Adaptive control of nonlinear systems using neuro-fuzzy networks with B-spline functions

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ABSTRACT

This article explores the problem of adaptive control for nonlinear dynamic systems operating under uncertainty. It presents a model reference adaptive control (MRAC) method that integrates a neuro-fuzzy network with B-spline basis functions. The proposed approach allows effective approximation of nonlinear behaviors and ensures high control accuracy despite external disturbances and structural uncertainties within the system. The paper compares the performance of conventional linear MRAC with the neuro-fuzzy controller. Simulation results demonstrate that the neuro-fuzzy MRAC achieves superior stability and accuracy in closed-loop control. Additionally, the study examines the system's local stability under specific conditions of the learning rate. To address the challenge of computational complexity, a decomposition strategy dividing the controller into smaller sub-models is introduced, effectively mitigating the "curse of dimensionality." The findings support the applicability of neuro-fuzzy controllers for the intelligent control of a wide range of nonlinear systems.

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1. INTRODUCTION

Most industrial control systems operate under uncertainty, where complex nonlinear interactions between technological variables, parameter variations, external disturbances, and random noise complicate the application of linear adaptive methods. When the mathematical model of the plant is not known a priori, adaptive control techniques are typically applied [1], [2]. Classical adaptive methods usually assume that the order of the system is known and remains constant during operation [3], [4]. While such approaches have proven effective for linear plants and have therefore found widespread use, their performance is generally superior to fixed-parameter proportional integral derivative (PID) control due to improved adaptability [5], [6]. However, linear model reference adaptive control (MRAC) performs well only near an operating point where the plant can be approximated by a linear model. For highly nonlinear, nonminimum-phase, or disturbance-rich systems, the performance of linear MRAC degrades and nonlinear adaptive control becomes necessary [7].

Neural networks are capable of approximating nonlinear functions with arbitrary accuracy and therefore have been widely applied in adaptive control of nonlinear systems [8]. Indirect adaptive neural control strategies were introduced in [9], and direct neural MRAC schemes for nonlinear systems with structural uncertainties were developed in [10]. Neuro-fuzzy controllers further extend this concept: once trained for certain operating conditions, they enable smooth transitions between local models without requiring reconfiguration of parameters, unlike conventional self-tuning controllers [11]-[13].

The novelty of this study lies in the integration of B-spline basis functions into the MRAC framework, which allows for smooth nonlinear approximation with reduced computational complexity. In contrast to conventional fuzzy and RBF-based adaptive controllers, the proposed B-spline MRAC introduces a decomposition strategy that separates global adaptation dynamics from local spline parameter learning. This hierarchical design significantly improves convergence speed and stability while maintaining flexibility in handling nonlinearities.

This study focuses on the synthesis of a nonlinear MRAC implemented using a neuro-fuzzy network structure. The proposed controller operates as a direct self-tuning scheme in which the parameters of the neuro-fuzzy network are updated online, while local stability of the closed-loop system is ensured under appropriate learning conditions. Although classical MRAC techniques are well established and widely used for linear time-invariant systems, their effectiveness significantly deteriorates when applied to strongly nonlinear plants. In such cases, linear fuzzy controllers remain applicable only within a limited operating region, which restricts their practical use. Existing methods for nonlinear control, including gain scheduling and Takagi–Sugeno fuzzy controllers, rely on sets of local linear models, whereas neuro-fuzzy controllers generalize this concept by providing smooth transitions between locally valid models across the operating space [14]-[16]. From a practical perspective, the proposed control framework is well-suited for power electronic and electromechanical applications, such as inverter-based energy conversion systems, vector-controlled induction and permanent magnet motors, and servo drives operating under nonlinear load conditions. These applications are characterized by parameter variation, external disturbances, and uncertain dynamics, where the smooth local approximation properties of B-spline basis functions improve control stability and response quality [17].

