International Journal of Power Electronics and Drive System (IJPEDS)
Vol. 5, No. 2, October 2014, pp. 237~243
ISSN: 2088-8694

Modelling of Variable Frequency Synchronous Buck Converter

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Article Info

Article history:

Received Jul 11, 2014
Revised Sep 9, 2014
Accepted Sep 25, 2014

Keyword:

DC-DC converter,
Enhanced Small Signal Analysis (ESSA),
Equivalent Series Inductance (ESL)

ABSTRACT

In this paper, novel small-signal averaged models for dc–dc converter operating at variable switching frequency are derived. This is achieved by separately considering the on-time and the off-time of the switching period. The derivation is shown in detail for a synchronous buck converter. The Enhanced Small Signal (ESSA) Model is derived for the synchronous buck converter. The equivalent series inductance (ESL) is also considered in this modelling. The buck converter model is also simulated in MATLAB and the result is also presented.

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1. INTRODUCTION

In the control of dc–dc converters, two control objectives are apparent: performance and efficiency. On the one hand, the research focus in the community has been put on the optimization of the conversion efficiency. For instance, the switching frequency can be reduced at low loads, or the control scheme could be switched between a constant on-time and a constant off-time control scheme depending on the load conditions. On the other hand, a strong interest can be found in the optimization of the dynamic performance.

When a variation of the switching period is tolerable during converter operation, this additional degree of freedom offers the opportunity of tight (near optimum) voltage regulation. Nevertheless, to fully exploit the switching period modulation in terms of dynamic transient performance and to ensure stability in all conditions, accurate models, which cover the dynamics of the power conversion system under variable frequency operation, are needed. A frequency-selective averaging is applied such that the switching frequency appears in the dynamic system model. The derivations give an accurate model of the converter dynamics also for situations when the traditional small-ripple conditions are not satisfied, but yield a nonlinear time-varying system formulation. As previous classic control theories largely depend on a linearized representation of the system under exam, the resulting model is of limited interest for the targeted design objective.

In this paper, an alternative formulation of the SSA model is presented, which yields a linearized small-signal representation of the power conversion circuit, where the on-time, as well as the off-time of the pulse-width modulation (PWM) signal are treated as distinct control inputs. In this manner, one can study the dynamics under variable switching frequency operation. The correctness of the enhanced converter representation is also discussed in this paper.

2. CONVERTER MODELLING

In this section, a small-signal averaged and linearized model will be derived, in which the on-time ˆton and the off-time ˆtoff of the PWM signal driving the power stage of the dc–dc converter appear as additional inputs to the system. This is in contrast to the conventional SSA model, where the small-signal duty cycle dˆ is the only control variable. The on-time is defined as the time period during which the binary PWM signal is "H", and the off-time is defined as the period of time during which the PWM signal is "L". Accordingly, for a synchronous buck converter in continuous conduction mode (CCM), during the on-time the high-side switch S1 is conducting and the low-side switch S2 is open. Whereas during the off-time, the high-side switch S1 is open and the low-side switch S2 is conducting. Similar considerations are valid for a boost converter. The relation between duty cycle, on-time, off-time, and switching period in equilibrium is given by:

$$D = T_{\_on}/T_{\_sw} = T_{\_on}/(T_{\_on} + T_{\_off}) \quad (1)$$

$$D' = (1\text{-}D) \quad (2)$$

Thus, a variation of the duty cycle corresponds to a variation of the on-time of the switching cycle, when the switching period Tsw is assumed to be constant: d(t) = ton(t) / Tsw . When, additionally, a variation of the switching period Tsw is allowed, the following equation is readily obtained:

$$d(t) = (T_{\_on}(t)/T_{\_sw}(t)) = (T_{\_on}(t)/T_{\_on}(t) + T_{\_(off}(t))$$

The state-space models, as considered in this study, are defined as:

$$dx(t))/dt = Ax(t) + Bu(t) \quad (3)$$

$$y(t) = Cx(t) + Du(t) \quad (4)$$

This is the basic small signal equation for the synchronous buck converter.

