Sliding Mode Backstepping Control of Induction Motor

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ABSTRACT
This work treats the modeling and simulation of non-linear system behavior of an induction motor using backstepping sliding mode control (BACK-SMC). First, the direct field oriented control IM is derived. Then, a sliding for direct field oriented control is proposed to compensate the uncertainties, which occur in the control. Finally, the study of Backstepping sliding controls strategy of the induction motor drive. Our non linear system is simulated in MATLAB SIMULINK environment, the results obtained illustrate the efficiency of the proposed control with no overshoot, and the rising time is improved with good disturbances rejections comparing with the classical control law.

1. INTRODUCTION
The development of induction motor drives has considerably accelerated in order to satisfy the increasing need of various industrial applications in low and medium power range. Indeed, induction motors have simple structure, high efficiency and increased torque/inertia ratio. However, their dynamical model is nonlinear, multivariable, coupled, and is subject to parameter uncertainties since the physical parameters are time-variant. The design of robust controllers becomes then a relevant challenge [1]-[2].

Induction motor drives control has been an active research domain over the last years. Different control techniques such as Field-Oriented control (FOC), feedback linearization control, sliding mode control passivity approach, and adaptive control have been reported in the literature [3]. The FOC ensures partial decoupling of the plant model using a suitable transformation and then PI controllers are used for tracking regulation errors. The high performance of such strategy may be deteriorated in practice due to plant uncertainties [4]-[5]. Exact input-output feedback linearization of induction motors model can be obtained using tools from differential geometry. This method cancels the nonlinear terms in the plant model which fails when the physical parameters varies [6]-[7]. By contrast, passivity-based control does not cancel all the nonlinearities but enforce them to be passive, i.e. dissipating energy and hence ensuring tracking regime [8]-[10]. Sliding Mode Control (SMC) is widely applied because of its easiness and attractive robustness properties [11]-[12].

Otherwise, the conventional PI controllers are the most common algorithms used in industry today. Their attractiveness is due to their structure simplicity and the industrial operators acquaintance with them. Several PI controllers have been proposed in the literature for linear and nonlinear processes [5], [15]. Nevertheless, PI controllers fundamental deficiency is the lack of asymptotic stability and robustness proofs for a given nonlinear system.
Therefore, this paper proposes to deal with this deficiency by proposing a robust nonlinear PI controller for an induction motor drive with unknown load torque. The controller is derived by combining a backstepping procedure with a sliding mode. More precisely, the controllers are determined by imposing the current-speed tracking recursively in two steps and by using appropriate gains that are nonlinear functions of the system state. The advantage of Backstepping sliding mode control is its robustness and ability to handle the non-linear behaviour of the system.

The model of the induction motor, and shows the direct field-oriented control (FOC) of induction motor in Section (2). Section (3) shows the development of sliding technique control design. Section (4) shows the development of Backstepping technique control design. The Speed Control of induction machine by Backstepping sliding mode controllers design is given in Section (5). Simulation results using MATLAB SIMULINK of different studied cases is defined in Section (6). Finally, the conclusions are drawn in Section (7).

2. MATHEMATICAL MODE OF IM

The used motor is a three-phase induction motor type (IM) supplied by an inverter voltage controlled with Pulse Modulation Width (PWM) techniques. A model based on circuit equivalent equations is generally sufficient in order to make control synthesis. The dynamic model of three-phase, Y-connected induction motor can be expressed in the d-q synchronously rotating frame as [13]:

\[
\begin{align*}
\frac{di_{\alpha}}{dt} &= a_1 i_{\alpha} + w_r i_{\phi_{\alpha}} + a_2 \phi_{\alpha} + b V_{\alpha} \\
\frac{di_{\phi_{\alpha}}}{dt} &= a_3 i_{\phi_{\alpha}} - w_r i_{\phi_{\alpha}} + a_4 \phi_{\phi_{\alpha}} + b V_{\phi_{\alpha}} \\
\frac{d\phi_{\alpha}}{dt} &= a_5 i_{\phi_{\alpha}} + a_6 \phi_{\alpha} \\
\frac{dw_{\alpha}}{dt} &= a_7 (i_{\phi_{\alpha}} \cdot \phi_{\alpha}) + a_8 \phi_{\alpha} + a_9 C_r
\end{align*}
\]

