International Journal of Power Electronics and Drive System (IJPEDS)
Vol. 7, No. 3, September 2016, pp. 800~815
ISSN: 2088-8694, DOI: 10.11591/ijpeds.v7.i3.pp800-815

Compensator Based Performance Enhancement Strategy for a SIQO Buck Converter

S. Augusti Lindiya[1], S. Palani[2], K. Vijayarekha[3]
[1,3] Department of Electrical and Electronics Engineering, SASTRA University, India
[2] Department of Electrical and Electronics Engineering, Sudharsan Engineering College, India

Article Info

Article history:

Received Nov 12, 2015
Revised Mar 23, 2016
Accepted Apr 24, 2016

Keyword:

Compensators
Controllers
Rise time
Single inductor multiple output converters
State space averaging

ABSTRACT

This paper attempts to design lag and lag-lead compensators for improving the performance of a Single Inductor Quad Output (SIQO) dc-dc buck converter in terms of time domain and frequency domain specifications. It develops the state space averaged model to find the duty ratio of the desired output voltages at steady state. The exercise arrives at the transfer function model from the state space averaged model through the use of its lumped small signal equivalent circuit which allows analyzing the performance of the system in frequency and time domains. The responses are derived in the MATLAB/Simulink® using discrete components incorporating compensators of the converter. The hardware results obtained using Data Acquisition Module DT9834® interfaced to MATLAB/Simulink® and prototype model establish the performance of the compensator based converter and further emphasize its ability to minimize the ripples over a range of operating loads.

Copyright © 2016 Institute of Advanced Engineering and Science.
All rights reserved.

Corresponding Author:

S. Augusti Lindiya,
Department of Electrical and Electronics Engineering,
SASTRA University,Thirumalaisamudram, Thanjavur 613401, India.
Email: lindiya@eee.sastra.edu

1. INTRODUCTION

The present day consumer electronics require multiple levels of supply voltages for their core modules. Besides in light of the view that the low drop out regulators reduce the conversion efficiencies at heavy load, they are replaced by the switch mode converters which provide the desired voltage levels in both magnitude and polarity. However the inductors present in each converter occupy large space and increase the cost of the whole system. Single Inductor Multiple Output (SIMO) converters offer advantages of using a single inductor which usually occupies less space for any number of outputs. However there is a need for an efficient controller which actually time shares the inductor current among all the outputs at different voltage/current levels such that the desired time domain specifications are obtained with reduced cross regulation and ripple.

A substantial work on SIMO converters are seen in the literature in the field of designing controllers. In [1], for a SIMO converter, load independent control through the sum of each output's phase difference and the freewheeling current have been used to generate the control signal such that the output of the converter has less under shoot or over shoot at load transients. The efficiency has been seen to be maximum at steady state since the freewheeling current duration is zero. Converters with single inductor capable of providing Buck and Boost configuration with independent control of each output is used to minimize the cross regulation between the outputs and obtain the desired regulatory characteristics using separate regulation of common mode and differential mode voltages by developing the non- linear functional gain blocks [2]. The converter has been able to regulate both buck and boost outputs by controlling the conducting period of the devices. In [3], the current accumulation problem was reduced along with high

conversion efficiency by delivering the input energy to the outputs through a lossless inductor continually. The steady state and the dynamic responses with nominal load change have been improved by designing the non- linear controller considering the saturation of control signals and the switching losses [4]. A hybrid topology has been developed by combining the class AB amplifier with the switching converter in order to enhance the transient response and minimize the cross regulation. The class AB amplifier has been seen to act faster than the switching converter and feed load transient current to suppress the effects of cross regulation [5]. A single shared hysteresis comparator has been used to regulate the output voltages and the time limited power distribution control used to suppress the cross regulation [6]. A power multiplexed discontinuous conduction mode switching scheme has been proposed for SIMO output dc-dc power converter to nullify the cross regulation and improve the dynamic and steady state performances of the system [7]. The analog implementation of PWM based sliding mode controller [8] along with internal integral control loop has been implemented for suppressing the cross regulation and steady state regulation error in a single inductor dual output dc-dc buck converter. In [9], ripple based adaptive off time control circuit is designed which does not require error amplifier or PWM compensation. The operating frequency is locked to the reference clock. The controllers discussed above use complicated algorithms and are not obtained from a mathematical procedure and complex circuits are used to implement the developed algorithms.

Modeling of dc-dc converters plays vital role to analyze the performance of the converter. The developed model will help to determine the parameters of the conventional or linear or non-linear controllers using optimization techniques. AC modeling and small signal transfer function of a single input single output dc-dc converter [10] are developed by considering the non-ideal components of Buck converter. For the same buck converter [11], novel small signal averaged model is developed when it operates at variable switching frequency. A generalized state space averaging technique has been used to obtain the dynamic model of buck converter along with sliding mode controller. The use of artificial intelligent techniques has been used to obtain the optimum values of the controller and enable the reduction in computational simulation time [12].

For a dual output, a digital implementation of multivariable gain controller is developedusing a model developed in steady state in such a way that the diagonal elements are used to improve the regulatory performance of the converter and the off diagonal elements are used to suppress the cross regulations [13]. However the algorithm used to develop a model is complex and restricted to two outputs. A ripple based modeling of single inductor dual output buck converter is also developed for a cross derivative state feedback control methodology [14]. In another research, an inductor current ripple-based modeling approach has been proposed to accurately model and analyze the converter. The control, cross- coupling, and cross-regulation transfer functions, generated throughthe model has been portrayed [15]. But the procedure used is very complex and lengthy.

In this paper the mathematical modeling of the Single Inductor Quad Output (SIQO) converter is developed by state space averaging technique which can be extended for any number of outputs easily. Further transfer function model is developed from lumped small signal equivalent circuit. Additionally, SIMO converters operated on open loop do not give satisfactory results at transients as well as under steady state conditions. Therefore compensators are designed and incorporated in each output using frequency and time domain specifications. The converter is developed in MATLAB/SIMULINK® using discrete components and its performance is analyzed with and without compensators and it is established that the excellent results are obtained using lag and lag-lead compensators. The performance of the converter so designed is validated using DT 9834® DAQ module.

2. SYSTEM DIAGRAM AND OPERATION

Figure 1 show the circuit diagram of single input four output dc-dc buck converter with single inductor. The converter can be operated in the continuous conduction mode (CCM), discontinuous conduction mode (DCM) and pseudo continuous conduction mode (PCCM) of operation. The mode of operation is decided based on the amount of ripples allowed, the load current rating and the converter efficiency. The continuous mode of operation enables to handle large amount of current with less amount of ripples and produce the high conversion efficiency but suffers from the effects of cross regulation. The effort in this investigation revolves to arrive at compensators for enhancing its performance in this perspective. The switching scheme of the converter for one switching cycle operating in continuous conduction mode of operation is shown in Figure 2. S_p and S_p' form the main path and S_1, S_2, S_3 and S_4 form sharing paths. Figure 2(a) shows the switching pulses for the switches in sharing path and Figure 2(b) shows the switching pulses for the switches in main path for the SIQO converter. Figure 3 shows the timing diagram of inductor current for one switching cycle.