2. METHOD

2.1. Overview of the proposition

Let the input-output model of discrete nonlinear dynamic systems be described as (1):

$$y(k+1) = a_0 y(k) + \cdots + a_{n_y-1} y\left(k - n_y + 1\right) + \phi\begin{bmatrix} u(k), u(k-1), \ldots, \\ u(k-m+1) \end{bmatrix} + \eta(k) \quad (1)$$

where, $u(k)$ and $y(k)$ – control and output, respectively; m and n – known orders of the system; $\eta(k)$ – sequence of independent identically distributed random variables with zero mean and variance Var (X); $\phi[\cdot]$ - smooth nonlinear function, expandable into a Taylor series. At the same time:

$$\frac{\partial\phi}{\partial u(k+1)} = 0; \frac{\partial\phi}{\partial u(k)} \neq 0. \quad (2)$$

From in (2), it is clear that the Jacobian $\phi[\cdot]$ exists. Rewriting in (1) gives the in (3).

$$A(z^{-1})y(k) = z^{-1}\phi[u(k), u(k-1), \ldots, u(k-m+1)] + \eta(k) \quad (3)$$

The local linearized model in (1) at the operating point $u(k)$ is defined as in (4):

$$\bar{A}(z^{-1})y(k) = z^{-1}\bar{B}(z^{-1})u(k) + \eta(k) \quad (4)$$

where, $\bar{A}(z^{-1})$ and $\bar{B}(z^{-1})$ are polynomials of the inverse shift operator z^{-1}, corresponding to orders n'_y and n'_u. The coefficients of these polynomials are functions of the current operating point $O(t)$ [18], [19]. This form of the model allows us to move on to (5).

$$\left.\begin{matrix} x_1(k) = y(k - n_y + 1) \\ \vdots \\ x_{n_y-1}(k) = y(k-1) \\ x_{n_y}(k) = y(k) \end{matrix}\right\} \quad (5)$$

With this in mind, in (1) can be rewritten in the form of a state space as (6).

$$X(k+1) = AX(k) + B\phi[u(k), u(k-1), \ldots, u(k-m+1)],\ y(k) = CX(k) \quad (6)$$

where, $X = \left[x_1, x_2, \dots, x_{n_y}\right]^T \in R^n$ – state vector, and $A = \begin{bmatrix} 0 & 1 & \cdots & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \square & \ddots & \square & \cdots \\ 0 & \cdots & \cdots & 0 & 1 \\ a_0 & a_1 & \cdots & \cdots & a_{n_y-1} \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \vdots \\ 1 \end{bmatrix}$, $C = [0\ 0\ \cdots\ \cdots\ 1]$.

A schematic diagram of the network with fuzzy logic is shown in Figure 1.

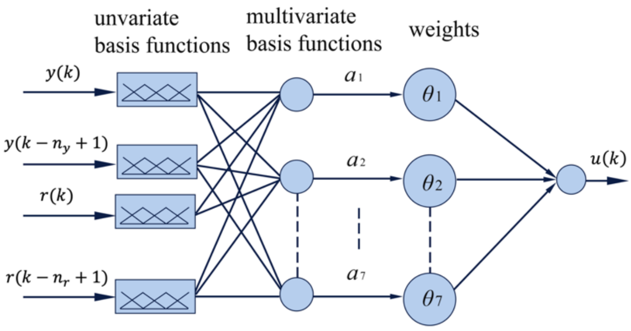

Figure 1. B-spline neuro-fuzzy network

The network input consists of a reference signal $\left[y(k), \dots, y\left(k-n_y+1\right), r(k-n_r+1)\right]$, which is blurred using one-dimensional B-spline basis functions. The network output is a linear combination of the fuzzyfied input. The development of a neurofuzzy B-spline network involves specifying the order of the one-dimensional basis functions v, the range of input and output variables, and the number of internal nodes w, or simply the number of basis functions that will be used in the fuzzifying process. The weights of neuro-fuzzy networks can be determined based on experimental data using the gradient descent method. The advantage of using B-spline functions is that the output data of the B-spline function can be calculated using a recursive relation for the data v and w. In addition, they have the following properties [20], [21]:

i) B-spline functions are defined on an organic carrier, and the output data of the basis function is compact and positive compared to its carrier, i.e., $\mu_i^j(x) = 0, x \notin [\lambda_{j-1}, \lambda_j]$ and $\mu_i^j(x) > 0, x \in [\lambda_{j-1}, \lambda_j]$. Consequently, the B-spline neurophase network stores information locally and can be trained based on local data, and only a small number of basis functions participate in the calculation of the network output data.

ii) The basis functions form a partition of unity, giving that the sum of the output data of the basis functions is always equal to unity, i.e., $\sum_j \mu_i^j(x) \equiv 1, x \in [x_{min}, x_{max}]$.

