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Multi-Objective Optimization Scheme for PID-Controlled DC Motor

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ABSTRACT

DC Motor is the most basic electro-mechanical equipment and well-known for its merit and simplicity. The performance of DC motor is assessed based on several qualities that are most-likely contradictory each other, i.e. settling time and overshoot percentage. Most of controller's optimization problems are multi-objective in nature since they normally have several conflicting objectives that must be met simultaneously. In this study, the grey relational analysis (GRA) was combined with Taguchi method to search the optimum PID parameter for multi-objective problem. First, a L_9 (3^3) orthogonal array was used to plan out the processing parameters that would affect the DC motor's speed. Then GRA was applied to overcome the drawback of single quality characteristics in the Taguchi method, and then the optimized PID parameter combination was obtained for multiple quality characteristics from the response table and the response graph from GRA. Signal-to-noise ratio (S/N ratio) calculation and analysis of variance (ANOVA) would be performed to find out the significant factors. Lastly, the reliability and reproducibility of the experiment was verified by confirming a confidence interval (CI) of 95%.

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1. INTRODUCTION

PID speed controller for separately excited DC motor has been examined on the basis of output response, i.e. minimum settling time, and minimum overshoot for speed demand application of DC motor [1]-[3]. Since the characteristic responses are often contradictory each other, then PID tuning is of multi-objective optimization problem. Some methods have been applied to perform PID multi-objective problem, such as: Genetic Algorithm [4]-[7], Ant Colony Optimization [8], Particle Swarm Optimization [9], etc. Taguchi method is an effective way to optimize multi-parameter combination because it significantly reduces the number of experiments. However, Taguchi method is appropriate for single-objective optimization problems. Thus, some modifications have been done to overcome such problems [10]-[11].

The objective of this study is to combine grey relational analysis (GRA) with the Taguchi method to optimize PID parameter combination. First, a L_9 (3^3) orthogonal array was used to plan out the processing parameters that would affect the settling time and overshoot percentage. Then, the GRA was applied to overcome the drawback of single quality characteristics in the Taguchi method, and then the optimized PID parameter combination was obtained for multiple quality characteristics from the response table and the response graph from GRA. Signal-to-noise ratio (S/N ratio) calculation and analysis of variance (ANOVA) would be performed to find out the significant factors. From the ANOVA analysis, the factor that more significantly affected the settling time are K_P and K_D, meanwhile the factor that more significantly affected the overshoot percentage are K_P and K_I. Finally, the reliability and reproducibility of the experiment was verified by confirming a confidence interval (CI) of 95%.

2. DC MOTOR MODELING

The plant to be controlled in this study is the separately-excited DC motor [12]. The motor velocity relies on voltage across the armature while the field current is set constant. The modeling DC servo motor is carried to mimic the actual DC motor. The equivalent schematic circuit of the DC motor is shown in Figure 1. Table 1 gives the parameter and values for motor simulation used in this study [13].

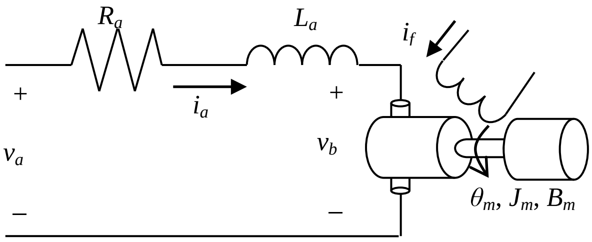


Figure 1. Equivalent Schematic Circuit of the DC Motor

Table 1. DC Motor Parameters and Values

Parameter	Nomenclature	Value
Moment of Inertia	J_m	0.022 kg.m^2
Friction coefficient	B_m	0.0005 N.ms
Back EMF constant	K_b	1.2 V/rad s^{-1}
Torque constant	K_a	0.015 Nm/A
Electric resistance	R_a	2.45 Ω
Electric inductance	L_a	0.035 H
Input voltage	v_a	
Back EMF voltage	v_b	
Armature Current	i_a	
Field Current	i_f	
Developed Torque	T_m	
Angle of motor shaft	θ_m	
Load Torque	T_L	