The state space model of the converter for each mode of operation is obtained by using Kirchhoff's voltage and current laws by considering the four output voltages (v_1, v_2, v_3 and v_4) and the inductor current (i_L) as state variables. The duty cycles d_1, d_2, d_3 and d_4 are the independent control variables. The disturbances are observed at the input voltage (v_{in}) and the load currents (i_1, i_2, i_3 and i_4) and R_{on} is the on state resistance of the conducting switches to represent the conducting losses.

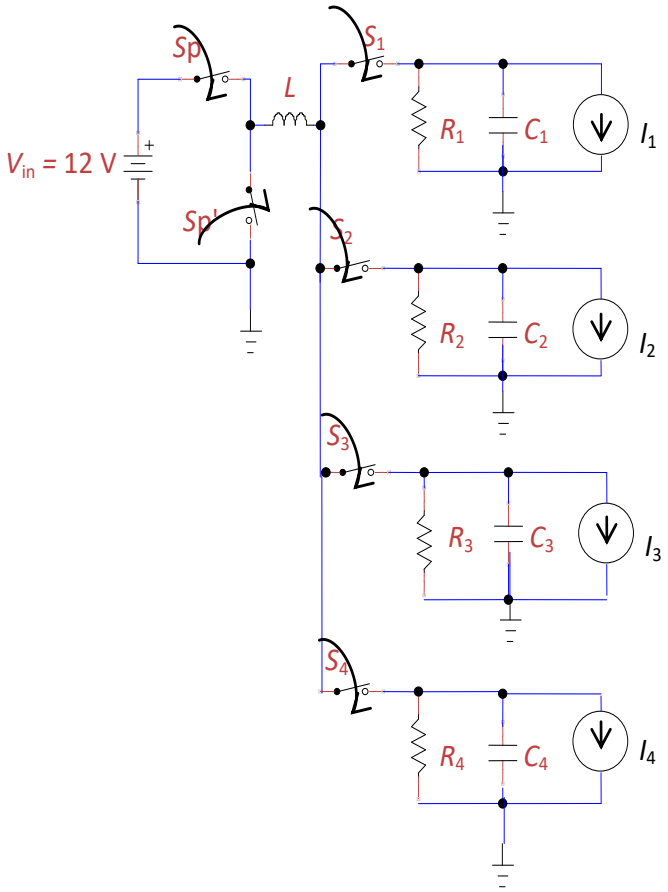


Figure 1. Circuit diagram of SIQO dc-dc converter

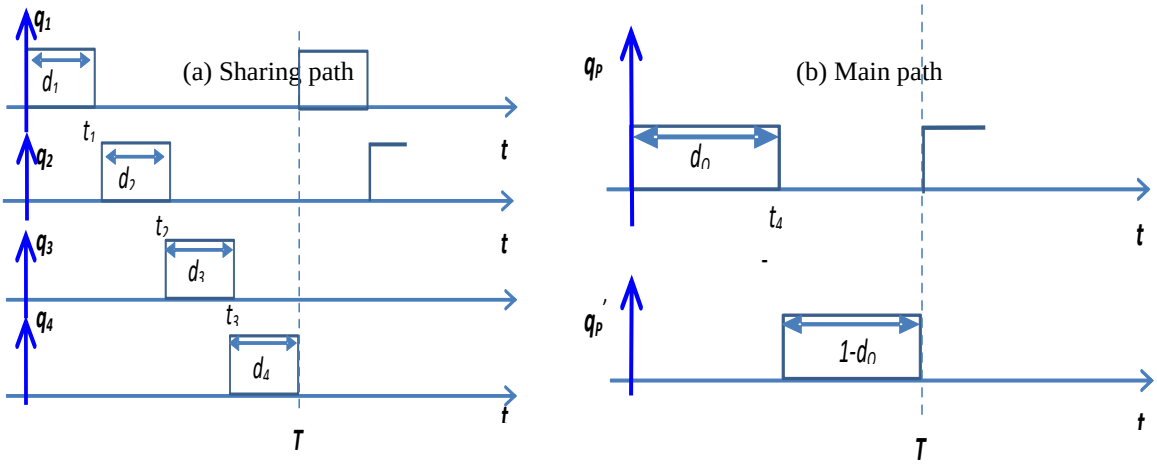


Figure 2. Switching pulses of the SIQO converter

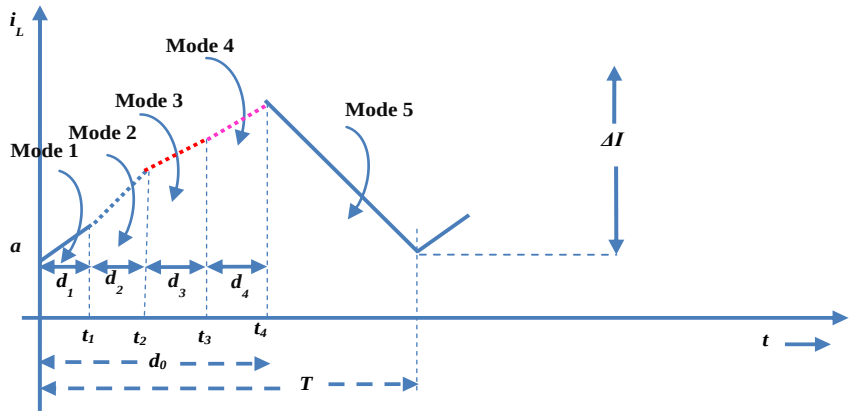


Figure 3. Timing diagram of inductor current

2.1. Mode 1 Operation

In this mode, the capacitor in output 1 gets charged from the input supply by turning on the switches S_1 and Sp and the differential equations obtaining for voltage across the inductor and current passing through the capacitor are written in state space form as in Equations (1) and (2).

$$\begin{bmatrix} \dot{i}_L \\ \dot{v}_1 \\ \dot{v}_2 \\ \dot{v}_3 \\ \dot{v}_4 \end{bmatrix} = \begin{bmatrix} -\frac{R_{on}}{L} & -\frac{1}{L} & 0 & 0 & 0 \\ \frac{1}{C_1} & -\frac{1}{R_1 C_1} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{R_2 C_2} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{R_3 C_3} & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{R_4 C_4} \end{bmatrix} \begin{bmatrix} i_L \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} + \begin{bmatrix} \frac{1}{L} & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{C_1} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{C_2} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{C_3} & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{C_4} \end{bmatrix} \begin{bmatrix} v_{in} \\ i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} \quad (1)$$

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_L \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} + [0]u \quad (2)$$

