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PID Controller Response to Set-Point Change in DC-DC Converter Control

Oladimeji Ibrahim, Nor Zaihar Yahaya, Nordin Saad
Departement of Electrical and Electronics Engineering, Universiti Teknologi PETRONAS, Malaysia

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ABSTRACT

Power converter operations and efficiency is affected by variation in supply voltage, loads current, circuit elements, ageing and temperature. To meet the objective of tight voltage regulation, power converters circuit module and the control unit must be robust to reject disturbances arising from supply, load variation and changes in circuit elements. PID controller has been the most widely used in power converter control. This paper presents studies of robustness of PID controller tuning methods to step changes in the set point and disturbance rejection in power converter control. A DC-DC boost converter was modelled using averaged state-space mothod and PID controllers were designed with five different tuning methods. The study reveals the transient response and disturbance rejection capability of each tuning methods for their suitability in power supply design applications.

Corresponding Author:

Oladimeji Ibrahim,
Departement of Electrical and Electronics Engineering,
Universiti Teknologi PETRONAS,
32610 Bander Seri Iskander, Perak Malaysia.
Email: reacholaibrahim@gmail.com

1. INTRODUCTION

Power converters change electrical energy from one voltage, current or frequency level to another. Distributed generations, communication systems, aviation and other electrical gadgets like computers, electronic appliance require power supply for their operation. Some of these systems are sensitive to voltage fluctuations while others tolerate certain level of variation that is within the international stipulated standard. The high efficiency of switching mode power supply has made it the most widely used for power supply design. Its output voltage is a function of the input line (supply) voltage, the duty cycle of the pulse width modulation (PWM) control signal, the load current, and also the converter circuit element values [1, 2].

Power supply is design to produce output voltage that will remain within a specified range in the face of load current change from no load to full load. To achieve constant output voltage, a negative feedback control loop is incorporated to automatically adjust the control of PWM duty cycle regardless of supply input variation, load change and variation of circuit elements. The error resulting from the difference between the set point and output voltage is compensated by the controller which determines the PWM duty cycle for ensuring that the output voltage is as close as possible to the reference value [3].

PID controller has widely been applied to compensate for error resulting from difference between the set point and feedback in power converters design due to its simplicity and ruggedness [4-7]. Though in recent time, research effort is towards investigating modern control theory like the fuzzy and state-space control that could better account for modelling inadequacy and variation in circuit element values. The PID control have shown high level of acceptance in terms the tight output voltage regulation in the face of disturbance from line and load variations. This work focuses on PID tuning methods for power converter applications by investigating their transient response to input supply variation and load disturbance. Five (5) different PID tuning methods; namely Ziegler-Nichol Frequency Domain Method (ZN-FDM), Modified

Ziegler-Nichol (MZN), Damped Oscillation Method (DOM), Tyreus-Luyben Tuning Method (TLM), and Good Gain Method (GGM) are investigated. A DC-DC boost converter is modelled using averaged state-space method and PID controllers are designed using the aforementioned tuning techniques. The Matlab-Simulink is used to simulate the system in time-domain for analysing the transient response and also their disturbance rejections capability.

2. BOOST CONVERTER MODELLING

A DC-DC boost converter is non-isolated power supply that produces output voltage greater than its input. It comprises of at least a FET power switch (MOSFET/IGBT, Q_1), a diode and an inductor (L) as energy storage. Filter capacitor (C) is usually added to reduce the output voltage ripple. The operation can be in continuous or discontinuous mode depending on the charging and discharging of the input inductor. The steady and dynamic states model of continuous current mode (CCM) operation of non-ideal boost converter is presented in subsections 2.1 and 2.2. The steady state model allows for design of circuit parameters while the dynamic model is used for controller design and dynamic analysis.

2.1 Steady State Model

The PWM scheme is the mostly widely used for gate signal controlof field effect transistor (FET) devicein power converters design. The power transfer stage of anon-isolated boost has the “on” and “off” state as shown in Figure 1 and Figure 2 respectively. The total duration of the ON state (T_{on}) and OFF states (T_{off}) in continuous conduction mode (CCM) is given by Eq. (1) and Eq. (2), where (d) is the duty cycle set by the control circuit, and (T_s) is the switch period for complete one cycle.

