

## Carrier Based Pulse Width Modulation Control of a Non-Square Direct Matrix Converter with Seven-phase Input and Three-phase Output

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### ABSTRACT

This paper presents pulse width modulation technique for a direct ac-ac converter. The converter has seven-phase input supply and three-phase output. The input supply may be obtained from a seven-phase wind energy generation system with variable output voltage magnitude and frequency. The output of the proposed converter topology may be fed to the three-phase stiff grid system. Thus the requirement of the modulation of the ac-ac converter is to produce fixed voltage and fixed frequency output while the input can be variable. Additionally the output voltage gain should be high. Simple carrier-based PWM technique is suggested and harmonic injection scheme is proposed to enhance the output voltage magnitude. The output voltage reaches 90% of the input supply voltage with the proposed technique. An additional control block is used to stabilize the output voltage and frequency of the converter. Simulation results are shown in the paper for the verification of the proposed scheme.

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## 1. INTRODUCTION

Multi-phase drive systems have been widely investigated in the literature and their reviews are presented in [1]. Multi-phase drive systems are shown to offer several advantages when compared with the three-phase drive system. This is possible due to the advanced development of the modern power electronic converters. The increased penetration of renewable energy generation especially wind energy generation system requires more robust, reliable and high power density generation system. The use of multi-phase generation system for renewable energy is reported recently [2]-[12]. Since the multi-phase machines offer the advantages of high reliability and high power density, they can be ideal choice of wind energy generation system. Normally, the interfacing of the renewable energy sources with the grid is done using a power electronic converter. Matrix converters are normally employed in the wind generation system to stabilize the voltage and frequency for grid integration [13]-[16].

The matrix converter or direct ac-ac converter offers several advantages over back-to-back bidirectional converters. The back-to-back converter offer bidirectional power flow by using fewer numbers of power switching devices and that too by using unidirectional IGBTs. However, the requirement of bulky dc link capacitor cannot be avoided. Moreover, the back-to-back converter needs extra feedback current control loop for controlling rectifier d-q axis current and also voltage control loop for controlling dc link voltage. Additionally, the matrix converter topology has the capability of providing active damping by injecting reactive power. The back-to-back converter has a limitation of reactive power injection due to limited component ratings. Further comparison can be done by looking at the per switch output current and it

is lower in the case of matrix converter and thus is better suited for high current start-up applications and for continuous low frequency operation.

A five-to-three-phase matrix converter topology is proposed in [17]-[18] for use in variable speed drives application. The proposed PWM technique in [17] is based on space vector PWM and the PWM reported in [18] is based on carrier-based scheme. The output voltage limit is not studied in these papers. Moreover, the output voltage is intended to have variable voltage and variable frequency. In contrast this paper focuses on obtaining fixed voltage and fixed frequency output for variable voltage and variable frequency input.

This paper proposes modulation and control techniques for a matrix converter with seven-phase input and three-phase output. Similar type of modulation and control technique is used in [19] for the case of five-to-three phase matrix converter. This matrix converter topology is intended to use for a seven-phase wind energy generation system. The major advantage of the proposed matrix converter is high output voltage value when compared to a conventional three-phase to three-phase output matrix converter. The output is limited to 86.6% in the conventional three-phase input to three-phase output matrix converter. This value is 90% in the matrix converter with seven-phase input and three-phase output. However, this limit can be further enhanced by employing over-modulation and at the cost of introducing low-order harmonics in the output. The block diagram of the proposed system is shown in Figure 1. This paper focuses on the pulse width modulation control of the matrix converter part only.

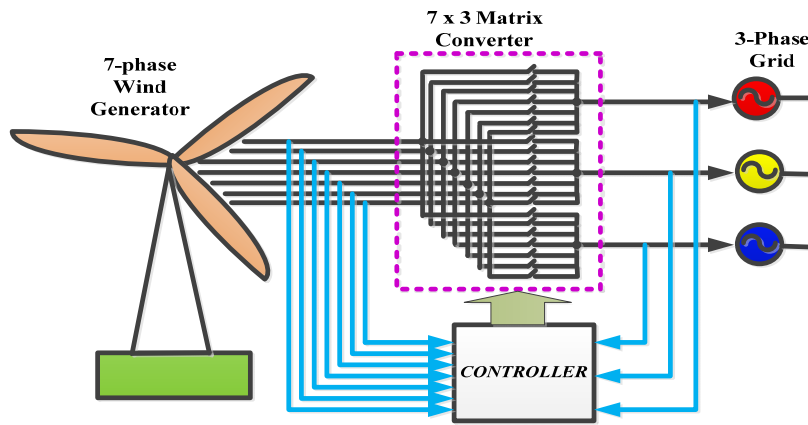


Figure 1. Block Schematic of the System under Investigation

## 2. CARRIER BASED PWM STRATEGY OF 7 BY 3 MATRIX CONVERTER

The input seven-phase system is assumed as.

