

## Variable Voltage Source Inverter with Controlled Frequency Spectrum Based on Random Pulse Width Modulation

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### ABSTRACT

This paper presents a new method for single phase variable voltage inverter based on Random Pulse Width Modulation. In Random Pulse Width Modulation based inverter, the frequency spectrum of the output current and voltage waveforms becomes continuous because of the randomization of the switching function of the devices controlling the output voltages. This paper establishes a theory that if the distributions of the random numbers generated by the random source are kept within certain limit with respect to the peak value of reference sinusoidal waveform, the frequency spectrum can be controlled. On the basis of the results, a novel drive using variable tap changing transformer (optional) and adaptive random number generator, to control the ratio between the numbers generated by the random source and the reference waveform has been suggested that will guarantee a better power quality profile for a broad range of output voltages.

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## 1. INTRODUCTION

Modern inverters use different types of modulation techniques for achieving different magnitude of output voltages. Variable output voltages are normally controlled by Pulse Width Modulation (PWM) [1]. But the problem with PWM based converters is that it introduces higher frequency components which are discrete in nature. This problem is originated by the periodic switching of the power electronics devices controlling the output voltages. These frequency components are discrete in nature because of the periodicity of the output current and voltage waveform. The entire spectrum energy is localized at discrete frequencies which cause problems like acoustic noise, electromagnetic interference, etc [2,3].

One solution to address the problems related to discrete frequency spectrum is to use Random Pulse Width Modulation (RPWM). The key property that differentiates random PWM from classic PWM is that the random PWM produces switching functions that have a non-deterministic (random) component [3]. If the switching function is non-deterministic then its frequency spectrum will be continuous because of non periodicity of the signal. Since the switching signal is random, its frequency spectrum would also change with time and hence the use Fourier Transform for the analysis of the signal would be required. Fourier transform of a particular realization of a random signal (of arbitrary length) is a random signal itself, i.e. it is a random variable at each frequency [3]. The continuity of the spectrum does not cause the energy of the signal to be concentrated on any particular frequency. And this spreading of the spectral energy eliminates the problems associated with classical PWM. But the problem arises when the randomization causes

excessive distortion at low values of modulation index and broad variations of output voltage swing. This paper works out a solution to control the spectrum such that it meets the power quality standards in all scenarios.

## 2. RESEARCH CRITERIA

The research findings in this paper are based upon the the following research criteria. The parameters in this section would help establish the problem and the results quantitatively.

### 2.1. The Control Parameters of a PWM drive

Among all the paremeters of an inverter, there are two important paremeters that are used to control the quality of the output voltage in the inverter:

1. Modulation Ratio
2. Modulation Index

Modulation Ratio is defined as:

$$M_r = f_c / f_s \quad (1)$$

where 'f<sub>c</sub>' is the carrier frequency (triangular in case of deterministic PWM) and 'f<sub>s</sub>' is the frequency of the modulating (reference) waveform (sinusoidal in this case).

Modulation Index 'Mi' is defined as:

$$Mi = \frac{\text{Amplitude of the Reference Waveform}}{\text{Amplitude of the Carrier Waveform}}$$

or

$$V_{in} = M_i \times V_{dc} \quad (2)$$

V<sub>in</sub> is the output voltage of inverter and V<sub>dc</sub> is the DC link voltage.

Typical Values of 'Mi' fall in the range: 0.7 – 0.95

### 2.2. Deterministic PWM Drive

The model given in figure 1 has been used to perform the analysis for deterministic pulse width modulation by varying the aforementioned parameters.

In figure 1, there is a PWM drive based upon Insulated Gate Bipolar Transistors 'IGBTs' named as 'universal bridge'. Two power supplies each of 200V DC are placed. There is a PWM Generator (Controller) used for driving the IGBTs. The load is connected with RL = 1ohm and L = 5mH. Oscilloscopes are also connected to record different current and voltage waveforms. The purpose of 'POWER GUI' module is to perform the spectral analysis by taking FFT of the waveform.