$$T_{on} = dT_s \tag{1}$$

$$T_{off} = (1-d)T_s \tag{2}$$

Mode 1, when switch Q_1 is in *on* state at time $0 < t < t_{on}$

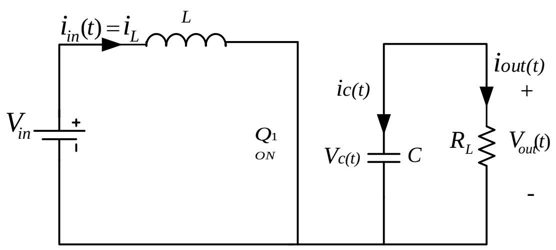

Figure 1. Equivalent circuit of boost converter at $0 < t < t_{on}$

The changes in inductor current ΔI_L is given by:

$$\Delta I_L = \frac{V_{in} t_{on}}{L} \tag{3}$$

Mode 2 when switch Q_1 is in *off* state at time $t_{on} < t \le T$

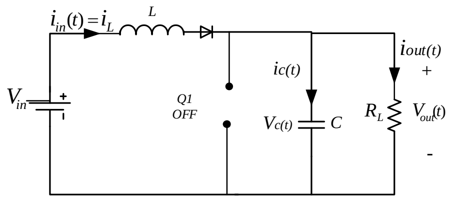

Figure 2. Equivalent circuit of boost converter at $t_{on}<t\leq T$

The inductor voltage changes by $V_{in} - V_{out}$ and the current falls by $\Delta I_L/T_{off}$, therefore the inductor current change is given by:

$$\Delta I_L = \frac{(V_{out} - V_{in})t_{off}}{L} \quad (4)$$

Thus, the steady state input to output voltage conversion ratio is obtained as:

$$V_{out} = \frac{V_{in}}{1-d} \quad (5)$$

The magnitude of peak-to-peak inductor current ripple ΔI_L is given by:

$$\Delta I_L = \frac{V_{in}\, d}{f_s L} \quad (6)$$

And, also the output capacitor voltage ripple ΔV_c is:

$$\Delta V_c = \Delta V_{out} = \frac{I_o d}{f_s C} \quad (7)$$

Table 1. Converter steady state parameters

Parameters	Symbol	Values
Input voltage	V_{in}	12 [V]
Output voltage	V_{out}	48 [V]
Output voltage ripple	ΔV_o	500 [mV]
Rated power	P_o	200 [W]
Load resistance	R_L	11.5 [Ω]
Average output current	I_o	4.17 [A]
Duty cycle	d	0.75
Inductor	L	0.5 [m H]
Output capacitor	C	125 [μF]
Switching frequency	f_s	50 [kHz]
MOSFET on resistance	R_{on}	40 [mΩ]
Inductor parasitic resistance	r_L	10 [mΩ]
Capacitor ESR	r_C	10 [mΩ]

2.2 Dynamic State Model

The state space averaged method is used to model a DC-DC boost converter considering parasitic parameters of the system components. State space averaged technique gives complete model with both steady state (DC) and dynamic (AC) quantities making it easy to obtain the system transfer function for dynamic state analysis [1, 8, 9].

The general state space averaged equation is given by Eq. (8) and Eq. (9)

$$\dot{x}(t) = \bar{A}x(t) + \bar{B}u(t) \tag{8}$$

$$y(t) = \bar{C}x(t) \tag{9}$$

Where, the state averaged matrix $\bar{A} = A_1 d + A_2(1-d)$, $\bar{B} = B_1 d + B_2(1-d)$, and $\bar{C} = C_1 d + C_2(1-d)$. The term $x(t)$ is the converter DC state vector defined as inductors currents and capacitors voltages, $u(t)$ is converter DC input vector and $y(t)$ is the converter DC output vector.

Considering the converter in mode 1 (Figure 1) when the switch Q_1 is in *on*-state at time interval dT_s, applying KVL and KCL the differential state variables for the boost converter are obtained as follows:

$$\begin{bmatrix} \frac{di_L(t)}{dt} \\ \frac{dV_c(t)}{dt} \end{bmatrix} = \begin{bmatrix} \frac{-(R_{on}+r_L)}{L} & 0 \\ 0 & -\frac{1}{C(R_L+r_c)} \end{bmatrix} \begin{bmatrix} i_L(t) \\ V_c(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} V_{in}(t) \tag{10}$$

$$\left[V_{out}(t)\right] = \begin{bmatrix} 0 & \frac{R_L}{(R_L+r_c)} \end{bmatrix} \begin{bmatrix} i_L(t) \\ V_c(t) \end{bmatrix} \tag{11}$$