Because the rotational speed, ω, is proportional to the back EMF, v_b, we have:

$$v_b(t) = K_b \frac{d\theta}{dt} = K_b \omega(t) \tag{1}$$

The electrical equation on the mesh yields:

$$v_a(t) = R_a i_a(t) + L_a \frac{di_a}{dt} + v_b(t) \tag{2}$$

Meanwhile, the electro-mechanical equation derived from Newton's law is:

$$T_m - T_L = J_m \frac{d^2\theta}{dt^2} + B_m \frac{d\theta}{dt} = K_a i_a \tag{3}$$

By using Laplace transformation to (1), (2), and (3), the equations can be rearranged as:

$$V_b(s) = K_b \omega(s) \tag{4}$$

$$V_a(s) = R_a i_a(s) + L_a s I_a(s) + V_b(s) \tag{5}$$

$$T_m(s) - T_L(s) = J_m s^2 \theta(s) + B_m s\theta(s) = K_a I_a(s) \quad (6)$$

Hence, the block diagram of DC motor relating the motor speed, ω, to input voltage, v_a, is described in Figure 2:

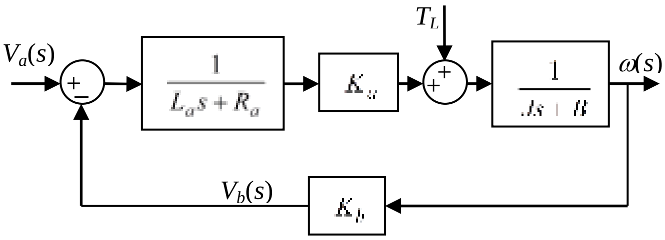


Figure 2. Block Diagram of the DC Motor in *s*-domain

3. RESEARCH METHODS

3.1. The Taguchi Method

The Taguchi method is an effective technique in single-objective optimization problem [14]. The characteristic of the Taguchi method is to utilize an orthogonal array to plan out the experiment and to use the signal-to-noise ratio (SN ratio) to analyze the data obtained from the experiment. The orthogonal array is used to reduce the number of design experiments required in any design process. In this study, there are three factors each at three levels, so the Taguchi method needs only nine experiments (L_9) while the full factorial design method needs 3^3 = 27 experiments.

Since the settling time and overshoot percentage have a direct effect on the performance with respect of DC motor's speed, therefore this study primarily looks into the relationship between the PID parameters and quality characteristics, using the Taguchi method and GRA to find the optimum PID parameter combination for DC motor's speed, with the goal of making the settling time and overshoot percentage as close to the ideal as possible. First, the quality characteristics are set for the DC motor's speed, and then the control factors and their levels are planned out. After choosing a suitable orthogonal array, the experiments according to the orthogonal array are conducted. Next, ANOVA is performed, at the same time performing relational analysis on double quality characteristics using GRA, and then the optimum PID parameter combination can be obtained through main effect analysis. Finally, a verification experiment is carried out, and the reliability and reproducibility of the experiment are verified with a 95% CI.

3.1.1. Orthogonal Array

In establishing an orthogonal array, three factors A, B, and C are considered. A is the proportional gain, B is the integral gain, and C is the derivative gain. To determine the factor levels, an initial guess must be established from the traditional Ziegler-Nichols method using Matlab's SISO design tool. The Ziegler-Nichols method gives the values of K_p, K_i, and K_d of 1.5865, 1.0018, and 0.62811, respectively. Then, the three levels of three control factors were identified as presented in Table 2.