### 2.3. Results by varying parameters for aforementioned PWM drive

The parameters, modulation index and modulation ratio are varied. In case of Deterministic Modulation, since the frequency spectrum is discrete in nature (because of the periodicity of the switching function by the PWM Generator), Total Harmonic Distortion 'THD' to characterize the strength of the harmonics and quality of the inverter is measured. Greater the value of total harmonics distortion, poor it is the quality of the inverter and vice versa. The mathematical value of THD is given in [1] (3).

$$THD = \sqrt{\left( \left( I_s / I_{s1} \right)^2 - 1 \right)} \quad (3)$$

'I<sub>s</sub>' and 'I<sub>s1</sub>' are the RMS value of the supply current waveform and the fundamental harmonic respectively. The value of 'I<sub>s</sub>' is given by (4) [1].

$$I_s = \sqrt{I_{s1}^2 + I_{s2}^2 + I_{s3}^2 + I_{s4}^2 + \dots} \quad (4)$$

*By varying Modulation Ratio:* Figure 2 shows the variation in the value of THD by varying the modulation ratio. It can be observed that if the switching speed is increased by increasing modulation ratio, THD is

decreased. It is also observed that by varying modulation, the peak value of the output current and voltage remains the same. In figure 2, it is also evident that if the numerical value of the modulation ratio is continuously increased beyond 25, there is no substantial improvement in THD, hence it is the typical value of modulation ratio.

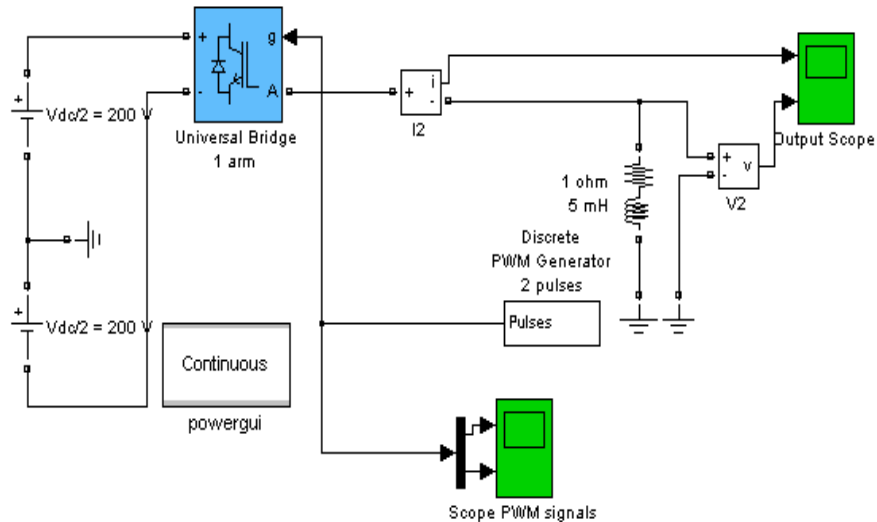


Figure 1. Model used for analysis of deterministic pulse width modulation.

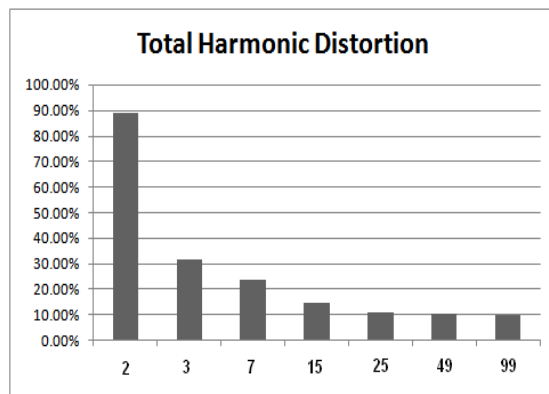


Figure 2. Percentage Variation in THD by varying varying Modulation Ratio.

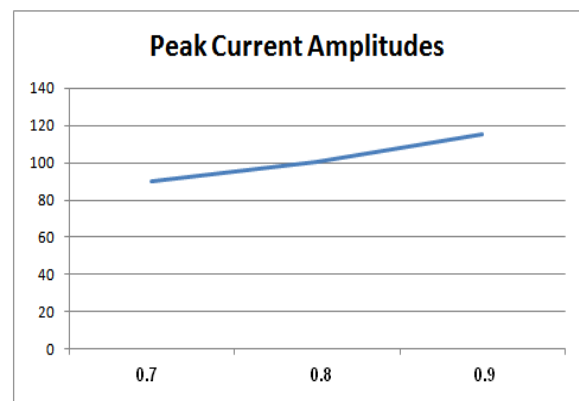


Figure 3. Variation in peak current amplitude by varying modulation index.