The equations (1) and (2) can be written in a simplified form as,

$$\dot{x} = A_1 x + B_1 u$$
$$y = C_1 x + D_1 u$$

2.2. Mode 2 Operation

In Mode 2, the output 2 gets charged from the input supply by turning on the switches S_p and S_2 and the same procedure is followed to represent the differential equations in state space form shown in Equations (3) and (4).

$$\begin{bmatrix} i_L \\ \dot{v}_1 \\ \dot{v}_2 \\ \dot{v}_3 \\ \dot{v}_4 \end{bmatrix} = \begin{bmatrix} -\frac{R_{on}}{L} & 0 & -\frac{1}{L} & 0 & 0 \\ 0 & -\frac{1}{R_1C_1} & 0 & 0 & 0 \\ \frac{1}{C_2} & 0 & -\frac{1}{R_2C_2} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{R_3C_3} & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{R_4C_4} \end{bmatrix} \begin{bmatrix} i_L \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} +$$

$$\begin{bmatrix} \frac{1}{L} & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{C_1} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{C_2} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{C_3} & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{C_4} \end{bmatrix} \begin{bmatrix} v_{in} \\ i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} \quad (3)$$

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_L \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} + [0]u \quad (4)$$

The Equations (3) and (4) can be written in a simplified form as,

$$\dot{x} = A_2x + B_2u$$
$$y = C_2x + D_2u$$

2.2. Mode 3 Operation

In this mode the capacitor in output 3 is getting charged by closing the switches S_p and $S_3$and the associated differential equations are shown in state space form as in Equations (5) and (6).

$$\begin{bmatrix} i_L \\ \dot{v}_1 \\ \dot{v}_2 \\ \dot{v}_3 \\ \dot{v}_4 \end{bmatrix} = \begin{bmatrix} -\frac{R_{on}}{L} & 0 & 0 & \frac{1}{L} & 0 \\ 0 & -\frac{1}{R_1C_1} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{R_2C_2} & 0 & 0 \\ \frac{1}{C_3} & 0 & 0 & -\frac{1}{R_3C_3} & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{R_4C_4} \end{bmatrix} \begin{bmatrix} i_L \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} +$$

$$\begin{bmatrix} \frac{1}{L} & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{C_1} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{C_2} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{C_3} & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{C_4} \end{bmatrix} \begin{bmatrix} v_{in} \\ i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} \quad (5)$$

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_L \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} + [0]u \quad (6)$$

The above equations (5) and (6) can be written in a simplified form as,

$$\dot{x} = A_3 x + B_3 u$$
$$y = C_3 x + D_3 u$$

2.3. Mode 4 Operation

In Mode 4 operation, the charging of capacitor in output 4 is done by closing the switches S_p and S_4 and the related differential equations are shown in Equations (7) and (8) in state space form.

$$\begin{bmatrix} i_L \\ \dot{v}_1 \\ \dot{v}_2 \\ \dot{v}_3 \\ \dot{v}_4 \end{bmatrix} = \begin{bmatrix} -\frac{R_{on}}{L} & 0 & 0 & 0 & -\frac{1}{L} \\ 0 & -\frac{1}{R_1 C_1} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{R_2 C_2} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{R_3 C_3} & 0 \\ \frac{1}{C_4} & 0 & 0 & 0 & -\frac{1}{R_4 C_4} \end{bmatrix} \begin{bmatrix} i_L \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} +$$

$$\begin{bmatrix} \frac{1}{L} & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{C_1} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{C_2} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{C_3} & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{C_4} \end{bmatrix} \begin{bmatrix} v_{in} \\ i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} \quad (7)$$

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_L \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} + [0]u \quad (8)$$

The above equations (7) and (8) can be written in simplified form as,

$$\dot{x} = A_4 x + B_4 u$$
$$y = C_4 x + D_4 u$$

2.4. Mode 5 Operation

In Mode 5 operation the inductor current is discharged through the output 4 by closing the switches S_p' and S_4 and the related differential equations are given in state space formin Equations (9) and (10).

$$\begin{bmatrix} i_L \\ \dot{v}_1 \\ \dot{v}_2 \\ \dot{v}_3 \\ \dot{v}_4 \end{bmatrix} = \begin{bmatrix} -\frac{R_{on}}{L} & 0 & 0 & 0 & -\frac{1}{L} \\ 0 & -\frac{1}{R_1 C_1} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{R_2 C_2} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{R_3 C_3} & 0 \\ \frac{1}{C_4} & 0 & 0 & 0 & -\frac{1}{R_4 C_4} \end{bmatrix} \begin{bmatrix} i_L \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} +$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{C_1} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{C_2} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{C_3} & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{C_4} \end{bmatrix} \begin{bmatrix} v_{in} \\ i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} \quad (9)$$

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_L \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} + [0]u \quad (10)$$

The above equations (9) and (10) are written in a simplified form as,

$$\dot{x} = A_5 x + B_5 u$$
$$y = C_5 x + D_5 u$$

As the converter switches among the five modes for each switching cycle, the averaged representation of the system is written through equation (11)

$$\dot{x} = Ax + Bu \quad (11)$$
$$y = Cx$$

where

$$A = A_1 D_1 + A_2 D_2 + A_3 D_3 + A_4 D_4 + A_5 (1 - D_1 - D_2 - D_3 - D_4)$$

$$B = B_1 D_1 + B_2 D_2 + B_3 D_3 + B_4 D_4 + B_5 (1 - D_1 - D_2 - D_3 - D_4)$$

$$C = C_1 D_1 + C_2 D_2 + C_3 D_3 + C_4 D_4 + C_5 (1 - D_1 - D_2 - D_3 - D_4)$$

$$A = \begin{bmatrix} -\frac{R_{on}}{L} & -\frac{D_1}{L} & -\frac{D_2}{L} & -\frac{D_3}{L} & -\frac{1}{L}(1 - D_1 - D_2 - D_3) \\ \frac{D_1}{C_1} & -\frac{1}{R_1 C_1} & 0 & 0 & 0 \\ \frac{D_2}{C_2} & 0 & -\frac{1}{R_2 C_2} & 0 & 0 \\ \frac{D_3}{C_3} & 0 & 0 & -\frac{1}{R_3 C_3} & 0 \\ \frac{1}{C_4}(1 - D_1 - D_2 - D_3) & 0 & 0 & 0 & -\frac{1}{R_4 C_4} \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{(1 - D_1 - D_2 - D_3)}{L} & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{C_1} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{C_2} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{C_3} & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{C_4} \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

3. SMALL SIGNAL PERTURBATION AND LINEARIZATION

To analyze the stability of non- linear circuit for normal operating condition with small deviations, linear models are developed by approximately linearizing the circuit. The linear model which also can be called as small signal ac model is developed for non-ideal SIQO dc-dc buck converter at the quiescent operating point (*I*, *V*) assuming that *U* and the duty ratios are equal to the chosen quiescent values together with superimposed small ac variations. The Table 1 gives the list of system variables considered to develop small signal ac model.