$$\begin{aligned} v_a &= |V| \cos(\omega t), & v_b &= |V| \cos(\omega t - 2\pi/7), & v_c &= |V| \cos(\omega t - 4\pi/7), & v_d &= |V| \cos(\omega t - 6\pi/7) \\ v_e &= |V| \cos(\omega t + 6\pi/7), & v_f &= |V| \cos(\omega t + 4\pi/7), & v_g &= |V| \cos(\omega t + 2\pi/7) \end{aligned} \quad (1)$$

Since the matrix converter outputs voltages with frequency decoupled from the input voltages, the duty ratios of the switches are to be calculated accordingly. The three-phase output voltage duty ratios should be calculated in such a way that output voltages remains independent of input frequency. In other words, the three-phase output voltages can be considered in synchronous reference frame and the seven-phase input voltages can be considered to be in stationary reference frame, so that the input frequency term will be absent in output voltages. Considering the above, duty ratios of output phase  $j$  is chosen as:

$$\begin{aligned} \delta_{aj} &= k_j \cos(\alpha - \rho), & \delta_{bj} &= k_j \cos(\alpha - 2\pi/7 - \rho), & \delta_{cj} &= k_j \cos(\alpha - 4\pi/7 - \rho), & \delta_{dj} &= k_j \cos(\alpha - 6\pi/7 - \rho), \\ \delta_{ej} &= k_j \cos(\alpha + 6\pi/7 - \rho), & \delta_{fj} &= k_j \cos(\alpha + 4\pi/7 - \rho), & \delta_{gj} &= k_j \cos(\alpha + 2\pi/7 - \rho), \end{aligned} \quad (2)$$

The input and output voltages are related as:

$$\begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} \delta_{aA} & \delta_{bA} & \delta_{cA} & \delta_{dA} & \delta_{eA} & \delta_{fA} & \delta_{gA} \\ \delta_{aB} & \delta_{bB} & \delta_{cB} & \delta_{dB} & \delta_{eB} & \delta_{fB} & \delta_{gB} \\ \delta_{aC} & \delta_{bC} & \delta_{cC} & \delta_{dC} & \delta_{eC} & \delta_{fC} & \delta_{gC} \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \\ v_d \\ v_e \\ v_f \\ v_g \end{bmatrix} \quad (3)$$

Therefore the phase  $j$  output voltage can be obtained by using the above duty ratios as:

$$\begin{aligned} V_j = k_j |V| & [\cos(\omega t) \bullet \cos(\omega t - \rho) + \cos(\omega t - 2\pi/7) \bullet \cos(\omega t - 2\pi/7 - \rho) + \cos(\omega t - 4\pi/7) \\ & \bullet \cos(\omega t - 4\pi/7 - \rho) + \cos(\omega t - 6\pi/7) \bullet \cos(\omega t - 6\pi/7 - \rho) + \cos(\omega t + 6\pi/7) \bullet \cos(\omega t + 6\pi/7 - \rho) \\ & + \cos(\omega t + 4\pi/7) \bullet \cos(\omega t + 4\pi/7 - \rho) + \cos(\omega t + 2\pi/7) \bullet \cos(\omega t + 2\pi/7 - \rho)] \end{aligned} \quad (4)$$

In general Equation (4) can be written as:

$$V_j = \frac{7}{2} k_j |V| \cos(\rho) \quad (5)$$

In Equation (5),  $\cos(\rho)$  term indicates that the output voltage is affected  $\rho$ . Thus, the output voltage  $V_j$  is independent of the input frequency and only depends on the amplitude  $|V|$  of the input voltage and  $k_j$  is a reference output voltage time-varying modulating signal for the output phase  $j$  with the desired output frequency  $\omega_o$ . The three phase reference output voltages can be represented as:

$$k_A = m \cos(\omega_o t), k_B = m \cos(\omega_o t - 2\pi/3), k_C = m \cos(\omega_o t - 4\pi/3) \quad (6)$$

Therefore, from (5), the output voltages are obtained as;

$$V_A = \left[ \frac{7}{2} m |V| \cos(\rho) \right] \cos(\omega_o t), V_B = \left[ \frac{7}{2} m |V| \cos(\rho) \right] \cos(\omega_o t - 2\pi/3), V_C = \left[ \frac{7}{2} m |V| \cos(\rho) \right] \cos(\omega_o t - 4\pi/3) \quad (7)$$

