Modeling and Analysis of Transformerless High Gain Buck-boost DC-DC Converters

Vu Tran* and Mufeed MahD*

*Department of Electrical Engineering, University of Massachusetts, Lowell

ABSTRACT
This paper proposes a transformerless switched capacitor buck boost converter model, which provides higher voltage gain and higher efficiency when compared to the conventional buck boost converter. The averaged model based on state-space description is analyzed in the paper. The simulation results are presented to confirm the capability of the converter to generate high voltage ratios. The comparison between the proposed model and the traditional model is also provided to reveal the improvement. The proposed converter is suitable for a wide application which requires high step-up DC-DC converters such as DC micro-grids and solar electrical energy.

1. INTRODUCTION
In recent years, as the demand for power is significantly increasing, renewable energy sources have received a lot of attention as an alternative way of generating direct electricity. Using renewable energy system has a lot of advantages. One of those is that it can eliminate harmful emissions from polluting the environment while also offering inexhaustible resources of primary energy. There are many sources of renewable energy, such as solar energy, wind turbines, and fuel cells. However, some sources such as fuel cells and solar cells have low output voltage [1], [2], [3]. Thus, a high efficiency and step-up DC-DC converter is desired in today’s system designs to increase the voltage supplied to the grid or be compatible in other applications.

Basically, by using an extremely high duty cycle, the traditional boost DC-DC converter can provide a very high voltage gain. However in actual applications, for a very high duty cycle, the voltage gain is reduced due to the non ideal elements in circuits such as inductors, capacitors, switches, diodes, etc. Moreover, extremely high duty cycle can create electromagnetic interference [4] [5], which might diminish the efficiency of the operation of circuits. Several researchers have designed models that can achieve high voltage gain while overcoming these disadvantages of traditional converters. High gain can be achieved by cascading two or more step-up converter stages [16] [17] [18] [19]. Extreme duty cycles can be avoided by setting an intermediate voltage between the two stages. However, additional components are required, the control circuit is more sophisticated and the total efficiency is reduced [5] [20].

Several step-up converter models using transformers are presented in [6] [7] [8]. In these models, the voltage gain is controlled by creating a conversion ratio function of the duty ratio and the transformer turns ratio. However, its efficiency will dramatically degrade by losses associated with the leakage inductance,
and may cause power losses and heat dissipation problems [9] [10]. Another disadvantage is the big size and heavy weight of the transformer, which is often desired to be as compact as possible in actual design. In [4] [9] [11] [12] [13] [14] [15], high step up converters using coupled-inductor technique is introduced. Coupled inductors were modeled to provide a high step up voltage and reduce the switch voltage stress, and the reverse recovery problem of the diode was reduced. However, the electromagnetic interference and efficiency is reduced due to the leakage inductance [16] [14], and the designing of the converter is relatively complex [17].

Another way of increasing voltage gain is integrating a switched-capacitor (SC) circuit with a boost converter, which is proposed in [1] [14] [21] [22]. The voltage gain can be improved by increasing number of charge pumps. However, the voltage gain will be reduced significantly if the input voltage is as small as the voltage dropping on the two diodes [23]. The other limitation causes from charge pump itself, if the switching frequency is not sufficiently fast, the capacitors will block the DC current, making the system less efficiency.

In order to deal with low-voltage photovoltaic (PV) arrays and the required higher voltage of the grid, a novel buck boost converter is proposed in this paper based on the traditional buck boost converter. The model is simple, which includes only one inductor, two capacitors and four power switches and two diodes, and thus, it is very easy to implement. With this model, we are able to save the wasted energy in the OFF state of the switches used in the circuit. Therefore, the proposed converter can not only provide high voltage gain, but also reduce the extremely high duty cycles of power switches, and increase the efficiency of the converter.

The paper is organized as follow. Section II describes the proposed circuit schematic. Section III analyzes steady-state topologies by using state-space approach. Section IV presents the simulation results for averaged model and pulse width modulation (PWM) model. Section V concludes the paper.