Table 1. System Variables

Input voltage	Output voltages	Load currents	Duty ratios
$v_{in} = V_{in} + \hat{V}_{in}$	$v_1 = V_1 + \hat{V}_1$	$i_1 = I_1 + \hat{\imath}_1$	$d_1 = D_1 + \hat{d}_1$
	$v_2 = V_2 + \hat{V}_2$	$i_2 = I_2 + \hat{\imath}_2$	$d_2 = D_2 + \hat{d}_2$
	$v_3 = V_3 + \hat{V}_3$	$i_3 = I_3 + \hat{\imath}_3$	$d_3 = D_3 + \hat{d}_3$
	$v_4 = V_4 + \hat{V}_4$	$i_4 = I_4 + \hat{\imath}_4$	$d_4 = D_4 + \hat{d}_4$

It is known that the small signal analysis follows the equation (11) and appropriate substitutions lead to Equations (12) and (13) and yield the final lumped equivalent small signal circuit of the SIQO converter in Figure 9.

$$x = x + \hat{x} \tag{12}$$

$$\dot{x} + \hat{\dot{x}} = A_1(D_1 + \hat{d}_1) + A_2(D_2 + \hat{d}_2) + A_3(D_3 + \hat{d}_3) + A_4\left(D_4 + \hat{d}_4\right) + A_5(1 - D_1 - D_2 - D_3 - D_4 - \hat{d}_1 - \hat{d}_2 - \hat{d}_3 - \hat{d}_4)(x + \hat{x}) + B_1(D_1 + \hat{d}_1) + B_2(D_2 + \hat{d}_2) + B_3(D_3 + \hat{d}_3) + B_4\left(D_4 + \hat{d}_4\right) + B_5(1 - D_1 - D_2 - D_3 - D_4 - \hat{d}_1 - \hat{d}_2 - \hat{d}_3 - \hat{d}_4)(u + \hat{u}) \tag{13}$$

$$\hat{\dot{x}} = A\hat{x} + B\hat{u} + [(A_1 - A_5)x + (B_1 - B_5)u]\hat{d}_1 + [(A_2 - A_5)x + (B_2 - B_5)u]\hat{d}_2 + [(A_3 - A_5)x + (B_3 - B_5)u]\hat{d}_3 + [(A_4 - A_5)x + (B_4 - B_5)u]\hat{d}_4 \tag{14}$$

$$\hat{\dot{x}} = A\hat{x} + [BE_1E_2E_3E_4]\begin{bmatrix} \hat{u} \\ \hat{d}_1 \\ \hat{d}_2 \\ \hat{d}_3 \\ \hat{d}_4 \end{bmatrix}$$

where

$$E_1 = \begin{bmatrix} \frac{V_{in} - V_1 + V_4}{L} \\ \frac{I_L}{C_1} \\ 0 \\ 0 \\ -\frac{I_L}{C_4} \end{bmatrix} E_2 = \begin{bmatrix} \frac{V_{in} - V_2 + V_4}{L} \\ \frac{I_L}{C_2} \\ 0 \\ 0 \\ -\frac{I_L}{C_4} \end{bmatrix} E_3 = \begin{bmatrix} \frac{V_{in} - V_3 + V_4}{L} \\ \frac{I_L}{C_3} \\ 0 \\ 0 \\ -\frac{I_L}{C_4} \end{bmatrix} E_4 = \begin{bmatrix} \frac{V_{in}}{L} \\ \frac{I_L}{C_1} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The developed small signal equivalent circuit helps to obtain the control to output transfer functions.To obtain the transfer function of output1 to d_1,the reduced equivalent circuit is drawn by making the other duty ratios and the other load currents as zero. The numerical values of the parameters of the SIQO

converter in Table 2 are used to obtain the transfer function model shown in Equations (15) to (18), one for each output by using MATLAB® M-file coding with the help of small signal state space model.

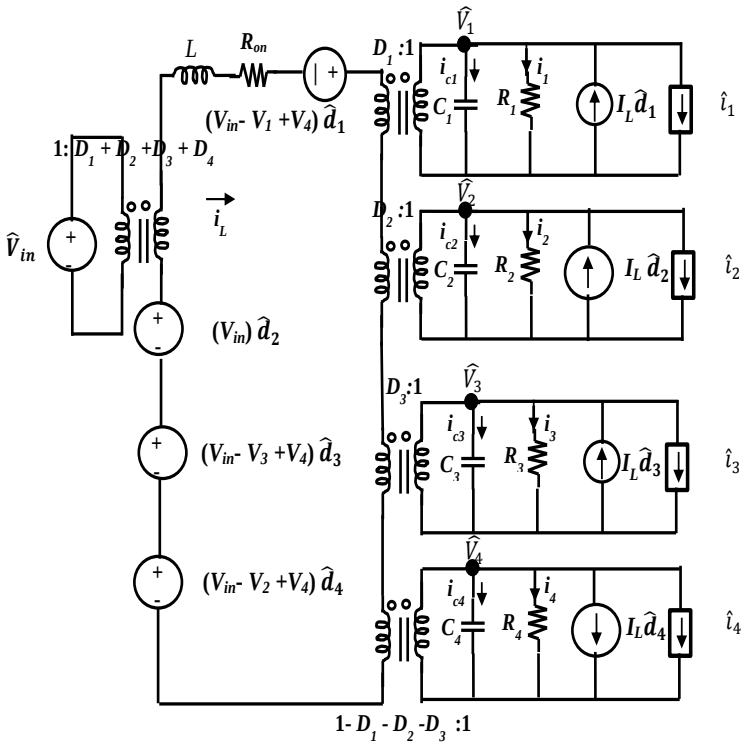


Figure 9. Lumped equivalent small-signal circuit of the converter

Table 2. The parameters of the SIQO Buck converter

Parameters	Symbol	Value
Input Voltage	V_{in}	12 V
Switching Frequency	f_S	33 kHz
Inductance	L	51 µH
Capacitance 1	C_1	2200 µF
Capacitance 2	C_2	1000 µF
Capacitance 3	C_3	1000 µF
Capacitance 4	C_4	3300 µF
Output Voltage 1	V_1	1 V
Output Voltage 2	V_2	1.2 V
Output Voltage 3	V_3	1.8 V
Output Voltage 4	V_4	3.3 V

$$G_1(s) = \frac{\hat{v}_1(s)}{\hat{d}_1(s)} = \frac{2273\,S^4+1.841*10^7S^3+3.591*10^{10}S^2+2.518*10^{13}S+5.517*10^{15}}{S^5+5976\,S^4+1.397*10S^3+1.423*10^{10}S^2+6.237*10^{12}S+9.708*10^{14}} \quad (15)$$