iii) The B-spline basis functions belong to the continuity class $C^{k-2}(x_{min}, x_{max})$, which guarantees that the membership functions $\mu_i^j(x)$ and their derivatives up to the corresponding order exist and are continuous over the entire input domain. This property ensures smooth transitions between neighboring fuzzy regions and prevents discontinuities in the generated control signal.

iv) The output of the neuro-fuzzy network is bounded due to its representation as a weighted sum of locally valid linear self-tuning controllers designed for selected operating points. Owing to the overlapping support of adjacent fuzzy sets, the resulting control law can be interpreted as a nonlinear self-tuning controller that produces smooth and well-behaved control actions across the operating range.

The neuro-fuzzy structure depicted in Figure 1 is constructed using a set of first-order fuzzy inference rules based on linguistic descriptions of the system output and reference signal. Each rule defines a local control contribution corresponding to a specific region of the input space. The overall control signal is obtained by aggregating the outputs of all activated rules and can be compactly expressed as (7).

$$u(t) = a^T(x)\theta \tag{7}$$

Where, $x(k)$ is the input vector:

$$x(k) = y\left[(k), \dots, y\left(k-n_y+1\right), r(k), \dots, r(k-n_r+1)\right] \tag{8}$$

where, $\theta = [\theta_1 \dots \theta_2 \dots \theta_p]^T$ – network weight vector, and p – denotes the total number of adjustable weights in the neuro-fuzzy network. For a given order of the B-spline basis functions v_i, and the corresponding number of internal nodes w_i, the total number of parameters is determined as (9).

$$p = \prod_{i=1}^{n}(w_i + v_i) \tag{9}$$

A multidimensional basis function is a transformed input vector obtained by tensor products of the output data of a one-dimensional basis spline B-function $\mu_{A_l^i}(x_l(k))$, i.e.

$$a_i(x) = \prod_{i=1}^{n} \mu_{A_l^i}(x_l(k)),\ i = 1,2, \dots, p \tag{10}$$

Where, $n = n_y + n_r$ – размерность входного вектора $x(k)$. According to (10), all the main properties of one-dimensional B-spline basis functions extend to multidimensional basis functions obtained by the tensor product. As follows from (7), the control $u(k)$, generated by the neuro-fuzzy network can also be represented as (11).

$$u(k) = \sum_{i=1}^{p} \theta_i a_i(x) = \sum_{i=1}^{p} \theta_i \prod_{l=1}^{n} \mu_{A_l^i}(x_l(k)) \tag{11}$$

2.2. Training a neuro-fuzzy network

Training of the neuro-fuzzy network is performed through online adaptation of the parameters associated with the fuzzy inference rules. In this process, the network weights are updated iteratively according to the following update law (12).

$$\theta_i(k+1) = \theta_i(k) - g\frac{\partial J}{\partial \theta_i} = \theta_i(k) - ge\frac{\partial e}{\partial \theta_i},\ i = 1,2, \dots, p \tag{12}$$

Where,

$$\frac{\partial e}{\partial \theta_i} = \frac{\partial y}{\partial \theta_i} = \frac{\partial y}{\partial u}\frac{\partial u}{\partial \theta_i} = \frac{\partial y}{\partial u} a_i(x) \tag{13}$$

Owing to the local support and compactness of the B-spline basis functions, the input space of the neuro-fuzzy network can be partitioned into q distinct regions, as defined in [22], where:

$$q = (w_y + 1)^{n_y}(w_r + 1)^{n_r} \tag{14}$$

where, w_y и w_r – the number of internal nodes for $y(k)$ and $r(k)$, respectively.

For each value of $x(k)$, only one region is activated in which the fuzzy input elements take nonzero values; in all other regions, they are equal to zero. The number of nonzero elements in the input vector $x(k)$ is determined as (15).

$$p' = v_y^{n_y} v_r^{n_r} \tag{15}$$

Where, v_y and v_r are the order of basis functions for $y(k)$ and $r(k)$, respectively.