2. CIRCUIT DESCRIPTION

![Figure 1. Traditional buck boost converter](image1)

The traditional buck boost converter is presented in Fig. 1. When the switch is ON, the energy from the power supply (PV panel) \( V_g \) is stored in the inductor. When the switch is OFF, it is easy to see that this energy is wasted. For example, when the duty cycle is 60%, at least \( 100\% - 60\% = 40\% \) of power is wasted. The new model of buck boost converter is proposed to save that wasted energy, which is illustrated in Fig. 2.

![Figure 2. Switched-capacitor buck-boost converter with high voltage gain](image2)

In the OFF state, energy from power source is stored in capacitors \( C \), then in the ON state, it is
pushed back to the circuit. Hence, the energy is saved and the efficiency is improved when compared with that of the traditional buck-boost converter.

![Figure 3. One stage switched-capacitor buck-boost converter](image)

The model can be extended to increase the voltage gain by simply increasing the stage, which includes the capacitor $C$, diode $D_1$, switch $S_b$, and two switches $S_r$. For simplicity, we consider the case of one stage as in Fig. 3. Adding more stages can be treated similarly.

![Figure 4. Switching topology of proposed circuit in OFF state](image)

In OFF state, the circuit is modeled in Fig. 4. Two switches $Q_1$ and $Q_2$ are OFF, $Q_3$ and $Q_4$ are ON, the current flows through $Q_3$ and $Q_4$, energy from power source $V_g$ is charged for the capacitor $C$. Diode $D_0$ is forward biased, which allows the current to go through the inductor to charge for capacitor $C_0$ and provide for the output simultaneously.

![Figure 5. Switching topology of proposed circuit in ON state](image)

Fig. 5 describes the circuit in the ON state. Two switches $Q_1$ and $Q_2$ are ON, $Q_3$ and $Q_4$ are OFF. Voltage source $V_g$ combined with the voltage on $C$ create a higher voltage, which will be pushed to the inductor $L$. More energy will be stored in the inductor compared with the traditional buck-boost converter. Diode $D_1$ acts as a free-wheeling diode.

### 3. STATE-SPACE MODEL OF THE SWITCHED CAPACITOR BUCK BOOST CONVERTER

In this section, the equivalent circuits of traditional model and proposed model are analyzed for ON and OFF states. In both models, when the diode operates in forward bias region, it is replaced by a voltage source $V_D$. When it is in reverse bias region, it blocks the current, hence it is replaced by an open circuit. MOSFET switches are controlled by PWM signals. When MOSFET is OFF, it works like an open circuit. When it is ON, it works like a resistor with the resistance $R_{ON}$.
Let the duty cycle of the PWM signal be $D$ and the DC voltage supply be $V_g$. The input is $u = [V_g \ V_D]^T$.

### 3.1. State-space description of the traditional model

Let the state of the traditional buck-boost converter be $X_T = [I_L \ V_0]^T$.

In $ON$ state, the equivalent circuit is described in Fig. 6

![Figure 6. Traditional buck boost converter’s equivalent circuit in $ON$ state](image)

Apply the KVL equations for this circuit, we have

\[
L \frac{dI_L}{dt} = V_g - (R_{ON} + R_L)I_L
\]

\[
C_0 \frac{dV_0}{dt} = -\frac{V_0}{R}
\]

The state space description of the circuit can be represented as $X'_T(ON) = A_{01}X_T + B_{01}u$

In which,

\[
A_{01} = \begin{bmatrix}
-\frac{R_{ON} + R_L}{L} & 0 \\
0 & -\frac{1}{RC_0}
\end{bmatrix}
\]

and

\[
B_{01} = \begin{bmatrix}
\frac{1}{\tau} \\
0
\end{bmatrix}
\]

In $OFF$ state, the equivalent circuit is described in Fig. 7

![Figure 7. Traditional buck boost converter’s equivalent circuit in $OFF$ state](image)

Similarly, applying the KVL equations for this circuit yields

\[
L \frac{dI_L}{dt} = V_0 - V_D - I_L R_L
\]

\[
C_0 \frac{dV_0}{dt} = -\frac{V_0}{R} - I_L
\]