*By varying Modulation Index:* It can be deduced from (2) that if the modulation index is changed, there will be a change in the magnitude of the output voltage and current. So to control the magnitude of the output voltage and current, the modulation index is varied. It has also been observed that changing modulation index has very small impact on the total harmonic distortion. Figure 3 shows the change in the output current by changing the modulation index.

#### 2.4. Deterministic PWM Vs Random PWM:

In case of deterministic PWM, It is seen that the spectrum is discrete in nature. This discrete spectrum generates the problems like acoustic noise in motors, input voltage waveform distortion, and electromagnetic interference. The most significant contributions to the EMI are the fundamental and its harmonics [4]. These problems are eliminated with the help of random PWM in which the switching function for universal bridge is randomized in nature and the significant noise producing frequencies are suppressed and spread through the spectra more uniformly [2,5]. This continuity in the spectrum eliminates the problem of large amplitudes at discrete frequencies. So, the entire spectrum spreads over entire range of frequencies with very small amplitudes at any particular frequency except the fundamental frequency.

In the larger scenario, the random modulation schemes can be categorized under four titles: random pulse position modulation (RPPM). Random pulse width modulation (RPWM), random carrier frequency modulation with fixed duty cycle (RCFMFD) and random carrier frequency modulation with variable duty cycle (RCFMVD) [6]. Also, a random space vector modulation scheme which effectively reduces the acoustic noise by smoothly distributing the tonal energy throughout the spectrum was introduced in [7]. Strategies that are based on an amalgamation of both the random modulation and deterministic modulation schemes are also discussed in the literature [8]. [9-13] present different optimization techniques used to control the spectral contents. In the analysis of the deterministic PWM drive, it has been observed that if the switching speed is kept sufficiently high by increasing the modulation ratio, an improvement in the power quality of the inverter is achieved because of the reduction in the THD. In this work, the switching speed for random modulation would be kept constant (on the average) for all variations in the magnitude of the output voltage.

### 3. ANALYSIS AND RESULTS FOR RANDOM PULSE WIDTH MODULATION

#### 3.1. Model Used for Random Pulse Width Modulation:

The simulations are performed in the MATLAB. The model that has been used for random pulse width modulation is given in figure 4. This drive is different from the one given in figure 1 as it utilizes a Random Source which is a noise source. The value of this noise source is compared with the reference sine wave. The comparator is designed in two phases. In first phase there is just one subtractor that computes the difference between the reference sine wave and random noise signal. If this difference is greater than zero, then one set of IGBTs (serving as switches) is turned 'on'. If the difference is 'less than zero' then the other set of IGBTs is turned on. The load conditions are similar to the one in case of Deterministic PWM shown in figure 1.