### 3. APPLICATION OF OFFSET DUTY RATIO

In the above discussion, duty-ratios become negative (see Equation (6)) which are not practically realizable. For the switches connected to output phase- $j$ , at any instant, the condition  $0 \leq d_{aj}, d_{bj}, d_{cj}, d_{dj}, d_{ej}, d_{fj}, d_{gj} \leq 1$  and  $d_{aj} + d_{bj} + d_{cj} + d_{dj} + d_{ej} + d_{fj} + d_{gj} = 1$  should be valid. Therefore, offset duty ratios should be added to the existing duty-ratios, so that the net resultant duty-ratios of individual switches are always positive. Furthermore, the offset duty-ratios should be added equally to all the output phases to ensure that the effect of resultant output voltage vector produced by the offset duty-ratios is null in the load. That is, the offset duty-ratios can only add the common-mode voltages in the output. Considering the case of output phase- $j$ ;

$$\begin{aligned} d_{aj} + d_{bj} + d_{cj} + d_{dj} + d_{ej} + d_{fj} + d_{gj} &= k_j \cos(\omega t - \rho) + k_j \cos(\omega t - 2\pi/7 - \rho) + k_j \cos(\omega t - 4\pi/7 - \rho) \\ &+ k_j \cos(\omega t - 6\pi/7 - \rho) + k_j \cos(\omega t + 6\pi/7 - \rho) + k_j \cos(\omega t + 4\pi/7 - \rho) + k_j \cos(\omega t + 2\pi/7 - \rho) = 0 \end{aligned} \quad (8)$$

The sum of all the duty ratios is zero because the duty ratios contain equal amount of positive and negative components. Absolute values of the duty-ratios are added to cancel the negative components from individual duty ratios. Thus the minimum individual offset duty ratios should be:

$$\begin{aligned}
D_a(t) &= |d_{aj}| = |k_j \cos(\omega t - \rho)|, \\
D_b(t) &= |d_{bj}| = |k_j \cos(\omega t - 2\pi/7 - \rho)|, \\
D_c(t) &= |d_{cj}| = |k_j \cos(\omega t - 4\pi/7 - \rho)|, \\
D_d(t) &= |d_{dj}| = |k_j \cos(\omega t - 6\pi/7 - \rho)|, \\
D_e(t) &= |d_{ej}| = |k_j \cos(\omega t + 6\pi/7 - \rho)|, \\
D_f(t) &= |d_{fj}| = |k_j \cos(\omega t + 4\pi/7 - \rho)|, \\
D_g(t) &= |d_{gj}| = |k_j \cos(\omega t + 2\pi/7 - \rho)|
\end{aligned} \tag{9}$$

The effective duty ratios are:

$$\begin{aligned}
\delta'_{aj} &= d_{aj} + D_a(t), \delta'_{bj} = d_{bj} + D_b(t), \delta'_{cj} = d_{cj} + D_c(t), \delta'_{dj} = d_{dj} + D_d(t), \delta'_{ej} = d_{ej} + D_e(t), \\
\delta'_{fj} &= d_{fj} + D_f(t), \delta'_{gj} = d_{gj} + D_g(t)
\end{aligned} \tag{10}$$

The net duty ratios  $0 \leq \delta'_{aj}, \delta'_{bj}, \delta'_{cj}, \delta'_{dj}, \delta'_{ej}, \delta'_{fj}, \delta'_{gj} \leq 1$  should be within the range of 0 to 1.

For the worst case in respect of seven phase input

$$0 \leq 2|k_j| \times \frac{1}{2\sin(\pi/14)} \leq 1 \tag{11}$$

The maximum value of  $k_j$  is equal to 0.2225. In any switching cycle the output phase should not be open circuited. Thus the sum of the duty ratios in (9) must equal unity. But the summation  $D_a(t) + D_b(t) + D_c(t) + D_d(t) + D_e(t) + D_f(t) + D_g(t)$  is less than or equal to unity. Hence another offset duty-ratio  $\left[1 - \{D_a(t) + D_b(t) + D_c(t) + D_d(t) + D_e(t) + D_f(t) + D_g(t)\}\right]/7$  is added to  $D_a(t), D_b(t), D_c(t), D_d(t), D_e(t), D_f(t)$  and  $D_g(t)$  in (11). The addition of this offset duty-ratio in all switches will maintain the output voltages and input currents unaffected. The above explanation finally derives the maximum modulation index for seven phase input with three phase output from Equation (7) as  $\frac{7}{2}k_j = \frac{7}{2} \times \sin\left(\frac{\pi}{14}\right) = 0.7788$  or 77.88% .