The state space description of the circuit can be represented as $X'_T(OFF) = A_{02}X_T + B_{02}u$
In which,
\[
A_{02} = \begin{bmatrix}
-\frac{R_L}{L} & \frac{1}{RC_0} \\
-\frac{1}{C_0} & -\frac{1}{RC_0}
\end{bmatrix}
\]
and
\[
B_{02} = \begin{bmatrix}
0 & -\frac{1}{\tau} \\
0 & 0
\end{bmatrix}
\]
Under the assumption of high frequency and ideal switching, the average model can be described as
\[
X_T = DX_T(ON) + (1 - D)X_T(OFF)
\]
\[
X_T' = (DA_{01} + (1 - D)A_{02})X_T + (DB_{01} + (1 - D)B_{02})u
\]
(1)
At steady state, the state of the circuits can be considered stable, or \(X'(t) = 0\). Solve the equations (1) with the condition of steady state \(X'_T = 0\), we have the state of the circuit \(X_T\).
\[
X_T = -(DA_{01} + (1 - D)A_{02})^{-1}(DB_{01} + (1 - D)B_{02})u
\]
(2)
From equation (1) we have the output voltage \(V_0\).
\[
V_0 = -\frac{R(1 - D)}{D R_{ON} + R_L + (1 - D)^2 R} (DV_g - (1 - D)V_D)
\]
(3)
We will not consider the full range \([0,1]\) of duty cycle due to the nonidealities. For very large or very small duty cycle, the averaged model does not reflect precisely the real circuit. Since there is no benefit in increasing the duty cycle beyond the value where the minimal output voltage is reached, we would prefer to limit the duty cycle in a smaller range. For the above example, we may limit \(D \in [0.1, 0.85]\).
Assumed that the resistors \(R_{ON}\) and \(R_L\) are much smaller than \(R\), and \(V_D\) is much smaller compared to \(V_g\). The equation (3) can be simplified as
\[
V_0 \approx -\frac{D}{1 - D} V_g
\]
(4)
3.2. State space description of the proposed model
Let the state of the traditional buck-boost converter be \(X_N = [I_L \quad V_0 \quad V_C]^T\).
In \textit{ON} state, the equivalent circuit is described in Fig. 8.

![Switched capacitor buck boost converter's equivalent circuit in ON state](image_url)

KVL equations for this circuit is as follows
\[
L \frac{dI_L}{dt} = V_g - (2R_{ON} + R_L)I_L + V_C
\]
\[
C_0 \frac{dV_0}{dt} = -\frac{V_0}{R}
\]
\[
C \frac{dV_C}{dt} = -I_L
\]
The state space description is \(X_N'(ON) = A_{11}X_N + B_{11}u\)
In which,
\[
A_{11} = \begin{bmatrix}
- \frac{2R_{ON} + R_L}{L} & 0 & 0 & \frac{1}{C} & 0 \\
0 & - \frac{1}{R_{ON}C} & \frac{1}{C} & 0 & 0 \\
- \frac{1}{C} & 0 & 0 & - \frac{1}{R_{ON}C} & 0 \\
0 & \frac{1}{C} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]
and
\[
B_{11} = \begin{bmatrix}
\frac{1}{L} \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]

In OFF state, the equivalent circuit is described in Fig. 9

KVL equations for this circuit gives
\[
L \frac{dI_L}{dt} = V_0 - R_LI_L - V_D
\]
\[
C_0 \frac{dV_0}{dt} = - \frac{V_0}{R} - I_L
\]
\[
C' \frac{dV_C}{dt} = \frac{V_D - V_C}{2R_{ON}}
\]
The circuit can be described as \(X'_N(\text{OFF}) = A_{12}X_N + B_{12}u\)

In which,
\[
A_{12} = \begin{bmatrix}
- \frac{R_L}{L} & 0 & 0 & \frac{1}{C} & 0 \\
0 & - \frac{1}{R_{ON}C} & \frac{1}{C} & 0 & 0 \\
- \frac{1}{C} & 0 & 0 & - \frac{1}{R_{ON}C} & 0 \\
0 & \frac{1}{C} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]
and
\[
B_{12} = \begin{bmatrix}
0 \\
0 \\
0 \\
\frac{1}{2R_{ON}C} \\
0
\end{bmatrix}
\]