$$G_2(s) = \frac{\hat{v}_2(s)}{\hat{d}_2(s)} = \frac{5000\,S^4+3.443*10^7S^3+6.968*10^{10}S^2+5.249*10^{13}S+1.247*10^{14}}{S^5+5976\,S^4+1.397*10S^3+1.423*10^{10}S^2+6.237*10^{12}S+9.708*10^{14}} \quad (16)$$

$$G_3(s) = \frac{\hat{v}_3(s)}{\hat{d}_3(s)} = \frac{5000\,S^4+4.39*10^7S^3+6.639*10^{10}S^2+3.443*10^{13}S+5.847*10^{15}}{S^5+5976\,S^4+1.397*10S^3+1.423*10^{10}S^2+6.237*10^{12}S+9.708*10^{14}} \quad (17)$$

$$G_4(s) = \frac{\hat{v}_4(s)}{\hat{d}_4(s)} = \frac{6.118*10^7S^3+1.145*10^{11}S^2+6.488*10^{13}S+1.159*10^{16}}{S^5+5976\,S^4+1.397*10S^3+1.423*10^{10}S^2+6.237*10^{12}S+9.708*10^{14}} \quad (18)$$

To test the accuracy of the developed mathematical model of the converter the output voltage V_4 and Inductor Current (Average value) I_L are observed by varying D_0 and D_4 while keeping D_1=0.32, D_2=0.024 and D_3=0.072. The observations as variations in output voltage and inductor current due to change in duty ratiosare shown in Table 3.

It is inferred from Table 3 that a linear increase is observed in I_L and V_4 due to increase in the duration of inductor current charging. Similarly the output voltages one and four are observed by varying $D_1$and D_4keeping D_0 = 0.32, D_2= 0.024 and D_3= 0.072 and the readings are tabulated in Table 4.

Table 3. Variation in output voltage and inductor current due to change in duty ratios

	D_0 =0.28		D_0 =0.30		D_0 =0.32		D_0 =0.34		D_0 =0.36	
	D_4 =0.138		D_4 =0.158		D_4 =0.178		D_4 =0.198		D_4 =0.218	
	I_L	V_4	I_L	V_4	I_L	V_4	I_L	V_4	I_L	V_4
Model	4.4	3	4.6	3.3	4.8	3.3	4.8	3.5	5	3.8
Simulation	4.0	2.9	4.2	3.1	4.6	3.1	4.6	3.3	4.8	3.5

Table4. Variation in output voltages due to change in duty ratios:

	D_1= 0.042		D_1 = 0.044		D_1 = 0.046		D_1 = 0.048		D_1 = 0.05	
	D_4 = 0.182		D_4 = 0.180		D_4 = 0.178		D_4 = 0.176		D_4 = 0.174	
	V_1	V_4	V_1	V_4	V_1	V_4	V_1	V_4	V_1	V_4
Model	0.8	3.5	0.8	3.5	0.8	3.52	0.76	3.52	0.75	3.53
Simulation	0.9	3.3	1.01	3.3	1.0	3.3	1.0	3.3	1.0	3.3

It is inferred from Table 4 that the output voltages are almost remain same as the charging period of the inductor current maintained constant. This proves that the system represented by mathematical model developed and discrete components are same with small deviations observed due to non-ideal components. Therefore a model is developed in this research for four outputs and verified.

4. COMPENSATOR DESIGN

The design of compensator revolves around two significant issues that include an enhancement in the stability of the converter system and therefrom derive an acceptable time response. In order to obtain the desired phase margin and to increase the steady state accuracy of the system, a compensator $G_c(s)$ is added in between the error detector and the systems $G(s)$. The block diagram representation of the closed loop system is shown in Figure 10 [16].

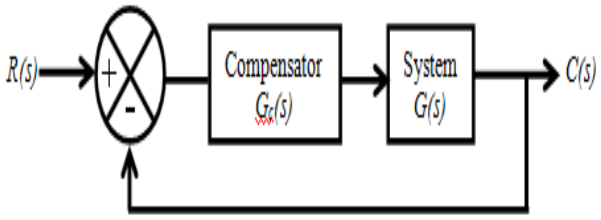


Figure 10. Block diagram representation of closed loop system

In Figure 10$R(s)$ is the reference input and $C(s)$ is the system output. The design of compensator attempts to remove the steady state error without disturbing the stability properties.The main contribution from the user end reiterates the need to focus on the choice of compensators for extricating an enhancement in both time and frequency domain responses. The codes written in the MATLAB® portal enable to obtain the Bode plots and the step response for the Equations (15), (16), (17) and (18) which help to obtain the classical stability criteria's such as phase and delay margins and gain crossover frequency and time response specifications. Compensators are designed with the help of MATLAB® SISO tool GUI to obtain the reasonable values of the performance specifications. Lag compensators are added to all the outputs such that

steady state error is removed [16]. The low frequency gain of the system is changed until the desired phase margin which should be in between 45 degree to 60 degree is obtained. The zero of the compensator is placed such that the zero dB gain cross over frequency should be in between 1/5 to 1/8 of switching frequency. The Equations (19), (20), (21) and (22) are the lag compensators obtained for four outputs respectively. The time and frequency domain specifications of the system with lag compensators are given in Table 5. It is inferred from Table 5 that the outputs have peak overshoot which is in the order of 20%.

$$G_{c1} = 20 * \frac{s+7320}{s+732} \quad (19)$$

$$G_{c2} = 5.5 * \frac{s+6320}{s+632} \quad (20)$$

$$G_{c3} = 8 * \frac{s+8240}{s+824} \quad (21)$$

$$G_{c4} = 0.5 * \frac{s+2000}{s+200} \quad (22)$$

In order to reduce the peak overshoot, lag-lead compensators are designed as given in Equations (23), (24), (25) and (26) for all the outputs and the performance specifications are tabulated in Table 5. As lead compensator is also added along with lag compensator to reduce the peak overshoot, there will be increase in phase margin as lead compensator contributes phase into the system.

$$G_{c1}(s) = 4.5 * \frac{(S+7320)}{(S+732)} \quad (23)$$

$$G_{c2} = 11 * \frac{s+7071}{s+1.414e04} * \frac{s+6320}{s+632} \quad (24)$$

$$G_{c3} = 16 * \frac{s+7071}{s+1.414e04} * \frac{s+8240}{s+824} \quad (25)$$

$$G_{c1} = 11 * \frac{s+7320}{s+732} * \frac{s+7071}{s+1.414e04} \quad (26)$$

Figure 11 and Figure 12a, b, c and d show the Bode plots and step responses of the dc-dc converter for four outputs with and without lag-lead compensator respectively.From the Bode plots shown in Figure 11,it is clear that the addition of lead compensator along with lag compensator will increase the phase margin but the desired peak overshoot and gain crossover frequency are obtained.