Where \( \sigma \) is the coefficient of dispersion and is given by:

\[
\sigma = 1 - \frac{L_{m}^{2}}{L_{r} L_{qr}}, \quad b = \frac{1}{\text{sig} \cdot L_{r}}, \quad a_1 = -b \left( R + \frac{L_{m}^{2}}{L_{r}} \right) R, \quad a_2 = \frac{L_{m} R}{\text{sig} L_{r} L_{qr}},
\]

\[
a_3 = -b \frac{L_{m}}{L_{r}}, \quad a_4 = \frac{L_{m} R}{L_{r}}, \quad a_5 = -R \frac{1}{L_{r}}, \quad a_6 = \frac{P L_{m}}{J L_{r}}, \quad a_7 = \frac{f_c}{J}, \quad a_8 = \frac{p}{J}
\]

3. SLIDING MODE CONTROL

A Sliding Mode Controller (SMC) is a Variable Structure Controller (VSC) [10]. Basically, a VSC includes several different continuous functions that can map plant state to a control surface, whereas switching among different functions is determined by plant state represented by a switching function [7]. Without lost of generality, consider the design of a sliding mode controller for the following first-order System.

\[
x^* = A(x, t) \cdot x + B(x, t) \cdot U
\]

Where \( U \) the input to the System the following is a possible choice of the structure of a sliding mode controller

\[
U = U_{eq} + K \cdot \text{Sign} \left[ s(x, t) \right]
\]

Where stands for equivalent control used when the System state is in the Sliding mode [2], \( K \) is a constant, being the maximal value of the controller output. \( S \) is switching function since the control action switches its sign on the two sides of the switching surface \( S(x) = 0 \), \( S \) is defined as [14]:
\[ S(x) = \left( \frac{\partial}{\partial t} + \lambda \right)^{-1} e(x) \] (5)

Where:

\[ e(x) = x^* - x, \]

\( x^* \) being the desired state. \( \lambda \) is a constant. Concerning the development of the control law, it is divided into two parts, the equivalent control \( U_{eq} \) and the attractively or reachability control \( U_s \). The equivalent control is determined off-line with a model that represents the plant as accurately as possible. If the plant is exactly identical to the model used for determining \( U_{eq} \) and there are no disturbances, there would be no need to apply an additional control \( U_s \). However, in practice there are a lot of differences between the model and the actual plant. Therefore, the control component \( U_s \) is necessary which will always guarantee that the state is attracted to the switching surface by satisfying the condition [13], [14].

\[ \dot{S}(x) \cdot S(x) < 0 \]

Therefore, the basic switching law is of the form:

\[ U = U_{eq} + U_{sw} \] (6)

\( U_{eq} \) is the equivalent control, and \( U_{sw} \) is the switching control. The function of \( U_{eq} \) is to maintain the trajectory on the sliding surface, and the function of \( U_{sw} \) is to guide the trajectory to this surface.

The surface is given by:

\[ S_1 = z_1 = w_m^* - w_m \] (7)

The derivative of the surface is:

\[ \dot{S}_1 = \dot{z}_1 = \dot{w}_m^* - \dot{w}_m \] (8)

In a conventional variable structure control, \( U_n \) generates a high control activity. It was first taken as constant, a relay function, which is very harmful to the actuators and may excite the model dynamics of the System. This is known as a chattering phenomenon. Ideally, to reach the sliding surface, the chattering phenomenon should be eliminated [13], [14]. However, in practice, chattering can only be reduced.

The first approach to reduce chattering was to introduce a boundary layer around the sliding surface and to use a smooth function to replace the discontinuous part of the control action as follows:

\[
\begin{align*}
U_{nw} &= \frac{K}{\varepsilon} S(x) & \text{if } |S(x)| < \varepsilon \\
U_{nw} &= K \cdot \text{sgn}(S(x)) & \text{if } |S(x)| > \varepsilon
\end{align*}
\] (9)

The constant \( K \) is linked to the speed of convergence towards the sliding surface of the process (the reaching mode). Compromise must be made when choosing this constant, since if \( K \) is very small the time response is important and the robustness may be lost, whereas when \( K \) is too big the chattering phenomenon increases.
4. BACKSTEPPING CONTROL DESIGN

In this section, we use the Backstepping algorithm to develop the speed control law of the induction motor. This speed will converge to the reference value from a wide set of initial conditions.