Similarly, mode 2 (Figure 2) when switch Q_1 is in off-state at time interval $(1\text{-}d)T_s$, applying KVL and KCL yields differential state variables:

$$\begin{bmatrix} \frac{di_L(t)}{dt} \\ \frac{dVc(t)}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{r_L + R_L // r_c}{L} & \frac{-R_L}{L(R_L+r_c)} \\ \frac{R_L}{C(R_L+r_c)} & -\frac{1}{C(R_L+r_c)} \end{bmatrix} \begin{bmatrix} i_L(t) \\ V_c(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} V_{in}(t) \tag{12}$$

$$\left[V_{out}(t)\right] = \begin{bmatrix} R_L // r_c & \frac{R_L}{(R_L+r_c)} \end{bmatrix} \begin{bmatrix} i_L(t) \\ V_c(t) \end{bmatrix} \tag{13}$$

The averaged state matrices for both "*on*"and "*off*"state with duty cycle (d) as weighting factor is obtained as:

$$\bar{A} = \begin{bmatrix} -\frac{(R_{on}+r_L)d + (r_L + R_L // r_c)1-d}{L} & -\frac{R_L(1-d)}{L(R_L+r_c)} \\ \frac{R_L(1-d)}{C(R_L+r_c)} & -\frac{1}{C(R_L+r_c)} \end{bmatrix} \tag{14}$$

$$\bar{B} = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} \tag{15}$$

$$\overline{C} = \begin{bmatrix} R_L \,//\, r_c(1-d) & \dfrac{R_L}{(R_L + r_c)} \end{bmatrix} \quad (16)$$

To carry out dynamic state analysis and controller design for the DC-DC boost converter, the Matlab state-space (*ss*) to transfer function (*tf*) command is used to obtain the converter control-to-output transfer function $G_{vd}(s)$ using the steady state parameters presented in Table I.

$$G_{vd}(s) = \frac{4.996\,s + 3.997\,e^{06}}{s^2 + 780\,s + 1.057\,e^{06}} \quad (17)$$

3. PID CONTROLLER DESIGN

A proportional integral and derivative (PID) controller is considered as the most widely studied and used in both academics and industries due to its simplicity and robustness. The PID controller is the aggregates of the three sub-control units, the proportional, integral and derivative control modes. The effective control signal $u(t)$ by a PID controller in Laplace domain is given by Eq. (18) [10, 11].

$$u(t) = k_p e(t) + K_i \int_0^t e(t)dt + k_d \frac{de(t)}{dt} \quad (18)$$

The relationship of Eq. (19) is the continuous *s*-domain transfer function:

$$C(s) = K_p + \frac{K_i}{S} + K_d S \quad (19)$$

Where, K_p, K_i and K_d are the proportional gain, integral gain and derivative gain respectively, T_i is the integral time constant and T_d derivative time constant.

PID controllers are designed using five (5) different tuning methods in order to improve the converter transient response and eliminate steady state error. The procedures for each tuning methods Ziegler-Nichol Frequency Domain Method (ZN-FDM), Modified Ziegler-Nichol (MZN), Damped Oscillation Method (DOM), Tyreus-Luyben Tuning Method (TLM), and Good Gain Method (GGM) are briefly explained in the subsections 3.1 to 3.5

3.1 Ziegler-Nichol Frequency Domain Method (ZN-FDM)

The Ziegler-Nichole frequency method was proposed by Ziegler and Nichole in the 1942 based on sustained oscillation of the system response [10, 12]. The closed-loop system under proportional controller (K_p) is driven to critically stable state by increasing the proportional gain with integral time constant (T_i) set to infinity and derivative constant (T_d) set to 0. The corresponding gain and period at this point are referred to as ultimate gain K_u and ultimate period P_u obtained as 1 and 0.003s respectively for the converter. The proportional gain (K_p), integral (T_i) and derivative constants (T_d) for the PID controller are 0.6, 0.0015 and 0.000375 respectively using the Ziegler-Nichol tuning parameters in Table II.

Table 2. Ziegler-Nichol PID tuning parameters

Type of Controller	K_p	T_i	T_d
P	$0.5\,K_u$	∞	0
PI	$0.45\,K_u$	$0.8\,P_u$	0
PID	$0.6\,K_u$	$0.5\,P_u$	$0.125\,P_u$

3.2 Modified Ziegler-Nichol (MZN)

The Ziegler-Nichol frequency response method forces a system process to marginal stability leading to unsafe operation for sensitive systems [10]. A more conservative method is proposed for $1/4$ decay ratio oscillation. The system is operated with low proportional gain control mode with integral time constant (T_i) set to (∞) and derivative time (T_d) set to (0). The gain is gradually adjusted until a decay ratio of $(1/4)^{th}$ is obtained in the successive output response. The modified Ziegler-Nichol settings for response with overshoot and no overshoot is presented in Table III. The proportional gain (K_p), integral (T_i) and derivative constants (T_d) for the PID controller are obtained as 0.194, 0.0015 and 0.00099 respectively.