The step responseof the fur outputs with and without compensator is shown in Figure 12 plots reveal that the reduced peak overshoot, less rise time and settling time and zero steady state error are obtained due to the addition of lag lead compensators. Introduction of lag compensator bring the system phase margin to an acceptable value of around 55 degree and desired time response characteristics exceptpercentage peak overshoot. However, the addition of lag-lead compensator will further improve the time response parameters, gain crossover frequency, delay margin and also reduce the peak overshoot but contribute a considerable change in phase margin. The power circuit developed in MATLAB/Simulink® using discrete components with the designed compensator and the observed output voltages is shown in Figure 13.

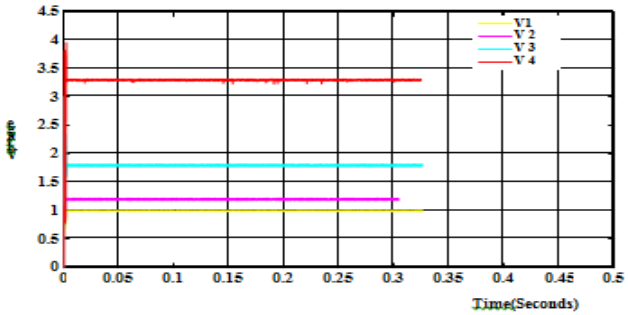


Figure 13. Simulated output Voltages response of the SIQO converter

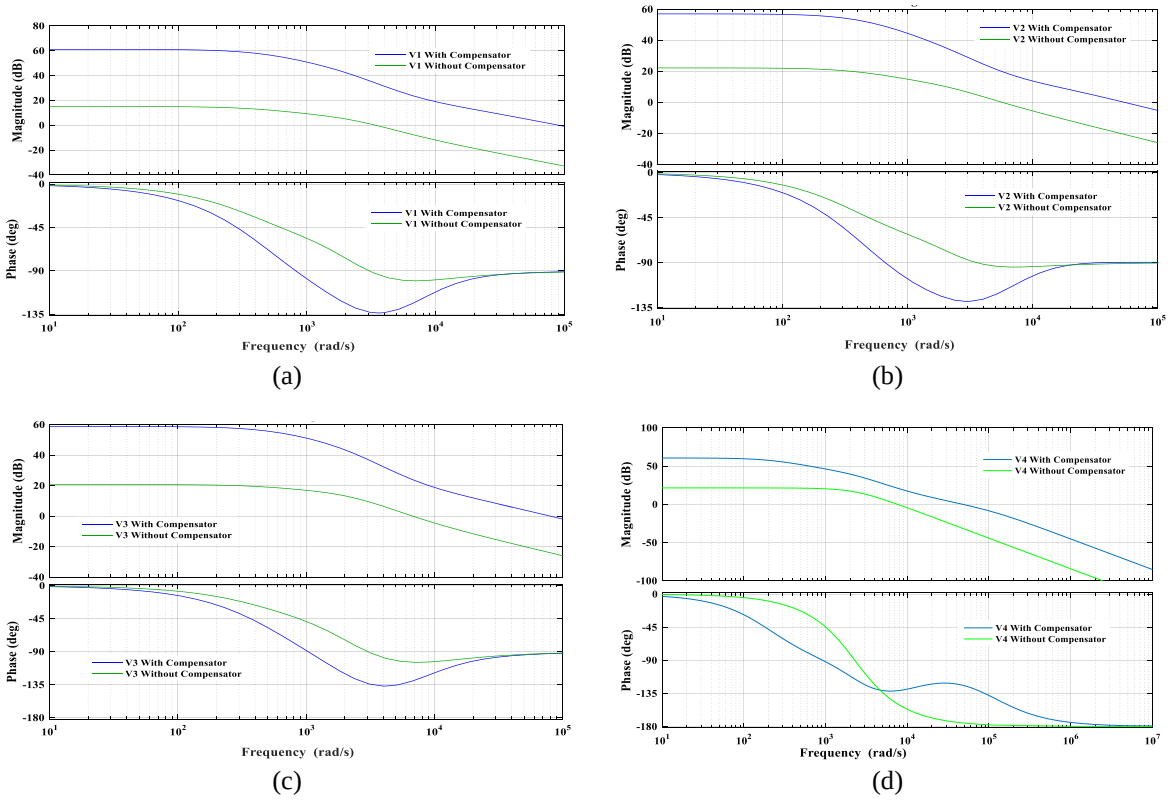


Figure 11. Magnitude and Phase response of the outputs with and without compensator
(a) V_1 (b) V_2 (c) V_3 (d) V_4

The closeness of the four output voltagesresponse obtained through the use of discrete components from simulation in Figure 13 with that in Figure 12 serves to validate the modeling of the converter and design of the compensator.

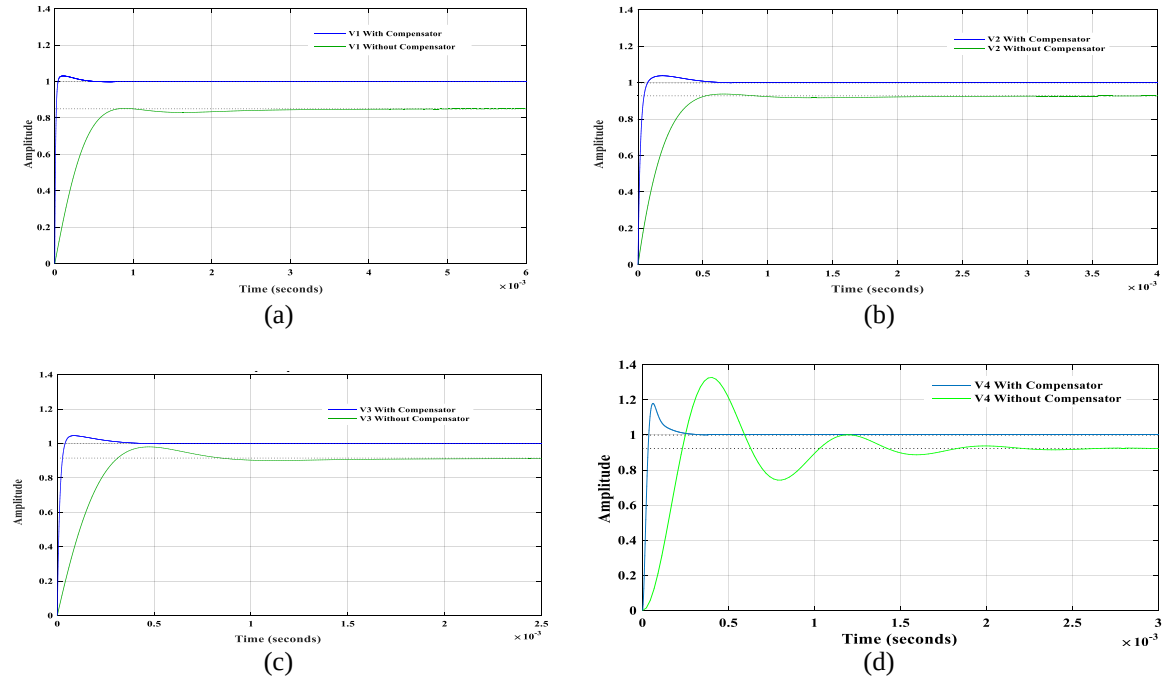


Figure 12. Step response of the outputs with and without compensator (a) V_1 (b) V_2 (c) V_3 (d) V_4