Under the assumption of high frequency and ideal switching, the average model can be described as
\[
X_N = DX_N(\text{ON}) + (1 - D)X_N(\text{OFF})
\]
and
\[
X'_N = (DA_{11} + (1 - D)A_{12})X_N + (DB_{11} + (1 - D)B_{12})u
\]

Under the same above assumption, the state of the circuit is represented in (6).
\[
X_N = -(DA_{11} + (1 - D)A_{12})^{-1}(DB_{11} + (1 - D)B_{12})u
\]

From (1), the output voltage \(V_0\) is calculated in (7)
\[
V_0 = - \frac{R(1 - D)(2DV_D - (1 - D)V_D)}{2D R_{ON} + R_L + D^2 L^2 R_{ON} + (1 - D)^2 R}
\]
$V_0$ can be approximated as

$$V_0 \approx -\frac{2D}{1-D}V_g \quad (8)$$

Hence, with the proposed switched capacitor buck boost circuit, the expected gain is doubled when compared with traditional buck boost converter. Generally, when we have $n$ stages, by similarly calculation, the output gain is $n$ times than the traditional one. The following part will demonstrate how the simulated circuit is working.

4. SIMULATION RESULTS

In this section, we constructed two simulation models, one is a state-space model based on the averaged circuit, which is called the averaged model. The other is a simulated circuit using SimPower in MatLab, which is called the PWM model. We will use this two circuits to demonstrate the theoretically results we obtain from section III. The parameters for the circuit are given as follow: $L = 0.1mH$, $R_L = 0.2\Omega$, $C_0 = 1mF$, $C = 0.47mF$, $R = 20\Omega$, $R_{ON} = 0.01\Omega$, $V_g = 6V$, $V_D = 0.3V$.

![Figure 10. Output of averaged model](image)

Fig. 10 shows results for the averaged model based on the averaged equations (3) and (7). In not a very high duty cycle region, energy from power source is stored in capacitor $C$, the voltage $V_C$ is equal to $V_g$. When it is pushed back to the circuit, the input becomes $V_g + V_C = 2V_g$. The output voltage of the proposed circuit is almost doubled compared with that of traditional buck-boost converter.

![Figure 11. System’s output with different load and duty cycle](image)

Fig. 11 presents outputs of the averaged model for traditional and proposed circuit with different duty cycles (from 0% to 100%) and resistive load $R$ (from 1\Omega to 50\Omega). We can see clearly in the figure...
that the output voltages is increased when load is increased. For the proposed model, the voltage output is always double when compared with the traditional model for a given duty cycle and a specific load.

Fig. 12 shows the simulation result. In this figure, the output voltage of traditional buck-boost converter is plotted as the red curve; and the output of the proposed converter is represented as the blue curve. Four difference sub figures associated with the duty cycles being 30%, 50%, 70%, and 80% are plotted. We can see that the output voltages of the proposed model are almost doubled compared with the traditional one. This empirical result confirms the feasibility of the proposed model.

Figure 12. Output voltage of simulink model compared with traditional buck boost circuit with D=30%, 50%, 70%, and 80%

5. CONCLUSION

A novel buck boost converter with a switched capacitor for high step-up converter is presented in this paper. Adding one more stage of switched capacitor significantly improves the voltage gain compared to the traditional one. Efficiency is also improved through the process of storing energy in the capacitor and then pushing it back to the circuit. The simulation results validate the theoretical results. The proposed converter is applicable in many applications in which high efficiency model is required.

REFERENCES


BIOGRAPHIES OF AUTHORS

Vu Tran is a graduate research assistant at Umass Lowell with Master of Biomedical Engineering from Tufts University (2010) and Master of Electrical Engineering from Hanoi University of Technology (Vietnam) in 2002. He obtained Engineering Degree in Electronics and Telecommunications from Hanoi University of Technology (Vietnam) in 2000. His researches are in fields of power electronics and control.

Mufeed Mahd
His researches are in fields of electronics, digital systems, ultrasound sensor, signal processing, and biomedical instrumentation.

Further info on his homepage: http://faculty.uml.edu/MufeedMahd/index.html