Table 3. Modified Ziegler-Nichol tuning parameters

PID Controller	K_p	T_i	T_d
Some Overshoot	0.33 K_u	0.5 P_u	0.33 P_u
No Overshoot	0.2 K_u	0.5 P_u	0.33 P_u

3.3 Damped Oscillation Method (DOM)

Herriot proposed a slight modification to the closed loop Ziegler-Nichol tuning method [13]. This methoddoes not allow the system to undergo oscillations so the process is not driven to marginal stability. The closed loop system is operated initially with low gain proportional control mode with integral time constant (T_i) = ∞ and derivative time constant T_d = 0 as in the case of Ziegler Nichole ultimate gain method. The gain is increased or decreased slowly until a $(1/4)^{th}$ decay ratio of successive overshoot and undershoot is obtained. At this point, proportional gain (K_p) and the period of damped oscillation T_{ud} are used to calculate the PID settings based on tuning parameters of Table IV. The optimum settings for the controller are obtained as K_p = 0.09, (T_i) =0.002 and T_d = 0.0005.

Table 4. Herriot tuning parameters for quarter-of-amplitude decay

Type of Controller	K_p	T_i	T_d
P	K_{pd}	-	-
PID	Adjusted	$T_{ud}/1.5$	$T_{ud}/6$

3.4 Tyreus-Luyben Tuning Method (TLM)

The Tyreus-Luyben method follows the same procedure as the Ziegler-Nichol method up till the point of obtaining the ultimate gain and ultimate period. This method only propose settings for PI and PID controller as presented in Table V [14]. The ultimate gain K_u and ultimate period P_u obtained under Ziegler-Nichol method as 1 and 0.003s are used to calculate the PID tune parameters. The proportional gain (K_p), integral (T_i) and derivative constants (T_d) for the PID controller are obtained as 0.4545, 0.0066 and 0.000476 respectively

Table 5. Tyreus-Luyben tuning rules for PI and PID

Type of Controller	K_p	T_i	T_d
P	$K_u/3.2$	$2.2P_u$	-
PID	$K_u/2.2$	$2.2P_u$	$P_u/6.3$

3.5 Good Gain Method (GGM)

In the year 2010, Finn Haugen developed a new PID tuning method called the good gain method [15]. The process is brought close to specified operating point under proportional controller (K_p) only set to 0 or 1, integral time T_i to ∞ and derivative time T_d to 0. The proportional gain value K_p is adjusted until the control loop has some overshoot and a barely observable undershoot as shown in Figure 3. The response is assumed to represent good stability of the control system with the gain value K_{pGG}. The integral time T_i is computed from (20)

$$T_i = 1.5T_{ou} \quad (20)$$

Where (T_{ou}) is the time between the overshoot and undershoot of the step response with only proportional controller.

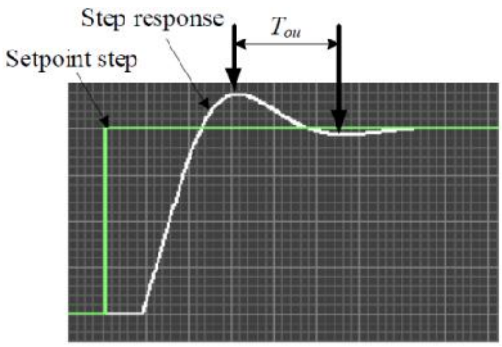

Figure 3. First overshoot and undershoot of the step response

The introduction of I-term in the loop having P-I controller in action might reduce the system stability. To compensate for this, the can be reduced to 80% of the original value.

$$K_p = 0.8K_{pGG} \tag{21}$$

The derivative-term can be added to make the controller a PID:

$$T_d = 0.25\,T_i \tag{22}$$

The proportional gain (K_p), integral constants (T_i) and derivative constants (T_d) for the controller are obtained as 0.04, 0.0045 and 0.001125 respectively.

4. RESULTS AND ANALYSIS

The performance analysis of each tuned PID controller using the five tuning methods on the modelled DC-DC converter are carried out in the Matlab-Simulink environment. The unit step input transient response in terms of the system rise time, settling time, steady state error and overshoot with unity feedback without controller (NOC) is presented in Figure 4.