2.3. Properties of a multidimensional neuro-fuzzy controller

The proposed neuro-fuzzy controller is composed of a static nonlinear mapping that preserves the topology of the input space and an adaptive linear component whose tuning mechanism is similar to that used in neural networks, thereby providing a consistent connection between fuzzy logic–based and classical control frameworks. Conventional fuzzy controllers are most often realized in a two-dimensional form, where the control action is generated using the tracking error and its variation, reflecting heuristic decision-making principles commonly applied by human operators to compensate for deviations from the reference value [23]. From a structural viewpoint, a two-dimensional fuzzy controller with linear inference rules can be represented as the superposition of a global multilevel relay and a local nonlinear PI controller, whereas an extension to three dimensions yields an equivalent structure combining a global relay mechanism with a local nonlinear PID controller [24], [25]. When normally distributed membership functions are employed, the resulting control law may be interpreted as a mixture of linear and nonlinear components, with the nonlinear contribution gradually approaching linear behavior as the number of fuzzy sets increases [26], [27]. Under these conditions, neuro-fuzzy controllers can be viewed as nonlinear self-tuning systems in which, for a sufficiently large set of linear fuzzy rules, the defuzzified control signal converges to a linear function of the controller inputs, including multi-input configurations [28], [29]. Because nonlinear dynamics are inherent in both the controlled plant and the neuro-fuzzy controller, guaranteeing global stability of the closed-loop system remains challenging. Nevertheless, by imposing appropriate constraints on the adaptation process, it is possible to ensure local stability in the vicinity of an equilibrium point. Based on this assumption, the state-space representation given in (6) can be reformulated as (16).

$$(k+1) = AX(k) + B\phi\begin{bmatrix} \sum_{i=1}^{p}\theta_i\prod_{l=1}^{n}\mu_{A_l^i}\big(x_l(k)\big), \sum_{i=1}^{p}{}^{(-1)}\theta_i \times \\ \times\prod_{l=1}^{n}\mu_{A_l^i}\big(x_l(k-1)\big),\dots,\sum_{i=1}^{p}{}^{-(m-1)}\theta_i \times \\ \times\prod_{l=1}^{n}\mu_{A_l^i}\big(x_l(k-m+1)\big)\end{bmatrix} \quad (16)$$

In this expression, the weights θ_i obtained in the previous update stages are denoted by the upper indices $(-1),\dots,-(m-1)$ to the left of it. Let the given point be $r=0$, and $\overline{X}=0$ be the equilibrium point of the system. Next, we select the Lyapunov function $V\{X(k)\}$, defined on the compact set S as (17).

$$V\{X(k)\} = \frac{1}{2}E^TE \quad (17)$$

Where, $E = X - \bar{X} = X$. Then in (18) and (19).

$$V\{X(k)\} = \frac{1}{2}X^TX = \frac{1}{2}\{y(k-n+1)^2 + \dots + y(k)^2\} \quad (18)$$

$$V\{X(k+1)\} = \frac{1}{2}\{y(k-n+2)^2 + \dots + y(k+1)^2\} \geq 0 \quad (19)$$

For a sufficiently small $\Delta\theta_i$, i.e., $|\Delta\theta_i| \leq d, \forall d \geq 0$, the change in the Lyapunov function is given by the expression:

$$V\{X(k+1)\} - V\{X(k)\} \cong \sum_i \frac{\partial V\{X(k+1)\}}{\partial\theta_i}\Delta\theta_i = y(k+1)\cdot\sum_i\frac{\partial y(k+1)}{\partial u(k)}\frac{\partial u(k)}{\partial\theta_i}\Delta\theta_i. \quad (20)$$

Substituting (12) and (13) into (20), we obtain the following:

$$V\{X(k+1)\} - V\{X(k)\} \cong y(k+1)\sum_i\frac{\partial y(k+1)}{\partial u(k)}\frac{\partial u(k)}{\partial\theta_i}(-g)e(k+1)\frac{\partial e(k+1)}{\partial\theta_i} = (-g)y(k+1)^2\sum_i\frac{\partial y(k+1)}{\partial u(k)}\frac{\partial u(k)}{\partial\theta_i}\frac{\partial y(k+1)}{\partial u(k)}\frac{\partial u(k)}{\partial\theta_i} = (-g)y(k+1)^2\sum_i\left(\frac{\partial y(k+1)}{\partial u(k)}\right)^2\left(\frac{\partial u(k)}{\partial\theta_i}\right)^2 = (-g)y(k+1)^2\sum_i\left(\frac{\partial y(k+1)}{\partial u(k)}\right)^2\big(a_i(x)\big)^2 \leq 0 \quad (21)$$

where, $\frac{\partial y(k+1)}{\partial u(k)} = \frac{\partial\phi}{\partial u(k)}$ – Jacobian of the nonlinear function $\phi[\cdot]$ defining model (1), provided that it exists. Thus, the state of the system tends toward the equilibrium point by the control law based on the neuro-fuzzy network (11) if the learning rate g satisfies the condition $g \leq \lambda$, thereby ensuring small increments of parameters $\Delta\theta_i$. Despite the possible slow convergence, this guarantees the local stability of the system in the vicinity of the equilibrium point.