Step 1: Firstly we consider the tracking objective of the direct current ($\phi_{dr}$). A tracking error $z_1 = w_{md} - w_m$ is defined, and the derivative becomes:

$$\dot{z}_1 = \dot{w}_{md} - \dot{w}_m$$  \hspace{1cm} (10)

$$\dot{z}_1 = \dot{w}_{md} - a_6 i_{qs} \phi_{dr} - a_7 w_m - a_8 C_r$$  \hspace{1cm} (11)

The proposed virtual command is:

$$\left(a_6 i_{qs} \phi_{dr}\right) = -a_7 w_m - a_8 C_r + c_1 z_1 + \dot{w}_{md}$$ \hspace{1cm} (12)

$$v(z_1) = z_1 \dot{z}_1 = z_1 \left[w_{md} - (-a_1 w_m - a_8 C_r + c_1 z_1 + \dot{w}_{md}) - a_7 w_m - a_8 C_r \right]$$ \hspace{1cm} (13)

With $c_1 > 0$

Step 2: The derivative of the error variable

$$z_2 = \left(a_6 i_{qs} \phi_{dr}\right) - a_6 i_{qs} \phi_{dr}$$ \hspace{1cm} (14)

$$z_2 = (-a_7 w_m - a_8 C_r + c_1 z_1 + \dot{w}_{md}) - a_6 i_{qs} \phi_{dr}$$ \hspace{1cm} (15)

$$\dot{z}_2 = \Phi_1 + \Phi_2 v_{qs}$$

$$\Phi_1 = \left[\dot{w}_{md} + c_1 \dot{w}_{md} - a_8 \dot{\phi}_{dr} + (-c_1 - a_7)(a_6 i_{qs} \phi_{dr} + a_7 w_m + a_8 C_r) \right]$$

$$- a_6 \left[a_4 i_{qs} - w_s i_{ds} + a_5 \phi_{dr} w_m + b v_{qs} \right] + i_{qs} \left(a_4 i_{qs} + a_5 \phi_{dr} \right)$$
\[ \Phi_2 = -a_6 b \phi_{dr} \]  
\( \dot{\psi}(z_2) = z_2 \dot{z}_2 = z_2 (\Phi_1 + \Phi_2 v_{qi}) \)  
\[ v_{qi} = -\frac{\Phi_1 - c_2 z_2}{\Phi_2} \]  
\[ \dot{\psi}(z_1) = -c_2 z_2^2 \]  

With \( c_2 > 0 \)

**Step 3:**

\[ z_3 = \phi_{dr}^* - \dot{\phi}_{dr} \]  
\[ \dot{z}_3 = \dot{\phi}_{dr}^* - \dot{\phi}_{dr} = \dot{\phi}_{dr}^* - a_4 i_{ds} - a_5 \phi_{dr} \]  

The proposed virtual command is:

\[ (a_4 i_{ds})^* = -a_5 \phi_{dr}^* + \dot{\phi}_{dr}^* + c_3 z_3 \]  
\[ \dot{z}_3 = \dot{\phi}_{dr}^* - (a_5 \phi_{dr}^* + \dot{\phi}_{dr}^* + c_3 z_3) - a_5 \phi_{dr} \]  
\[ \dot{\psi}(z_3) = z_3 \dot{z}_3 = -c_2 z_3^2 \]  

With \( c_3 > 0 \)

**Step 4:**

\[ z_4 = (a_4 i_{ds})^* - a_4 i_{ds} \]  
\[ z_4 = (a_4 i_{ds})^* - a_4 i_{ds} = -a_5 \phi_{dr}^* + \dot{\phi}_{dr}^* + c_3 z_3 - a_4 i_{ds} \]  
\[ \dot{z}_4 = \Phi_3 + \Phi_4 v_{ds} \]  

With,

\[ \Phi_3 = \dot{\phi}_{dr}^* + c_3 \dot{\phi}_{dr}^* - (a_5 + c_3) (a_4 i_{ds} + a_5 \phi_{dr}) - a_4 (a_4 i_{ds} + w_s i_{qs} + a_2 \Phi_{dr}) \]  
\[ \Phi_4 = -a_4 b \]  
\[ v_{ds} = -\frac{\Phi_4 - c_4 z_4}{\Phi_4} \]  
\[ \dot{z}_4 = \Phi_3 + \left( -\frac{\Phi_3 - c_4 z_4}{\Phi_4} \right) \Phi_4 \]
\[ \dot{v}(z_4) = z_4 \dot{z}_4 = -c_4 z_4^2 \]  
(26)

With \( c_4 > 0 \).