Table 2. PID Parameters with Their Ranges and Values at Three Levels

Factors	PID parameters	Range	Level 1	Level 2	Level 3
A	K_P	1 - 2	1	1.5	2
B	K_I	0.8 - 1.2	0.8	1	1.2
C	K_D	0.2 - 1	0.2	0.6	1

3.1.2. Signal-to-Noise Ratio

Taguchi experiment design employs quality loss as a base to plan a statistical measurement for the evaluation of performance, called the SN ratio. In this study, the performance indexes of PID optimization are the settling time and overshoot percentage. The aim of the optimization is to make the performance indexes as close as to their target values. So, the quality characteristics are chosen to be the smaller-the-better, and the SN ratio is:

$$SN = -10\log MSD \tag{7}$$

and the mean squared deviation is:

$$MSD = \frac{1}{n}\sum_{i=1}^{n} y_i^2 \tag{8}$$

where y_i is the value of the quality measurement and n is the total number of measurements.

3.1.3. Main Effect Analysis

After proceeding with the experiment plan according to the L_9 orthogonal array, the data produced by various processing parameter combinations can be obtained to obtain the response table and the response graph. The data obtained from the experiments are used to calculate the main effect values ΔF as follows:

$$F_i = \frac{1}{m}\sum_{j=1}^{m} y_i \tag{9}$$

$$\Delta F = \max(F_1, F_2, F_3) - \min(F_1, F_2, F_3) \tag{10}$$

where F_i is the mean SN ratio of the i-th level of factor F; m is the number of level i; ΔF is the value of the main effects of factor F and y_i is the SN ratio produced on each i level row Next, these data are sorted into a response table, and the effect of each factor is analyzed. If the main effect value of a factor ΔF is large, then that given factor's effect on the system is much greater than that of the other factors.

3.2. Grey Relational Analysis [15]

An appropriate mathematical model must be established to describe the relationship between the two objectives and their target values. The first step is to analyze the difference in each objectives caused by each processing parameter, and find their relationships with their target values. The GRA method used in this study is based on the Grey Theory to find the relationship between actual values and their target values throughout all trials in the orthogonal array. The GRA of the settling time and overshoot percentage is defined as follows:

The target value of settling time and overshoot percentage:

Reference sequence $X_0 = (x_0(1), x_0(2))$

The settling times and overshoots percentages obtained from the various experiments in the orthogonal array:

$X_i = (x_i(1), x_i(2)), \quad i = 1, 2, \ldots, 9$

The relation between the target value and the observed experiment value is the correlation coefficient

$$\gamma(x_0(k), x_i(k)) = \frac{\zeta \max\limits_{1\le m\le 9} \max\limits_{k} \Delta_{0,m}(k)}{\Delta_{0,i}(k) + \zeta \max\limits_{1\le m\le 9} \max\limits_{k} \Delta_{0,m}(k)} \tag{11}$$

$k = 1, 2, \quad i = 1, 2, \ldots, 9$

where $\gamma(x_0(k), x_i(k))$ describes the degree of correlation of X_0 and X_i at point k, and represents the local characteristic condition of the relation between X_0 and X_i. It is said that the average of $\gamma(x_0(k), x_i(k))$ is the relational grade of X_i relative to X_0, which is formulated as:

$$\gamma(X_0, X_i) = \frac{1}{n}\sum_{k=1}^{n} \gamma(x_0(k), x_i(k)) \tag{12}$$

4. RESULTS AND ANALYSIS

4.1. PID Optimization

The control factor and their levels were assigned into the L_9 orthogonal array to provide the plan and procedure for handling the experiment. Then the 9 experiments, as shown in Table 3 were proceeded with in accordance with the orthogonal array to find out both settling time and percentage overshoot. To obtain the values of settling times and overshoot percentages, the MATLAB-Simulink model of the DC motor system with the PID controller is used using motor parameters listed in Table I, as shown in Figure 3.