If  $k_A, k_B, k_C$  are chosen to be 3-phase sinusoidal references as given in Equation (6), the input voltage capability is not fully utilized for output voltage generation and the output magnitude remains only 77.88% of the input magnitude. To overcome this, an additional common mode term equal to  $\{\max(k_A, k_B, k_C) + \min(k_A, k_B, k_C)\}/2$  is added as in the carrier-based PWM principle as implemented in two-level inverters. Thus the amplitude of  $(k_A, k_B, k_C)$  can be enhanced from 0.2225 to 0.257.

### 3.1. Without Common-mode Voltage Addition

In the above section, two offsets are added to the original duty ratios to form the following effective duty ratio that can be compared to the triangular carrier wave to generate the gating signals for the bidirectional power switches. The output phase voltage magnitude will reach 77.88% of the input voltage magnitude with this method. To further enhance the output voltage magnitude, common mode voltage of the output reference signals are added to formulate the new duty ratios as discussed in the next section.

### 3.2. With Common-mode Voltage Addition

The duty ratios can further be modified by injection common mode voltage of the output voltage references to improve the output voltage magnitude. The output voltage magnitude increases and reaches its limiting value of 90% of the input magnitude. The common mode voltage that is added to obtain new duty ratios are;

$$V_{cm} = -\frac{V_{Max} - V_{Min}}{2} \tag{12}$$

Where,

$$V_{MAX} = \max\{k_A, k_B, k_C\}, V_{MIN} = \min\{k_A, k_B, k_C\} \tag{13}$$

The duty ratio for output phase p can be written as;

$$\begin{aligned} \delta_{aj} &= D_a(t) + (1 - \{D_a(t) + D_b(t) + D_c(t) + D_d(t) + D_e(t) + D_f(t) + D_g(t)\})/3 + [k_j + V_{cm}] \times \cos(\omega t - \rho) \\ \delta_{bj} &= D_b(t) + (1 - \{D_a(t) + D_b(t) + D_c(t) + D_d(t) + D_e(t) + D_f(t) + D_g(t)\})/3 + [k_j + V_{cm}] \times \cos(\omega t - 2\pi/7 - \rho) \\ \delta_{cj} &= D_c(t) + (1 - \{D_a(t) + D_b(t) + D_c(t) + D_d(t) + D_e(t) + D_f(t) + D_g(t)\})/3 + [k_j + V_{cm}] \times \cos(\omega t - 4\pi/7 - \rho) \\ \delta_{dj} &= D_d(t) + (1 - \{D_a(t) + D_b(t) + D_c(t) + D_d(t) + D_e(t) + D_f(t) + D_g(t)\})/3 + [k_j + V_{cm}] \times \cos(\omega t - 6\pi/7 - \rho) \\ \delta_{ej} &= D_e(t) + (1 - \{D_a(t) + D_b(t) + D_c(t) + D_d(t) + D_e(t) + D_f(t) + D_g(t)\})/3 + [k_j + V_{cm}] \times \cos(\omega t + 6\pi/7 - \rho) \\ \delta_{fj} &= D_f(t) + (1 - \{D_a(t) + D_b(t) + D_c(t) + D_d(t) + D_e(t) + D_f(t) + D_g(t)\})/3 + [k_j + V_{cm}] \times \cos(\omega t + 4\pi/7 - \rho) \\ \delta_{gj} &= D_g(t) + (1 - \{D_a(t) + D_b(t) + D_c(t) + D_d(t) + D_e(t) + D_f(t) + D_g(t)\})/3 + [k_j + V_{cm}] \times \cos(\omega t + 2\pi/7 - \rho) \end{aligned} \tag{14}$$

Where  $j \in A, B, C$

**4. SIMULATION RESULTS**

The proposed scheme is validated by simulation of seven to three phase Matrix converter using Matlab/Simulink. The model is simulated for seven phase input at 30Hz with 100V peak per phase. The output is set at 50Hz frequency. Simple R-L load is connected at the output. The value of R is 10 Ω and L is 10 mH. With this constraint the model is simulated for 60millisec or 0.06sec. The output voltages have constant frequency and constant voltage for all the time of simulation. Different simulation results are shown in the figures. The practical implementation of the proposed scheme is under study.