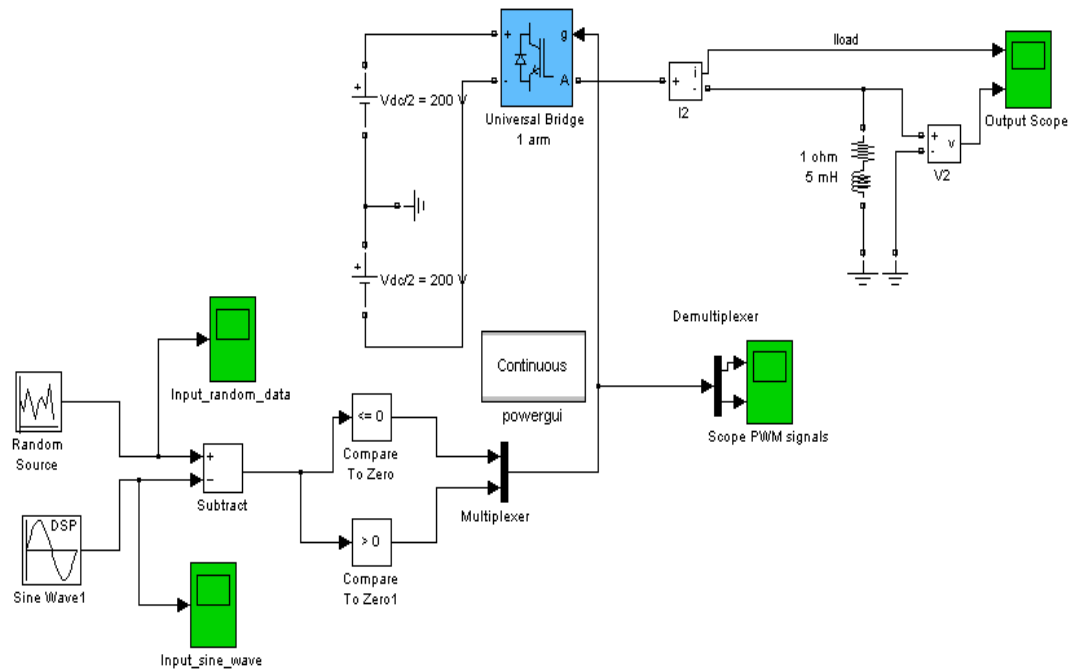


Figure 4: Model used for analysis of random pulse width modulation.

#### 3.2. Types of Analysis

In this analysis, the following two types of noise generators have been used: Gaussian noise generator and Uniform noise generator.

*Gaussian Noise Generator:* Probability density function (PDF) which gives the probability of all the random values generated by the Gaussian Noise Generator is given in (5) [14]:

$$\phi(x; \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x - \mu)^2}{2\sigma^2}\right) \quad (5)$$

In this equation, ‘ $\mu$ ’ represents the mean value (average value of the randomly generated numbers) and ‘ $\sigma$ ’ corresponds to the variance (which determines the spread of the randomly generated numbers). Greater the value of ‘ $\sigma$ ’, greater it is the spread of probability density function and vice versa.

*Uniform Noise Generator:* In uniform distribution, there are two parameters ‘a’ (minimum value) and ‘b’ (maximum value) that determine the characteristics of the uniform noise generator. The probability density function for Uniform Distribution is given in (6) [14].

$$f(x) = \begin{cases} \frac{1}{b-a} & \text{for } a \leq x \leq b, \\ 0 & \text{for } x < a \text{ or } x > b, \end{cases} \quad (6)$$

In uniform noise generator, all the numbers between ‘a’ and ‘b’ are equally probable.

### 3.2.1. Analysis for Gaussian Noise Generator

The time domain analyses for current waveform for Gaussian noise generator are given in figure 5. In these analyses, the value of variance is varied while keeping the value of mean fixed. It can be observed that if the value of variance is increased, the output waveform is a poor approximation of the ideal sine wave. The results from Gaussian noise generator are in accordance with the deterministic modulation. If the value of variance is increased, the numbers generated by the random source greater than the reference waveform have larger probability and the switching rate is reduced. There is one more observation from figure 6, that if the value of the variance is increased, the peak current amplitude is decreased so is the RMS value of the output voltages. Figure 7 contains the frequency domain analysis for the Gaussian noise generator. In this figure, it can be observed that if the value of variance is increased, the involvement of high frequency amplitudes is increased and result is poor approximation with the sinusoidal behavior.