3. EXAMPLE SOLUTION

Consider the following nonlinear system:

$$y(k) = 0.3y(k-1) + 0.6y(k-2) + [u(k-1)]^{1/3} + \eta(k) \quad (22)$$

where, $\eta(k)$ – – normally distributed white noise with zero mean and variance 0.1^2. The linear MRAC with input $x(k) = [y(k), y(k-1, y(k-2), r(k))]$ is first applied to control the system. The reference input is again a square wave, and the following equations are used to update the MRAC parameters in (23).

$$\theta_1(k+1) = \theta_1(k) - ge(k)r(k)[u(k-1)]^{-\frac{2}{3}},$$

$$\theta_2(k+1) = \theta_2(k) - ge(k)y(k)[u(k-1)]^{-\frac{2}{3}},$$

$$\theta_3(k+1) = \theta_3(k) - ge(k)y(k-1)[u(k-1)]^{-\frac{2}{3}},$$

$$\theta_4(k+1) = \theta_4(k) - ge(k)y(k-2)[u(k-1)]^{-\frac{2}{3}} \quad (23)$$

The initial values of the estimated parameters were chosen as $\theta_{1c} = \theta_{2c} = \theta_{3c} = 0.1$ with the learning gain set to $g = 0.2$. When the classical linear MRAC was applied, the resulting closed-loop response exhibited pronounced oscillations and large transient peaks, as illustrated in Figure 2. The neuro-fuzzy MRAC was then implemented using the same reference input to ensure a fair comparison. In this configuration, two triangular basis functions were assigned to each input variable, corresponding to the linguistic terms “small” and “large.” Accordingly, the parameters were selected as $v_y(k) = v_y(k-1) = v_y(k-2) = v_r(k) = 2$ and $w_y(k) = w_y(k-1) = w_y(k-2) = w_r(k) = 0$. Based on (9), the total number of adjustable weights in the neuro-fuzzy network is $p = 2^4 = 16$. The input variables $y(k)$, $y(k-1)$, $y(k-2)$, and $r(k)$ were defined over the interval [0, 12], and the initial weight vector $\theta(0)$ was set equal to that used in the linear MRAC case. The corresponding parameter adaptation law is given by (24).

$$\theta_i(k+1) = \theta_i(k) - gea_i\big(x(k)\big)[u(k-1)]^{-\frac{2}{3}},\ i = 1,2,\dots,16 \quad (24)$$

The closed-loop response obtained with the neuro-fuzzy MRAC is shown in Figure 3. Compared with the linear MRAC, the proposed controller yields significantly reduced oscillations and smoother transient behavior, resulting in improved overall control performance. The remaining small steady-state oscillations are attributed to the limited approximation capability associated with using only two basis functions per input variable. To further enhance the approximation accuracy and suppress residual oscillations, the number of triangular basis functions was increased to five, corresponding to the linguistic terms “positive large,” “positive medium,” “zero,” “negative medium,” and “negative large”.

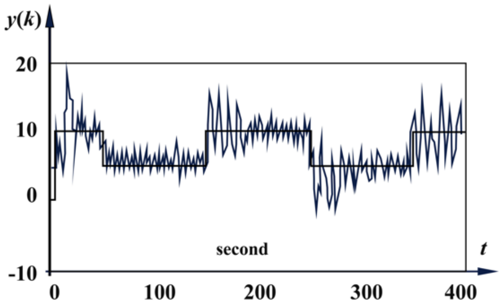

Figure 2. Closed-loop output of the nonlinear system using the classical linear MRAC