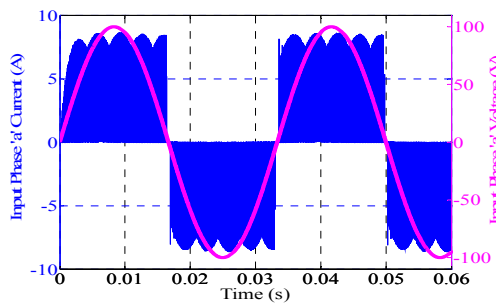


Figure 2. Input Phase Voltage and Current

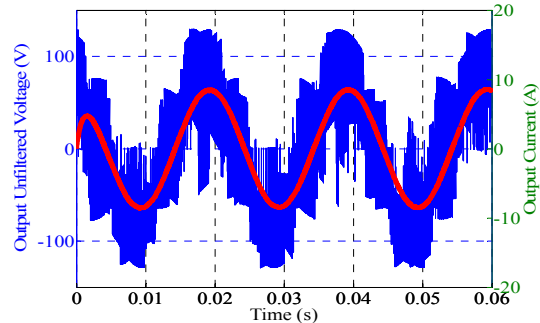


Figure 3. Output Current and Unfiltered Output Voltage

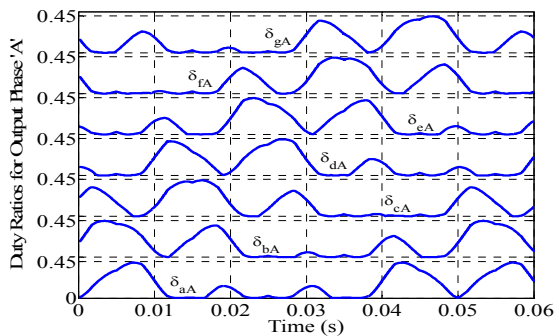


Figure 4. Output Duty Ratios

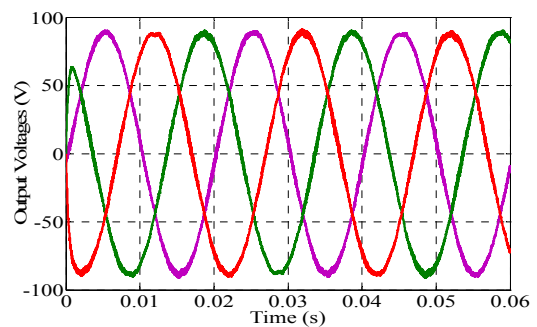


Figure 5. Filtered Output Voltage

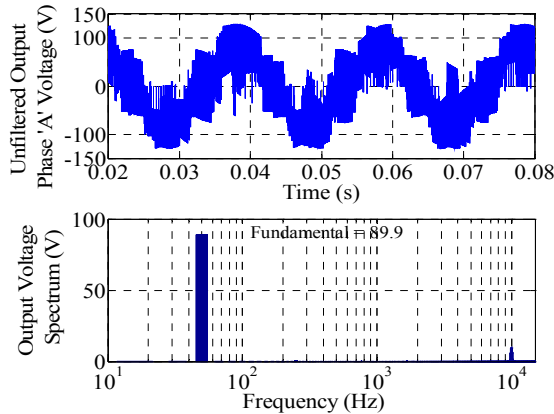


Figure 6. Spectrum of Output Voltage

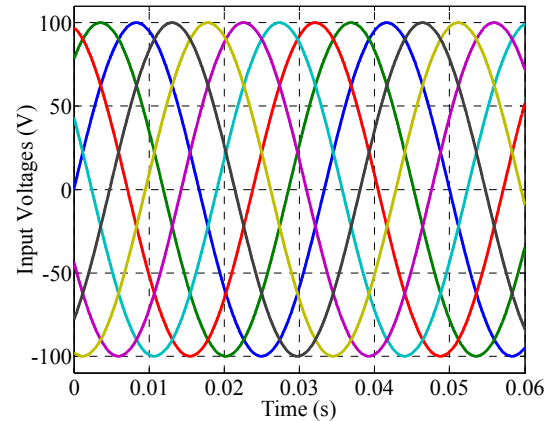


Figure 7. Seven-Phase Input Voltages

## 5. CONCLUSION

The paper proposes modulation and control of a seven-phase input and three-phase output direct ac-ac converter. The control objective is to obtain fixed voltage and fixed frequency three-phase output for variable voltage and variable frequency seven-phase input. Simple carrier-based scheme is employed along with offset addition to enhance the output voltage magnitude. The output voltage magnitude is higher when compared to a conventional three-phase to three-phase matrix converter. The proposed technique can be modified to obtain variable voltage and variable frequency for the adjustable drive application too. The simulation results are reported in the paper to validate the proposed scheme.

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