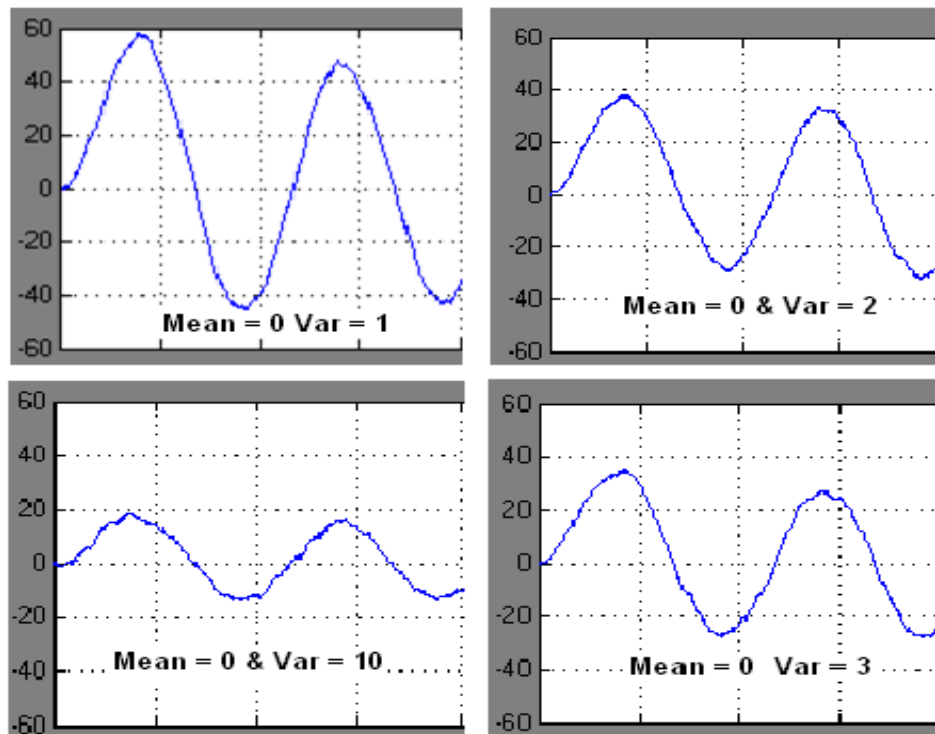


Figure 5. Time domain analysis for Gaussian noise generator.

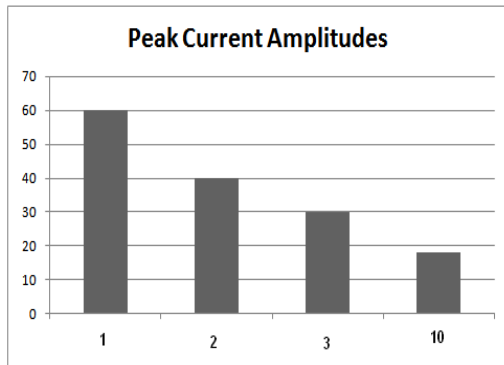


Figure 6. Peak current amplitude versus variance.

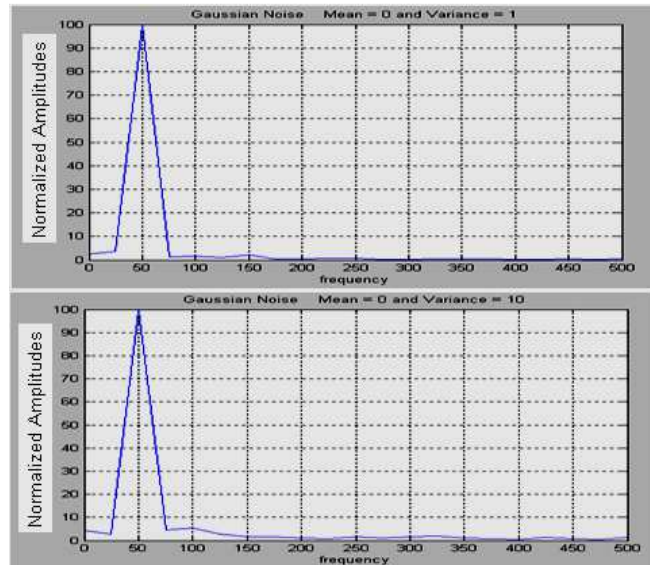


Figure 7. Frequency domain analysis by varying variance.

Notice that in the analysis for Gaussian noise generator, the value of mean is kept constant at '0'. If a different value of the mean is selected, then the time domain current waveform is shifted above or below the x-axis (A DC offset) depending upon the sign (positive or negative) of the mean. This scenario is not acceptable and hence not included in the analysis.

### 3.2.2. Analysis for Uniform Noise Generator

Figure 8 gives the time domain analysis for the current waveform based on uniform noise generator. It can be seen that if the difference between the maximum and minimum values generated by the noise generator is increased, the probability of the numbers greater than peak value of reference waveform is increased resulting in the reduction of the switching speed for the universal bridge and consequently the inverter shows a poor power quality profile with reduced peak amplitudes.