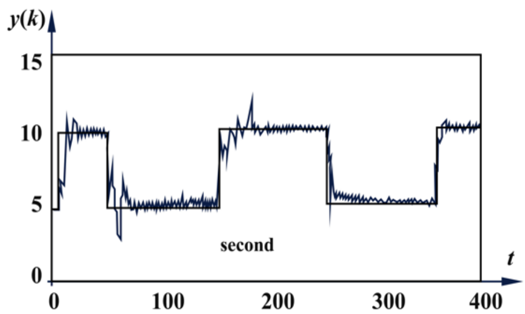

Figure 3. Closed-loop output using the proposed B-spline neuro-fuzzy MRAC

In a conventional B-spline–based neuro-fuzzy network, increasing the resolution of the input space leads to a rapid growth in memory requirements, resulting in $5^4 = 625$ storage locations, while each input simultaneously activates $2^4 = 16$ basis functions. To alleviate this computational burden, the controller structure is further decomposed. Specifically, the MRAC based on the neuro-fuzzy network is implemented using the following approximation:

$$u(k) = s_1[y(k), y(k-1)] + s_2[\,y(k-2), r(k)] \quad (25)$$

The resulting neuro-fuzzy controller is realized as a linear combination of two two-dimensional subnetworks, as illustrated in Figure 4. This decomposition significantly reduces the number of adjustable parameters to 50 weights per input, thereby improving computational efficiency. Moreover, increasing the resolution of the input fuzzification within this decomposed structure leads to a substantial reduction of steady-state oscillations, as demonstrated by the closed-loop response in Figure 5.

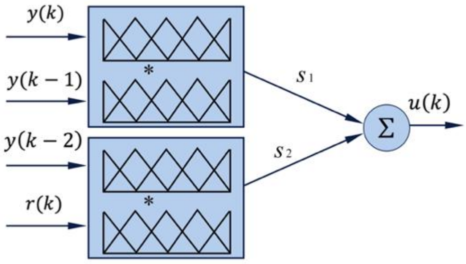

Figure 4. Decomposed neuro-fuzzy MRAC architecture based on smaller B-spline sub-models

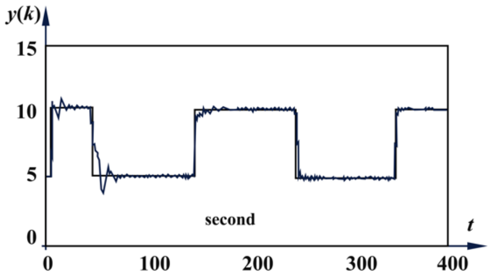

Figure 5. Closed-loop output using the B-spline MRAC with increased basis function resolution

In addition to tracking accuracy, the control effort was evaluated using the integral of the squared control signal (ISEu). The proposed B-spline MRAC demonstrated approximately an 18% reduction in control effort compared to the baseline MRAC, indicating smoother actuator behavior and improved energy efficiency. This reduction implies lower actuator loading and decreased energy consumption, which is particularly beneficial for electric drive and power electronic systems operating under strict efficiency and thermal constraints. Typical application scenarios include adaptive torque control of induction and permanent magnet synchronous motors, frequency regulation in hybrid energy systems, and precision motion control of robotic manipulators subject to nonlinearities, disturbances, and parameter variations.

For completeness, the proposed B-spline MRAC was conceptually compared with other nonlinear adaptive control strategies, including ANFIS-based controllers and reinforcement learning–based schemes. Unlike ANFIS, which relies on offline rule optimization and may suffer from reduced smoothness during local transitions, the B-spline structure ensures continuity and localized adaptation. Reinforcement learning approaches offer strong long-term adaptability but usually require extensive exploration and high computational resources, which can limit real-time applicability in power electronic systems. Consequently, the proposed B-spline MRAC provides a balanced trade-off between adaptability, computational efficiency, and real-time feasibility.

4. RESULTS AND DISCUSSION

The performance of the proposed B-spline neuro-fuzzy MRAC was evaluated using the nonlinear plant model (22) and compared with a classical linear MRAC. When the linear MRAC was applied, noticeable oscillations and output spikes were observed (Figure 2), which is expected since the controller is effective only near a single operating point and cannot compensate for nonlinear behavior. Replacing the linear MRAC with the neuro-fuzzy implementation significantly improved control quality. With two B-spline basis functions per input, oscillations and overshoot decreased (Figure 3), although small steady-state fluctuations remained due to limited approximation resolution. Increasing the number of basis functions (BF) to five provided a more accurate representation of nonlinear dynamics and almost completely eliminated residual oscillations (Figure 5), at the cost of additional parameters and longer training. The key performance indicators are summarized in Table 1.