3. ENHANCED SMALL SIGNAL MODELLING OF SYNCHRONOUS BUCK CONVERTER

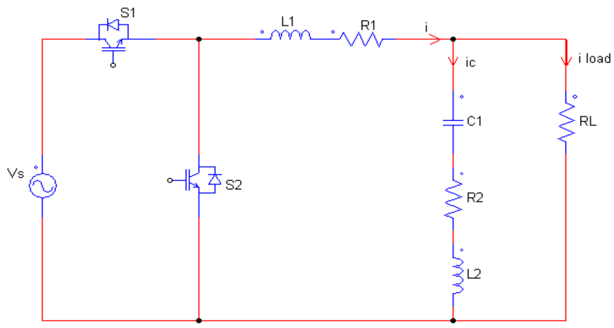

Figure 1. Buck converter model

From the above circuit two modes of operation are possible (i.e) when the switch is ON state and the switch is in OFF state. The state variables are given as:

$$X = \begin{bmatrix} v_c \\ i_L \\ i_c \end{bmatrix}$$

The input variables are given as U. They are given below.

$$U = \begin{bmatrix} v_i \\ t_{on} \\ t_{off} \end{bmatrix}$$

The output variable is given as:

$$Y = [v_o]$$

3.1. MODE 1

When the switch S1 is in ON state and the switch S2 is in OFF state, the circuit equations according to Kirchoff's voltage law is given as:

$$L\,\frac{di_L}{dt} = V_i - i_L R_L + t_{on} \tag{5}$$

$$C\,\frac{dv_c}{dt} = -\frac{V_o(t)}{R} \tag{6}$$

$$L_c\,\frac{di_c}{dt} = V_i - V_c - i_c R_c \tag{7}$$

By applying the above equations to the small signal analysis model equations, we get:

$$\frac{dx(t)}{dt} = \begin{bmatrix} 0 & 0 & \frac{1}{c} \\ 0 & -\frac{R_p}{L} & \frac{R_L}{L} \\ -\frac{1}{LC} & \frac{R_L}{LC} & \frac{R_c+R_L}{LC} \end{bmatrix} \begin{bmatrix} v_c \\ i_L \\ i_c \end{bmatrix} + \begin{bmatrix} o & \frac{1}{L} & o \end{bmatrix} \begin{bmatrix} v_i \\ t_{on} \\ t_{off} \end{bmatrix} \tag{8}$$

$$Y = \begin{bmatrix} o & R_L & -R_L \end{bmatrix} \begin{bmatrix} v_c \\ i_L \\ i_c \end{bmatrix} + [\mathrm{o\ \ o\ \ o}] \begin{bmatrix} v_i \\ t_{on} \\ t_{off} \end{bmatrix} \tag{9}$$

These above equations are similar to the basic small signal equations. Thus from the above equations,

$$A_1 = \begin{bmatrix} 0 & 0 & \frac{1}{c} \\ 0 & -\frac{R_p}{L} & \frac{R_L}{L} \\ -\frac{1}{LC} & \frac{R_L}{LC} & \frac{R_c+R_L}{LC} \end{bmatrix}$$

$$B_1 = \begin{bmatrix} o & \frac{1}{L} & o \end{bmatrix}$$

$$C_1 = \begin{bmatrix} o & R_L & -R_L \end{bmatrix}$$

$$D_1 = [\mathrm{o\ \ o\ \ o}]$$

Thus the four matrices are derived from mode 1. Similar calculations are made in mode 2 also.

3.2. MODE 2

When the switch S2 is in ON state and the switch S1 is in OFF state, the circuit equations according to Kirchoff's voltage law is given as:

$$L\,\frac{di_L}{dt} = V_i - i_L R_L \tag{10}$$

$$C\,\frac{dv_c}{dt} = -\frac{V_o(t)}{R} \tag{11}$$

$$L_c\,\frac{di_c}{dt} = V_i - V_c - i_c R_c \tag{12}$$

Similar to above mode, in this mode also we form the matrix equation as given below:

$$\frac{dx(t)}{dt} = \begin{bmatrix} 0 & 0 & \frac{1}{c} \\ 0 & -\frac{R_p}{L} & \frac{R_L}{L} \\ -\frac{1}{LC} & \frac{R_L}{LC} & \frac{R_c+R_L}{LC} \end{bmatrix} \begin{bmatrix} v_c \\ i_L \\ i_c \end{bmatrix} + [0 \quad 0 \quad 0] \begin{bmatrix} v_i \\ t_{on} \\ t_{off} \end{bmatrix} \tag{13}$$

$$Y = [o \quad R_L \quad -R_L] \begin{bmatrix} v_c \\ i_L \\ i_c \end{bmatrix} + [o \ \ o \ \ o] \begin{bmatrix} v_i \\ t_{on} \\ t_{off} \end{bmatrix} \tag{14}$$