5. HARDWARE IMPLEMENTATION

In the Section 4, the dc-dc converter and the compensators which are incorporated in four outputs are discussed and their performance study is done using MATLAB® M-file coding. However it is necessary to verify these results by hardware implementation and it is explained in this section. The output voltages of the converter are given to the analog input of the data acquisition module DT9834® which is interfaced to MATLAB/Simulink®. Then the output voltages are compared with the reference voltages and the error signal is applied to the compensator developed in MATLAB/Simulink® and the control voltage signals are applied to the analog output of the DAQ module. The pulse width modulation generator and driver circuit avails the data from DAQ module to generate the pulses for the IGBT switches in the power module. The experimental hardware setup of SIQO converter seen through the photograph in Figure 14 is constructed and tested for the same specifications as those used in simulation.

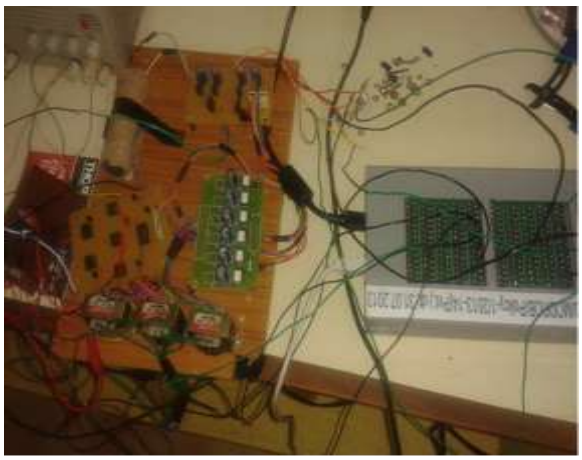


Figure 14. Hardware test setup of SIQO converter with compensator

Figure 15 represents the PWM pulses for the main path obtained from PWM generator and driver circuit using the reference signal obtained from Data Acquisition Module.

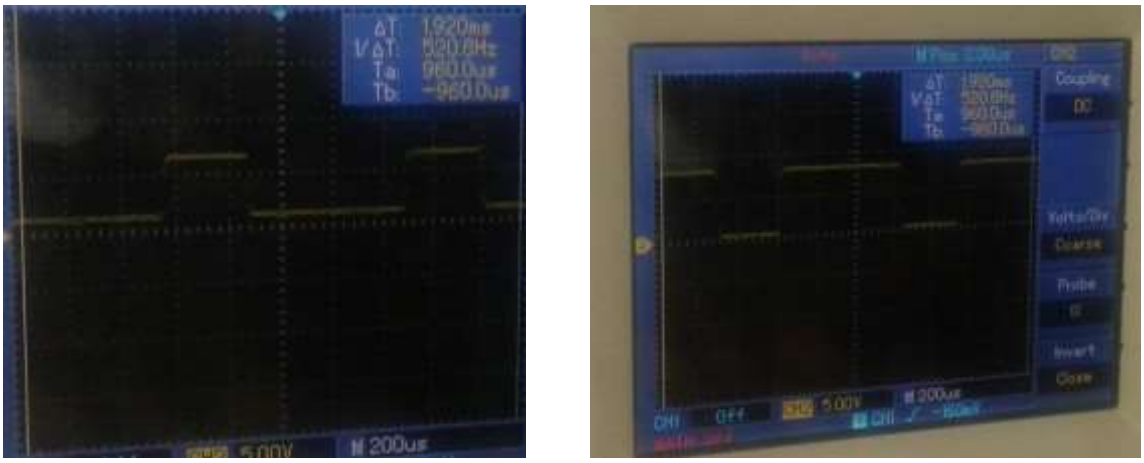


Figure 15. PWM pulses for main path

Figure 16 shows the output voltages obtained from the model which can be observed in the scope connected to the analog input of Data Acquisition Module. Table 6 compares the performance of the SIQO converter as a function of the output voltage, voltage regulation and ripple in the output voltages acquired

through simulation and from the hardware module to ensure the viability of the system to operate over a range of loads.

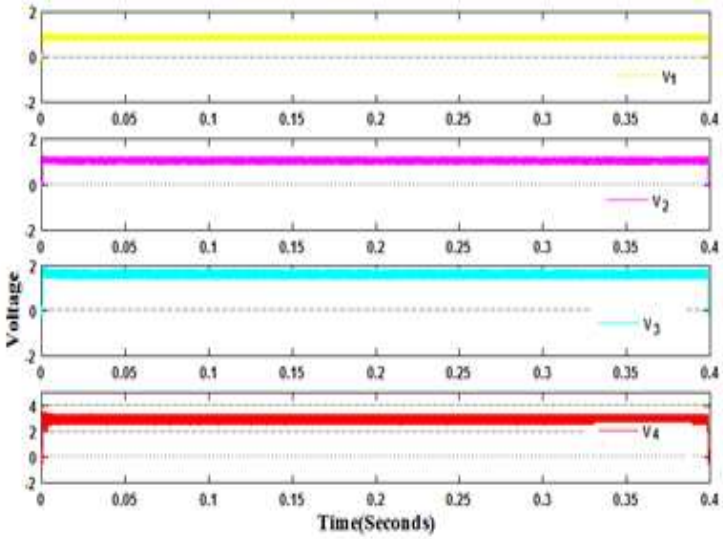


Figure 16. Output voltages of the SIQO converter from Prototype Model

Table 6. Performance of SIQO converter obtained by simulation and hardware

Sl.No.	Load power (w)	Reference Voltages		Simulation		Hardware	
				Mean Value(V)	Ripple (V)	Mean Value	Ripple
1	1	V_1	1	0.9997	0.02	1	0.1
	0.6	V_2	1.2	1.207	0.02	1.2	0.1
	1.8	V_3	1.8	1.799	0.02	1.8	0.1
	6.6	V_4	3.3	3.295	0.018	3.3	0.1
2	0.83	V_1	1	1.003	0.015	1	0.1
	0.55	V_2	1.2	1.208	0.015	1.19	0.1
	1.62	V_3	1.8	1.799	0.02	1.8	0.1
	5.88	V_4	3.3	3.289	0.02	3.3	0.1
3	1.2	V_1	1	1.001	0.015	1	0.1
	0.65	V_2	1.2	1.195	0.025	1.2	0.1
	2.025	V_3	1.8	1.792	0.03	1.79	0.1
	7.26	V_4	3.3	3.301	0.025	3.3	0.1

It is inferred that the compensator is able to maintain the output voltages constant against the load disturbances and the ripples are maintained at small value. Moreover the output voltages will not be disturbed by the disturbances present at other outputs.