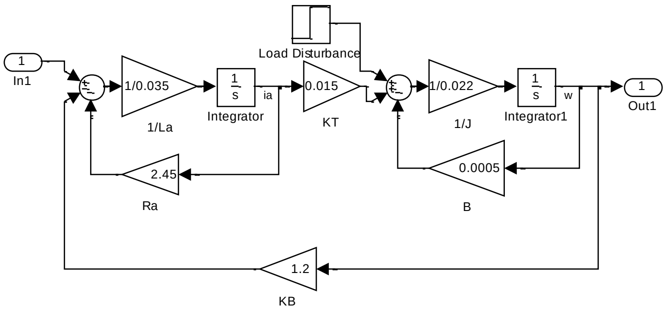


Figure 3. DC Motor schematic in MATLAB-Simulink

Table 3. Simulation Results

Trial	K_p	K_i	K_d	Settling time (s)	Overhoot (%)	S/N Ratio of Settling time (dB)	S/N Ratio of Overshoot Percentage (dB)
1	1	0.8	0.2	10.8506	8.4979	-20.7091	-18.5862
2	1	1	0.6	21.5923	14.3346	-26.6860	-23.1277
3	1	1.2	1	21.7593	19.6386	-26.7529	-25.8622
4	1.5	0.8	0.6	11.6622	5.1042	-21.3356	-14.1586
5	1.5	1	1	11.2332	10.1646	-21.0101	-20.1418
6	1.5	1.2	0.2	9.3057	10.0445	-19.3750	-20.0386
7	2	0.8	1	10.8026	2.9654	-20.6706	-9.4417
8	2	1	0.2	9.0183	3.3661	-19.1025	-10.5425
9	2	1.2	0.6	9.5636	7.4879	-19.6124	-17.4872

S/N ratios were computed in each of the trial combination and the values were displayed in the last two columns of Table 3. Since the experimental design is orthogonal, the effect of each factor is separable. Therefore the average S/N Ratio of each level (1, 2, 3) of three control factors (K_P, K_I and K_D) can be obtained using Equation 7.

The ANOVA table was calculated out of the SN ratios in Table 3, as shown in Table 4 and 5. From the ANOVA table that with regard to the settling time, control factors B have a smaller effect and are therefore categorized as pooled errors; on the other hand, control factors A and C have a profound F-ratio, meaning that the effects of these factors are all significant. Whereas, the significant factors for the overshoot percentage is factors A and B.

Table 4. ANOVA Analysis for Settling Time

Source	SS	d.f	MS	F-ratio	SS'	%P
A	41.9801	2	20.99005	8.336039	36.94413	53.11675
B*	2.993788	2	1.496894			
C	17.50063	2	8.750313	3.47512	12.46465	17.92116
error	7.078167	2	3.539084			
pooled error	10.07196	4	2.517989			28.96209
total	69.55269	8				100

*=pooled terms

Table 5. ANOVA Analysis for Overshoot Percentage

Source	SS	d.f	MS	F-ratio	SS'	%P
A	151.7815	2	75.89074	22.68487	145.0906	60.37541
B	75.15085	2	37.57543	11.23185	68.45998	28.48771
C*	7.922014	2	3.961007			
error	5.459721	2	2.729861			
pooled error	13.38173	4	3.345434			11.13687
total	240.3141	8				100

*=pooled terms

where SS = Sum of Square
d.f = degree of freedom
MS = Mean Square
F-Ratio = MS/MSE
SS' = Pure Sum of Square
%P = Percentage of Contribution

Table 6. The Deviation Sequences, Grey Relational coefficients, and Grey Relational grades

No	$\Delta_{0,i}(1)$	$\Delta_{0,i}(2)$	$\gamma_{0,i}(1)$	$\gamma_{0,i}(2)$	$\gamma_{0,i}$	Rank
1	0	0.783173	1	0.508279	0.754139	6
2	0	0.663876	1	0.549432	0.774716	4
3	0	0.902538	1	0.472842	0.736421	7
4	0	0.43767	1	0.649082	0.824541	3
5	0	0.904871	1	0.472198	0.736099	8
6	0	1.079392	1	0.428571	0.714286	9
7	0	0.274508	1	0.746776	0.873388	1
8	0	0.373252	1	0.684432	0.842216	2
9	0	0.782958	1	0.508347	0.754174	5

Table 7. The Response Table for The GRA

	A	B	C
Level 1	0.755092	0.817356	0.770214
Level 2	0.758309	0.784344	0.784477
Level 3	0.823259	0.73496	0.781969
Main effect	0.068167	0.082396	0.011756
Rank	2	1	3