Thus from the above equations we can also get the following matrices,

$$A_2 = \begin{bmatrix} 0 & 0 & \frac{1}{c} \\ 0 & -\frac{R_p}{L} & \frac{R_L}{L} \\ -\frac{1}{LC} & \frac{R_L}{LC} & \frac{R_c+R_L}{LC} \end{bmatrix}$$

$B_2 = [0 \quad 0 \quad 0]$

$C_2 = [0 \quad R_L \quad -R_L]$

$D_2 = [0 \ \ 0 \ \ 0]$

Considering small perturbations around the equilibrium point, the states **x**, input signals **u**, and the control variables t_{on} and $t_{off}$are given by,

$x = X + \hat{x}, u = U + \hat{u}$

$t_{on} = T_{on} + \hat{t}_{on}, \quad t_{off} = T_{off} + \hat{t}_{off}$

Accordingly, the state equation is given by,

$$\frac{d\hat{x}}{dt} = ((A_1\text{-}A_2)\frac{T_{on} + \hat{t}_{on}}{T_{on} + \hat{t}_{on} + T_{off} + \hat{t}_{off}} + A_2)\ (X + \hat{x}\ ) + ((B_1\text{-}B_2)\frac{T_{on} + \hat{t}_{on}}{T_{on} + \hat{t}_{on} + T_{off} + \hat{t}_{off}} + B_2)\ (U + \hat{u}) \tag{15}$$

And the output equation is,

$$Y + \hat{y} = ((C_1\text{-}C_2)\frac{T_{on} + \hat{t}_{on}}{T_{on} + \hat{t}_{on} + T_{off} + \hat{t}_{off}} + C_2)\ (X + \hat{x}\ ) + ((D_1\text{-}D_2)\frac{T_{on} + \hat{t}_{on}}{T_{on} + \hat{t}_{on} + T_{off} + \hat{t}_{off}} + D_2)\ (U + \hat{u}) \tag{16}$$

Collecting the direct-current (dc) terms and according to the condition of the equilibrium,

$0 = Ax(t) + Bu(t)$
$\mathbf{y}(t) = Cx(t) + Du(t)$

Where the averaged matrices are given by,

$A = DA_1 + D'A_2$
$\tilde{B} = DB_1 + D'B_2$
$C = DC_1 + D'C_2$
$\tilde{D} = DD_1 + D'D_2$

After calculation, we get:

$$A = \begin{bmatrix} 0 & 0 & \frac{1}{c} \\ 0 & -\frac{R_p}{L} & \frac{R_L}{L} \\ -\frac{1}{LC} & \frac{R_L}{LC} & \frac{R_c+R_L}{LC} \end{bmatrix}$$

$\tilde{B} = [o \quad \frac{D}{L} \quad o]$

$C = [o \quad R_L \quad -R_L]$

$\tilde{D} = [o \;\; o \;\; o]$

To calculate B and D, we use the procedure as given below,

$B = [\tilde{B} \quad b_{t_{on}} \quad b_{t_{off}}]$

$D = [\tilde{D} \quad d_{t_{on}} \quad d_{t_{off}}]$

$b_{t_{on}}$&$b_{t_{off}}$ and $d_{t_{on}}$&$d_{t_{off}}$ can be calculated from the formula below,

$$b_{t_{on}} = (A_1 X + B_1 U) / (T_{on} + T_{off})$$
$$b_{t_{off}} = (A_2 X + B_2 U) / (T_{on} + T_{off}) \quad (17)$$

And,

$$d_{t_{on}} = (C_1 X + D_1 U - Y) / (T_{on} + T_{off})$$
$$d_{t_{off}} = (C_2 X + D_2 U - Y) / (T_{on} + T_{off}) \quad (18)$$

