirre, "Analysis of Economic Time Series Using Narmax Polynomial Models," pp. 213–235, 2002, doi: 10.1007/978-1-4615-0931-8\_11.
- [31] A. Aguirre and L. A. Aguirre, "Time series analysis of monthly beef cattle prices with nonlinear autoregressive models," *Applied Economics*, vol. 32, no. 3, pp. 265–275, 2000, doi: 10.1080/000368400322697.
- [32] A. Ounissi, K. Yakoub, A. Kaddouri, and R. Abdessemed, "Robust adaptive displacement tracking control of a piezo-actuated stage," *2017 6th International Conference on Systems and Control, ICSC 2017*, pp. 297–302, 2017, doi: 10.1109/ICoSC.2017.7958695.
- [33] I. J. Leontaritis and S. A. Billings, "Input-output parametric models for non-linear systems Part I: Deterministic non-linear systems," *International Journal of Control*, vol. 41, no. 2, pp. 303–328, 1985, doi: 10.1080/0020718508961129.
- [34] S. A. Billings, *Nonlinear System Identification: NARMAX Methods in the Time, Frequency, and Spatio-Temporal Domains*. Chichester, UK: John Wiley & Sons, Ltd, 2013. doi: 10.1002/9781118535561.

## BIOGRAPHIES OF AUTHORS



**Ounissi Amor**    Received the B.Sc., the M.Sc. and the Ph.D. degrees in electrical engineering from Batna University in 1988, 2003, and 2013 respectively. Actually, he is an associate professor at Batna 2 University and an active member of L.E.B. research laboratory. Dr. Ounissi was a visiting professor at the university of Moncton, NB, Canada, several time. His current area of research includes piezoelectric actuators modeling and control. He can be contact at email: [ounissi\\_omar@yahoo.fr](mailto:ounissi_omar@yahoo.fr) or [amor.ounissi@univ-batna2.dz](mailto:amor.ounissi@univ-batna2.dz).



**Azeddine Kaddouri**    Received the B.Sc in electrical engineering from Batna University, Batna, Algeria in 1988 and the M. Sc. and the Ph.D. degrees in Electrical Engineering from Laval University, Quebec, QC, Canada, in 1993, and 2000 respectively. From 1993 to 1999, he was a Research Assistant with the Power Electronics and Industrial Control Research Group, École de technologie supérieure, Montreal, PQ, Canada. In 1999, he joined the University of Moncton, Moncton, NB, Canada, where he is currently a full Professor in the Electrical Engineering Department. Currently, he is a member of the Laboratoire de Robotique, Électronique et Industrie 4.0 at the University of Moncton. He can be contacted at email: [azeddine.kaddouri@umoncton.ca](mailto:azeddine.kaddouri@umoncton.ca).



**Rachid Abdessemed**    Received the M.Sc. in electrical engineering from Kiev Polytechnic Institute and Electrodynamics Research Institute in 1978 and the Ph.D. degree from Ukrainian Academy of Sciences in 1982. Currently, he is a full professor in University of Batna 2, Algeria and a director of the L.E.B. research laboratory in the same University. Pr. Abdessemed is an author of tens of research papers and several books. His research interests are the design and control of induction machines and converters, reliability, magnetic bearings, and renewable energy. He can be contacted at email: [rachid.abdessemed@gmail.com](mailto:rachid.abdessemed@gmail.com).