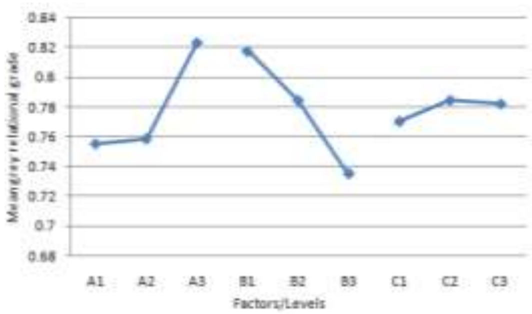


Figure 4. The Response Graph for the GRA

Then, GRA was utilized to carry out optimum PID parameter combination analysis. The target values for settling time and overshoot percentage were X_0 = (2, 0). The deviation sequence for the reference sequence and the sequences in the orthogonal array, the grey relational coefficients, and grey relational grade calculation results are shown in Table 6. Next, the GRA response table and response graph could be obtained using main effect analysis, as shown in Table 7 and Figure 4. From the response table and response graph, the optimum PID combination for DC motor's speed was determined, which is A3, B1, C2. In other words,the K_P of 2, K_I of 0.8, and K_D of 0.6. This PID combination was obtained beyond the search space within the L_9 orthogonal array. After the optimum PID parameter combination was obtained through GRA, the verification experiments were conducted. First, the theoretical predicted values could be figured out through the significant factors in the ANOVA table.

After the optimum PID control parameters were obtained through GRA, the confirmation simulations were conducted. First, the theoretical predicted values could be figured out through the significant factors in the ANOVA table using the following formula:

$$\hat{SN} = \bar{T} + \sum_{i=1}^{n}\left(F_i - \bar{T}\right) \tag{13}$$

where $\bar{T}$ is the average of all the SN ratios and F_i is the SN ratio of significant factor level values. In order to effectively evaluate the various observed values, their Confidence Interval (CI) must be calculated. Its calculation is shown below:

$$CI_1 = \sqrt{F_{\alpha,1,v_2} \times MSE \times \frac{1}{n_{eff}}} \tag{14}$$

whilst, The CI of the calculated experiment value is:

$$CI_2 = \sqrt{F_{\alpha,1,v_2} \times MSE \times \left(\frac{1}{n_{eff}} + \frac{1}{r}\right)} \tag{15}$$

where $F_{\alpha,1,v2}$ is the F-ratio of significance level α; α is the significance level, the confidence level is 1- α; v_2 is the DF of the mean square of the pooled error; MSE is the mean square of the pooled error; n_{eff} is the effective observed value, and r is the number of verification experiments.

According to (13), the predicted value for the settling time and overshoot percentage were is -20.6449 dB and -8.843 dB, respectively; whereas according to (14), the expected average value of 95% CI of the settling time and overshoot percentage were 3.2838 and 3.7851 dB, respectively. Then, the settling time and overshoot percentage under optimum PID parameter (K_P=2, K_I=0.8, K_D=0.6) were calculated. The simulation showed that the calculated settling time and percentage overshoot were -14.322 and -5.4266 dB, respectively; whereas according to (15), the expected average value of 95% CI of the settling time and overshoot percentage were 5.4948 and 6.3371 dB, respectively. The diagram for the CI of the settling time and overshoot percentage verification experiment values and theoretical predicted values are shown in Figure 5. From the diagrams, the CI from the verification experiment and the theoretical predictions did indeed coincide; therefore the results from our experiment are indeed reliable had been proven.