Thus after calculations we get the value of B & D matrices as given below,

$$b_{t_{on}} = \begin{bmatrix} 0 \\ D' \frac{V_i}{L(T_{on} + T_{off})} \\ 0 \end{bmatrix}$$

$$b_{t_{off}} = \begin{bmatrix} 0 \\ -D \frac{V_i}{L(T_{on} + T_{off})} \\ 0 \end{bmatrix}$$

$d_{t_{on}} = d_{t_{off}} = 0$

$$B = \begin{bmatrix} 0 & 0 & 0 \\ \frac{D}{L} & D' \frac{V_i}{L(T_{on} + T_{off})} & -D \frac{V_i}{L(T_{on} + T_{off})} \\ 0 & 0 & 0 \end{bmatrix}$$

$D = 0$

By splitting the switching period of the PWM signal into the on-time and the off-time of the period, the input vector size is $m + 2$, compared to $m + 1$ in case of the traditional SSA approach (where solely the duty cycle is added as control input). Nevertheless, the input components (*t*on and *t*off) do not *independently* affect the state vector. This is intuitively clear and reflected in the resulting models, as:

rank(**B**) $\leq$ (m + 1)

i.e., **B** does not have full rank.

4. SIMULATION MODEL

The buck converter model simulated in MATLAB is given below.

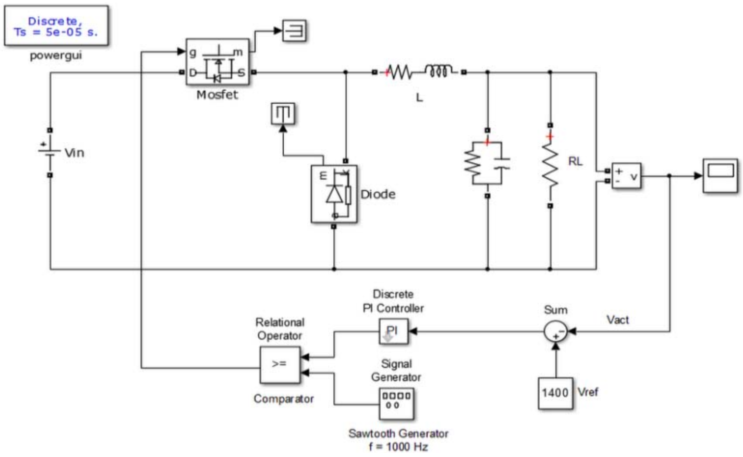

Figure 2. Simulation diagram of buck converter in MATLAB R2013A

Thus the buck converter model is simulated in MATLAB and the output is similar to that of the ideal output. The simulated output is presented below.

5. SIMULATED OUTPUT

The simulated output shows that a 2000V input DC source has been bucked to give an output of 1500V DC output. The simulated output is shown below.

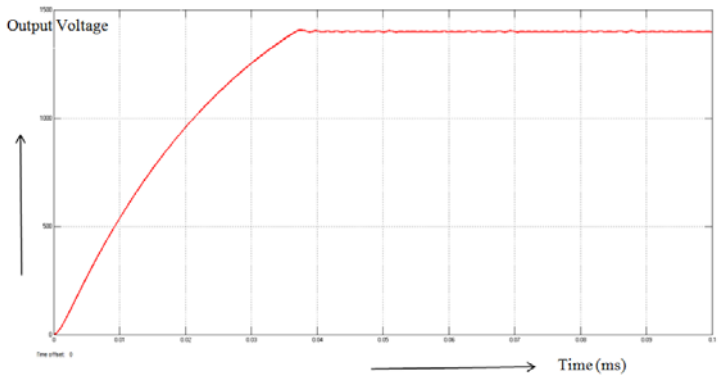

Figure 3. Simulated output of the buck converter

6. CONCLUSION

In this paper, an alternative and novel formulation of the linearized small-signal models for dc–dc converters has been presented. The derivation of the dynamic model has been shown in detail for a synchronous buck converter. Thus the duty cycle is also added as a control input to the converter topology discussed above. Thus the converter can also be controlled by using the duty cycle variations.

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