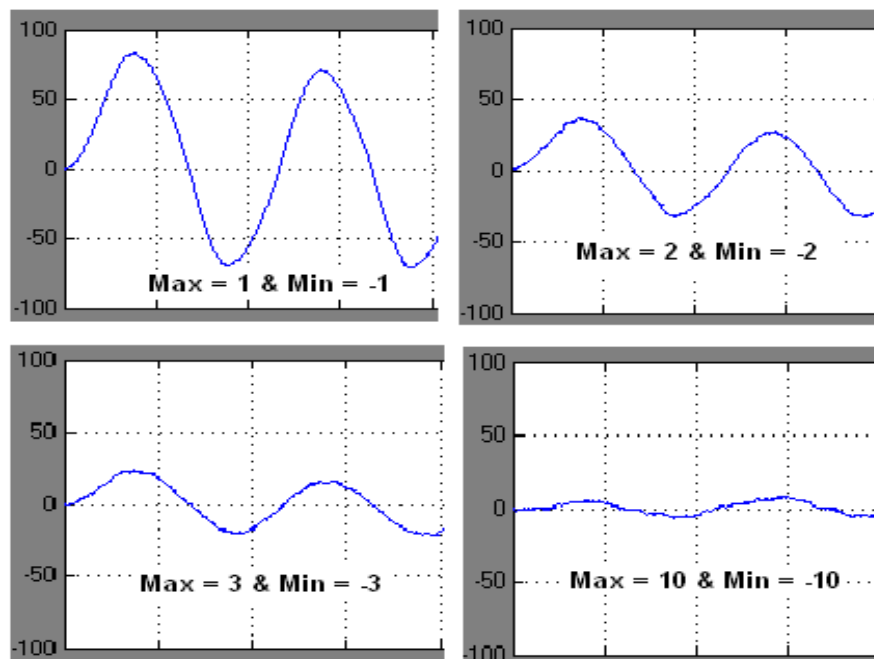


Figure 8. Time domain analysis for uniform noise generator.

The results obtained from uniform noise generator are exactly in accordance with those obtained from the Gaussian noise generator.

In case of uniform noise generator, if the magnitudes of 'a' and 'b' are not kept constant, then again the mean  $\neq 0$  i.e., the waveform of the load current is shifted above or below the x-axis and again the analysis of this situation is not included in this work.

### 3.3. Discussion on the Results:

In case of Gaussian and uniform generator, it is seen that if the output voltages are controlled by increasing the range of the numbers (by varying variance for Gaussian noise generator and increasing the difference between maximum and minimum value for Uniform Noise Generator) generated by noise generator, switching speed of universal bridge is reduced, causing degradation in the performance of the inverter in term of spectral contents of the current waveform i.e., the involvement of high frequency amplitudes to a greater extent.

### 3.4. Optimized RPWM drive

Hence to overcome the distortion in the output waveform caused when the 'variance' for Gaussian or the difference between maximum and minimum value for Uniform Noise Generator is increased, a technique is proposed that would help in changing the output voltages without causing the distortion in the output waveform of the inverter. Figure 9 gives the amended drive which optimizes the results.