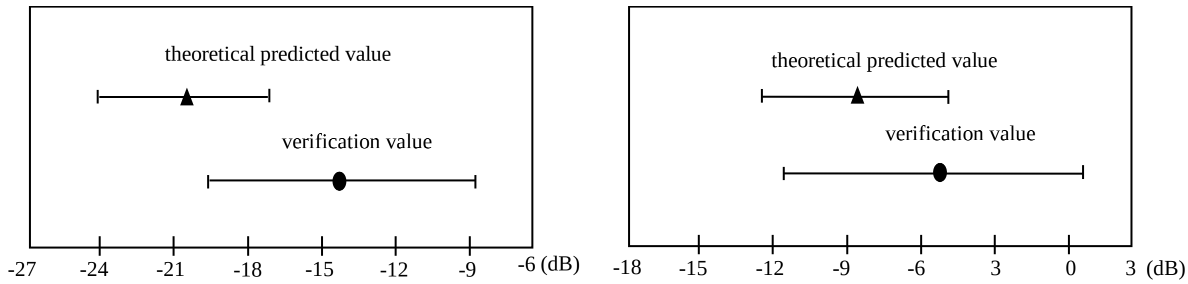
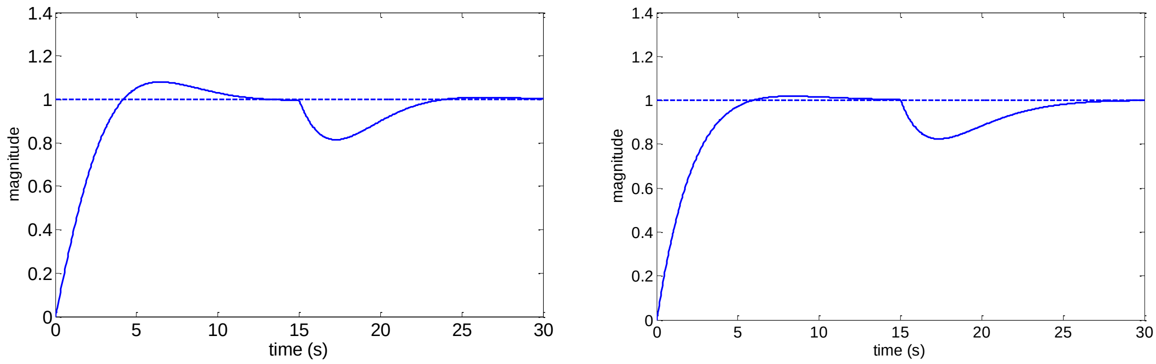


Figure 5. Diagram for the CI of the verification experimental value and theoretical predicted value for (a) settling time and (b) overshoot percentage

4.2. Time Response Performance

Figure 6(a) and (b) demonstrates the disturbed step responses of the DC motor speed with PID parameter obtained from the traditional Ziegler-Nichols method, and Taguchi-GRA, respectively. The load disturbance is 0.005 Nm at t=15 s.



(a) (b)

Figure 6. Unit step responses with disturbance using (a) Ziegler-Nichols and (b) Taguchi-GRA method

It is shown that the PID parameter optimization using Taguchi-GRA method improves the time response characteristic and system robustness against disturbance compared to that using Ziegler-Nichols conventional tuning.

5. CONCLUSION

A Taguchi method combined with the GRA was successfully employed to seek the optimum PID parameters beyond the search space in the orthogonal array. The proportional gain, the integral gain, and the derivative gain were chosen to define the search space for the multi-objective optimization problem, while the objective is to minimize the settling time and overshoot percentage of DC motor's speed. From the response table and response graph, the optimum PID parameter combination was concluded as: K_P of 2, K_I of 0.8, and K_D of 0.6. From the ANOVA analysis, the factor that more significantly affected the settling time are K_P and K_D, meanwhile the factor that more significantly affected the overshoot percentage are K_P and K_I. After carrying out verifying simulations, the 95% CI diagrams for the verification values coincides with that of the theoretical predicted values. It means that the results of this method possess reproducibility. The unit response using this method demonstrated the superiority over the traditional Ziegler-Nichols PID tuning, in terms of both the transient response and robustness.

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Gunawan Dewantoro received his Bachelor Degree in Electrical Engineering, Gadjah Mada University in 2008 and M.Sc.Eng in Automation and Control, National Taiwan University of Science and Technology in 2011. He has been with Satya Wacana Christian University since 2012. His research areas include control systems, intelligent systems, and optimizations.