6. CONCLUSION

A single input four output dc-dc buck converter with single inductor is designed which results in low cost and less space. SIMO converters in the open loop operation resultsin poor performance under steady state and transient conditions. To improve the performance a compensator is to be added to the converter. To design compensators the four output dc-dc buck converter has been modeled in the state space framework and its small signal equivalent circuit is derived over a chosen quiescent point. The transfer function model has also been obtained to realize the Bode plot for the system and there from enable the choice of compensators.

The simulation study has been presented to demonstrate the function of the compensators and benign its benefits through improvements in time response specifications. As the designed lag-lead compensators are not able to satisfy both phase margin and peak overshoot, it is recommended to design multivariable controller using optimization technique which is the extension of this work. With the compensators incorporated in each output steady state output voltages are observed using

MATLAB/Simulink®. The prototype using DT9834® DAQ module results have been garnered to correlate the simulated performance and evince a place for the use of the converter in the practical world.

REFERENCES

[1] D. Park, *et al.*, "*An 83% Peak Efficiency and 1.07W/mm2 Power Density Single Inductor 4-Output DC-DC Converter with Bang-Bang Zeroth-Order Control"*, IEEE Asian Solid-State Circuits Conference, pp. 61-64, 2014.

[2] D. Trevisan and P. Mattavelli, "Digital Control of Single-Inductor Multiple-Output Step-Down DC–DC Converters in CCM", *IEEE Transactions on Industrial Electronics* vol. 55, pp. 3476-3484, Feb 2008.

[3] Y.H. Ko, *et al*., "Non-Load-Balance-Dependent High Efficiency Single-Inductor Multiple-Output (SIMO) DC-DC Converters", pp.1-4, 2012.

[4] M. Sharifian and M. Delshad, "Improving the Steady and Transient Performance in the Buck Converter Using Sliding Mode Control", *International Transaction of Electrical and Computer Engineers System* 2, pp.1-6, 2014.

[5] M.Y. Jung, *et al*., *"An Error-Based Controlled Single-Inductor 10-Output DC-DC Buck Converter with High Efficiency at Light Load Using Adaptive Pulse Modulation", 2015* IEEE International Solid-State Circuits Conference, pp. 1-3, 2015.

[6] J. Kim, *et al*., "A Single-Inductor Eight-Channel Output DC–DC Converter with Time-Limited Power Distribution Control and Single Shared Hysteresis Comparator", *IEEE Transactions on Circuits and Systems—I: Regular Papers*, vol. 60, pp. 3354-3367, 2013.

[7] P. Patra, *et al*., "Control Scheme for Reduced Cross-Regulation in Single-Inductor Multiple-Output DC–DC Converters", *IEEE Transactions on Industrial Electronics*, vol. 60, pp. 5095-5105, 2012.

[8] Q. Liu, *et al*., "*Monolithic Quasi-Sliding-Mode Controller for SIDO Buck Converter in PCCM*", 2012 IEEE Asia Pacific Conference on Circuits and Systems (APCCAS), pp. 428 – 431, 2012.

[9] D. Lu, *et al.*, *"An 87%-Peak-Efficiency DVS-Capable Single-Inductor 4-Output DC-DC Buck Converter with Ripple-Based Adaptive Off-Time Control*", IEEE International Solid-State Circuits Conference, ISSCC 2014 , pp. 82-83, 2014

[10] Rahim Ildarabadi, and Azadeh Ahmadi, "A Review to AC Modeling and Transfer Function of DC-DC Converters", *TELKOMNIKA Indonesian Journal of Electrical Engineering*, Vol 13, No 2 , pp. 271-281.

[11] Jeya Selvan Renius, *et al.*, "Modelling and Simulation of Variable Frequency Synchronous DC-DC Buck Converter", *International Journal of Power Electronics and Drive Systems*, Vol 5, No 2, pp 237-243.

[12] S. Chonsatidjamroen, *et al.*, "Dynamic Model of a Buck Converter with a Sliding Mode Control," *World Academy of Science, Engineering and Technology*, vol. 60, pp. 1677- 1683, 2009.

[13] J.D. Dasika, *et al*., "Multivariable Control of Single-Inductor Dual-Output Buck Converters", *IEEE Transactions on Power Electronics*, 29, pp. 2061-2070, 2013

[14] P. Patra, *et al*.,"Control Scheme for Reduced Cross Regulation in Single Inductor Multiple Output DC-DC Converters", *IEEE Transactions on Industrial Electronics*, Vol. 60, No. 11, November 2013.

[15] P. Patra, *et al*., A Single-Inductor Multiple-Output switcher with simultaneous Buck, Boost, and Inverted Outputs", *IEEE Transactions on Power Electronics*, vol. 27, pp. 1936-1951, 2012.

[16] Sarun Soman and T.S. Sangeetha, Development of Digital Controller for DC-DC Buck Converter", *International Journal of Power Electronics and Drive System (IJPEDS)*, Vol. 6, pp. 788 – 796, December 2015.

BIOGRAPHIES OF AUTHORS



S. Augusti Lindiya, Assistant Professor, Department of Electrical & Electronics Engineering She obtained her Bachelor Degree in Electrical & Electronics Engineering in the year 1998 from MepcoSchlenk College of Engineering, Sivakasi, M.E. in Power Electronics & Drivesin the year 2000 from Shanmugha College of Engineering, Thanjavur and doing her Ph.D in SASTRA University. She has published three papers in international conferences and ten papers in international journals. She is working in the area of modeling and design of controllers for power converters.



Dr. S. Palani is Dean and Professor, Department of Electronics & Communication Engineering. He obtained his Bachelor Degree in Electrical Engineering in the year of 1966 (University of Madras), M. Tech in Control Systems Engineering from IIT, Kharagpur (1968) and Ph.D. in Control Systems Engineering from University of Madras in 1982. He has wide teaching experience over four decades. He has published more than 60 research papers in reputed national and international journals and conferences. Under his guidance 14 candidates were awarded Ph.D degree and 2 candidates are doing Ph.D in Anna University and 2 more candidates in SASTRA University



Dr. K. Vijayarekha, Associate Dean and Professor, Department of Electrical & Electronics Engineering. She obtained her Bachelor Degree in Electrical & Electronics Engineering in the year of 1990 from AlagappaChettiar College of Engineering and Technology, Karaikudi, M.E in Power Systems from NIT, Trichy (1992) and Ph.D. in 2008 from SASTRA University. She has teaching experience of 23 years. She has published more than 45 research papers in reputed national and international journals and conferences in the area of image processing, power converters, Non Destructive Testing, Interharmonics and FACTS. Under her guidance seven candidates are doing Ph.D in SASTRA University.