5. ASSOCIATION BACKSTEPPING SLIDING MODE CONTROL

The control law obtained is:

\[ \dot{z}_2 = \Phi_1 + \Phi_2 v_{qs} = -q_4 \text{sign}(z_4) - q_2 z_2 \]

Then;

\[ v_{qs} = \frac{(- \Phi_1 - q_4 \text{sign}(z_4) - q_2 z_2)}{\Phi_2} \]

(27)

And,

\[ \dot{z}_4 = \Phi_3 + \Phi_4 v_{ds} = -q_4 \text{sign}(z_4) - q_4 z_4 \]

Then;

\[ v_{ds} = \frac{(- \Phi_3 - q_4 \text{sign}(z_4) - q_4 z_4)}{\Phi_4} \]

(28)

The Figure 2 shows the backstepping sliding control strategy scheme for each induction motor.

![Figure 2. Block diagram speed control of IM by a combination of the BACK-SMC command](image)

6. SIMULATION RESULTS

The three controls adopted as PI, sliding mode, and Backstepping sliding mode are tested by numerical simulation for the values of these coefficients:

<table>
<thead>
<tr>
<th></th>
<th>( \rho_{op}=10 )</th>
<th>( \rho_{op}=10 )</th>
<th>( \rho_{op}=800 )</th>
<th>( \rho_{op}=800 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>PI</td>
<td>( L_d=800 )</td>
<td>( \text{eps}_1=10^{-4} )</td>
<td>( K_v=2500 )</td>
<td>( K_v=2500 )</td>
</tr>
<tr>
<td>SMC</td>
<td>( L_d=800 )</td>
<td>( \text{eps}_2=10^{-4} )</td>
<td>( K_v=1500 )</td>
<td>( K_v=1500 )</td>
</tr>
<tr>
<td>Back-SMC</td>
<td>( C_1=100 )</td>
<td>( q_1=100 )</td>
<td>( q_3=100 )</td>
<td>( \text{eps}_1=10^{-8} )</td>
</tr>
<tr>
<td></td>
<td>( C_2=300 )</td>
<td>( q_2=100 )</td>
<td>( q_4=100 )</td>
<td>( \text{eps}_2=10^{-8} )</td>
</tr>
</tbody>
</table>

The simulation results are showing Figure 3-Figure 5. Figure 3 shows the speed with PI, SMC And Back-SMC, Figure 4 the torque and Figure 5 shows the current \( I_{a,b,c} \).
Figure (3), (4) and (5) shows the evolution of electrical and mechanical parameters of the IM ideal voltage supplied to a load variation between 0.5 seconds and 1 second, and reverse speed set point at time 1.5 second 200 -200 [rad / s].

The results show a good response in IM alimented ideal tension, continuing with a very low response time and a static error to zero for Backstepping control mode by sliding the control input to the PI controller, and controller with sliding mode. The couple has a peak related to the start and fades during permanent regime. The load change has also allowed us to conclude on the rejection of the disturbance which is satisfactory.

![Figure 3. The speed of IM](image1)

![Figure 4. The torque](image2)

![Figure 5. The Current Ia,b,c](image3)

7. CONCLUSION

The sliding mode control of the field oriented induction motor was proposed. To show the effectiveness and performances of the developed control scheme, simulation study was carried out. Good results were obtained despite the simplicity of the chosen sliding surfaces. The robustness and the tracking qualities of the proposed control system using sliding mode controllers appear clearly.

Furthermore, in order to reduce the chattering, due to the discontinuous nature of the controller, backstepping controllers were added to the sliding mode controllers.

These gave good results as well and simplicity with regards to the adjustment of parameters. The simulations results show the efficiency of the sliding mode controller technique, however the strategy of backstepping sliding mode controller brings good performances.

REFERENCES


**BIOGRAPHIES OF AUTHORS**

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