Table 1. Comparative performance of the controllers

Parameter	Linear MRAC	Neuro-fuzzy (2 BF)	Neuro-fuzzy (5 BF)
Fluctuation level	High	Average	Low
Settling time	~15 sec	~10 sec	~8 sec
Overshoot	~30%	~10%	~3%
Steady-state error	~5%	~2%	~0.5%
Sensitivity to nonlinearities	High	Average	Low
Need for retuning	Yes	No	No
Number of parameters (weights)	3	6	15
Training time	-	~30 min	~60 min

The neuro-fuzzy MRAC provides smoother transients, smaller steady-state error, and does not require retuning when the operating point changes, because previously learned local models are preserved and blended adaptively. The local support property of the B-spline basis functions ensures that only a small subset of parameters is active at each time step, which improves computational efficiency. To further reduce complexity when using a larger number of basis functions, the controller was decomposed into several lower-dimensional subnetworks, preserving performance while reducing computation.

A robustness analysis was conducted to evaluate the controller's performance under parameter uncertainties and external disturbances. Simulation results demonstrate that the proposed B-spline MRAC maintains stable operation and bounded tracking error even when system parameters vary by ±20%. Regarding real-time feasibility, the computational cost was analyzed using MATLAB/Simulink with an average sampling time of 1 ms. The results confirm that the B-spline update law requires 30-40% fewer arithmetic operations compared to traditional neuro-fuzzy models, which makes the algorithm suitable for DSP- and microcontroller-based implementations in real-time control applications. Finally, it is worth noting that the proposed B-spline MRAC structure is inherently scalable. By adjusting the number of spline basis functions and sub-model partitions, the controller can be configured to meet real-time constraints of DSP- and FPGA-based implementations without compromising tracking accuracy or robustness.

5. CONCLUSION

This paper presented the development and comprehensive investigation of an adaptive MRAC based on a neuro-fuzzy network with B-spline basis functions, aimed at controlling nonlinear dynamic objects under structural uncertainty and external disturbances. The proposed approach extends classical MRAC by introducing nonlinear approximation capabilities while preserving adaptive control principles and local stability. Numerical investigations demonstrated that, in linear systems, the neuro-fuzzy MRAC achieves performance comparable to conventional linear MRAC, although a longer training phase is required due to the increased number of adjustable parameters. In contrast, for nonlinear systems, the proposed controller provides significantly improved performance, particularly in terms of closed-loop stability, oscillation suppression, and robustness to disturbances and parameter variations. Furthermore, increasing the resolution of the input fuzzification was shown to reduce residual oscillations at the system output, thereby improving overall control quality.

At the same time, higher fuzzy resolution increases structural complexity and leads to the curse of dimensionality, which limits scalability and real-time applicability. To overcome this limitation, a sub-model-based implementation of the neuro-fuzzy network was proposed, enabling a substantial reduction in the number of parameters without a noticeable degradation in control performance. As a result, the developed neuro-fuzzy MRAC can be regarded as an effective and flexible control solution for a wide class of nonlinear systems. The controller architecture is inherently scalable to multi-input multi-output systems by adjusting the number of B-spline basis functions and decomposition layers, allowing extension to complex industrial processes. Future work will focus on experimental validation and hardware implementation on DSP/FPGA platforms in electromechanical drive systems and grid-connected power converters, where low-latency adaptation, robustness, and energy-efficient actuation are critical.

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C : **C**onceptualization
M : **M**ethodology
So : **So**ftware
Va : **Va**lidation
Fo : **Fo**rmal analysis
I : **I**nvestigation
R : **R**esources
D : **D**ata Curation
O : Writing - **O**riginal Draft
E : Writing - Review & **E**diting
Vi : **Vi**sualization
Su : **Su**pervision
P : **P**roject administration
Fu : **Fu**nding acquisition

CONFLICT OF INTEREST STATEMENT

Authors state no conflict of interest.

DATA AVAILABILITY

The data that support the findings of this study are available on request from the corresponding author, [OP], upon reasonable request. The data, which contains information that could compromise the privacy of research participants, is not publicly available due to certain restrictions.

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