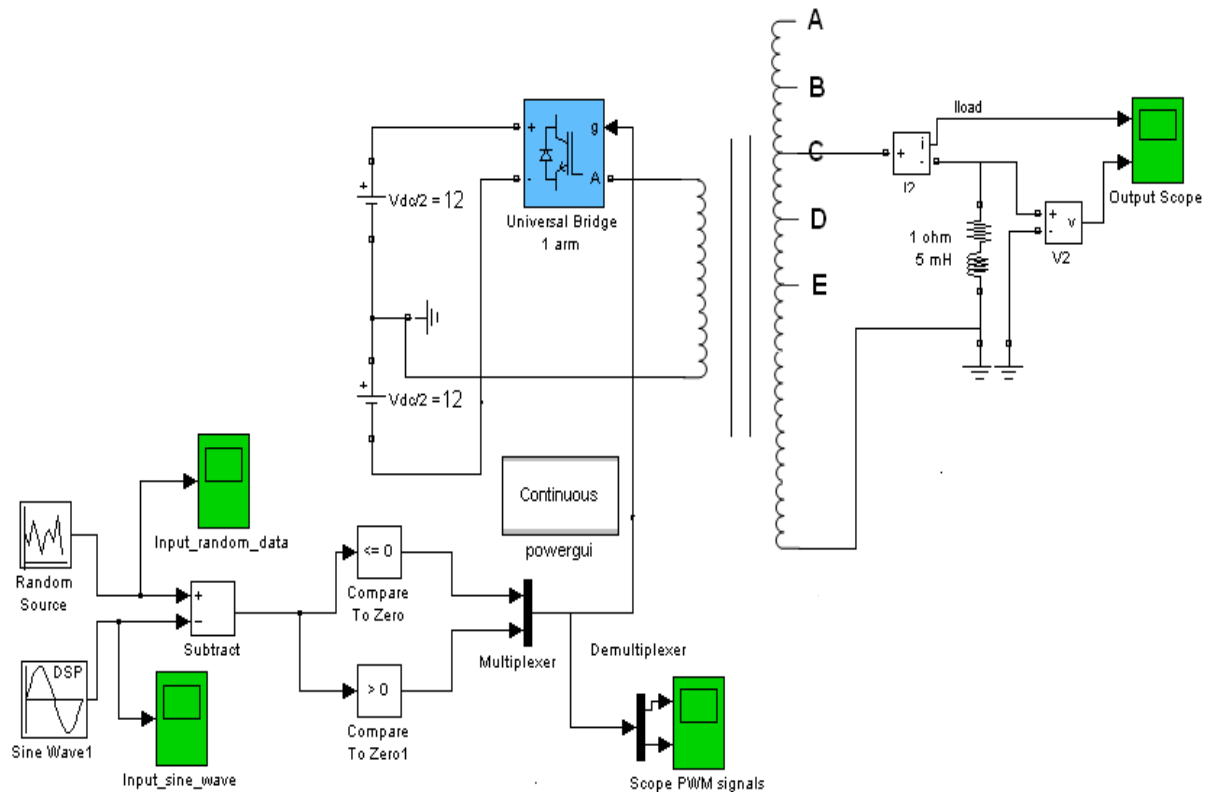


Figure 9. Optimized random pulse width modulation.

In this alteration, a transformer with variable taps has been used. In case of variable voltage inverter, whenever it is desired to change the output voltage, range of the numbers generated by the random generator and the position of the selector switch would be selected such that it provides the optimized performance for that particular situation. From Figure 5, comparing the shape of the output current in case of variance = 1 and variance = 2, there is not much problem as far as the degradation in the output sine wave is concerned. Same is the case in Figure. 8 where, by comparing the results for Max, Min =  $\pm 1$  and Max, Min =  $\pm 2$ , the degradation in the quality of the inverter is ignorable. From these figures, it can be concluded that the output voltage level can be changed by changing the range (within limits) of the number generated by the random source to some extent without much effect on the quality of the waveform.

While implementing the system in hardware, usually a lookup table of pregenerated random numbers is used. Empirical analysis of the spectrum suggests that certain areas in the spectrum are amplified if certain numbers in the random numbers list are used more often. Hence, by making prior measurements, a hybrid random number generator can be developed to get the desired shape of the spectrum. If it is desirable to change the output voltages beyond that limit, then the tap of the transformer would also be changed. The idea is that a minimum difference between the range of the numbers generated by the random source and the peak value of the reference sinusoidal waveform should be maintained in order to get the optimized results (which will guarantee a certain minimum switching speed). If difference tends to go below that minimum threshold, then the tap of the transformer should also be adjusted from the selector switch. Hence, a modified RPWM scheme has been introduced that provides the optimized performance for the whole range of modulation index.

The optimized strategy consists following steps:

- Prior statistical information of the relationship between the modulation index and the distortion factor should be available.
- Drive could be preconfigured to meet certain spectra shapes by making a hybrid random numbers table.
- If the prior information of the load is not available (hence the information of changing the random numbers table and its effect on the spectrum is not available), the computational intelligence can be incorporated in the drive control that would obtain the information of the spectral density and modify the lookup table as per demand.
- A threshold should be setup such that if the modulation index is too small (below that threshold), the entire spectrum is almost uniformly dense. Hence, instead of introducing further modifications in the random number generator (which would further worsen the power quality), the appropriate settings for the transformer's transfer ratio can be applied. The transformer may be servo/stepper motor controlled for improved accuracy.

#### 4. CONCLUSIONS

It has been observed that decrease in the switching speed reduces the power quality of the output of the inverter in both the cases, deterministic PWM and random PWM. Further, in case of random pulse width modulation, the increase in the range of the numbers generated by the random generator degrades the power quality of the output of an inverter for smaller values of modulation index. Also, certain numbers in the random numbers lookup tables add to certain high density areas in the spectrum. A novel strategy based on the use of hybrid random numbers list with restrictions on the ranges of numbers that change adaptively with operation, and an optional (only required when the prior two strategies are not good enough) variable tap transformer has been proposed. A few results with these strategies have been given that individually prove the efficacy of this technique. Hence, this technique will provide an optimal voltage source inverter which would maintain an excellent power quality profile for a broad range of output voltage swing.

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