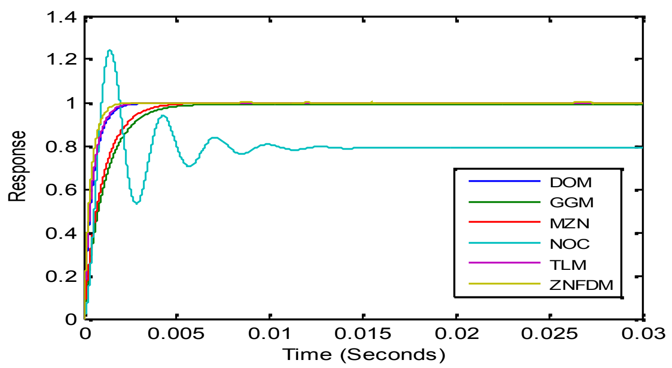

Figure 4. Tuned PID controller response to unit step input

The system time parametric response in Figure 4 shows that all the five controllers are able to improve the system transient response and eliminate steady state error with no overshoot. The unity feedback closed loop with no controller has a settling time of about 12 msec. while the good gain (GGM) PID controller has the slowest response of 5sec. among the tuned controllers. The time response also shows that the Ziegler-Nichol frequency domain, damped oscillation, and Tyreus-Luyben PID controllers has faster rise time of 1.5 msec, 2msec and 2.2 msec respectively at start up.

Toinvestigate the robustness of the tuned PID controllers, the closed loop system comprising of the modelled converter and the controllers are subjected to set point change. The parametric response in terms of percentage overshoot, settling time and the controller ability to restore the system back to normal operation to set point of 5V, 10V and 15V is presented in Figure 5.

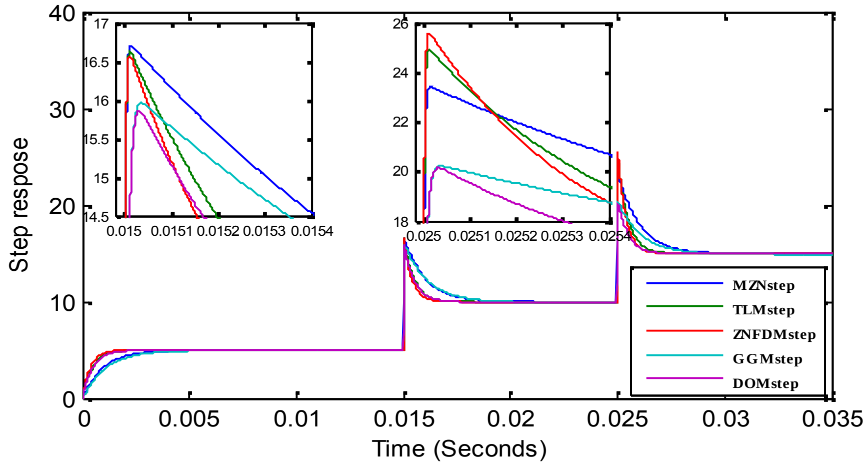

Figure 5. Tuned PID controller transient response to set point change

Table 6. PID controller auto-tuning response at set point 5V, 10V and 15V

Set point	Maximum overshoot (%)			Settling time (msec.)		
	5	10	15	5	10	15
Ziegler-Nichol	0	65.5	70.7	1.5	1.5	1.5
Damped oscillation	0	58	34	2	2	1.5
Tyreus-Luyben	0	66	65.3	2.2	2.5	2
Modified Ziegler-Nichol	0	67	56	4	4	4.5
Good Gain	0	60	34.7	5	4	4.5

Table VI shows the summary of the controller's response to set point change. At start up with 5V input, the transient response show that Ziegler-Nichol PID controller has faster rise time and a settling time of 1.5 msec. followed by damped oscillation and Tyreus-Luyben controllers with 2 msec. and 2.2 msec. respectively. The Modified Ziegler-Nichol and Good gain controllers have the slowest response of 4 msec. and 5 msec. respectively. It is observed that the five controller has no overshoot at start up but when set point changed to 10V, the Modified Ziegler-Nichol has highest overshoot of 67% and damped oscillation has the lowest of 58%. The set point is further increased to 15V, the Ziegler-Nichol has overshoot of 70.7 % and damped oscillation still has the lowest overshoot of 34%. Among the tuned controllers, the Ziegler-Nichol PID controller has the faster settling time but with high overshoot while the damped oscillation has the second fastest settling time with advantage of lowest overshoot to set point change.

A further studies is carried out on the controller's response to disturbance rejection capability. The system is subjected to unit step disturbance after 15 msec. and the systems response is presented in Figure 6. The Ziegler-Nichol PID controller is able to restore the system back to normal operating conditions in 2 msec while it took 2.2 msec for bothTyreus-Luyben and damped oscillation controllers. The Modified Ziegler-Nichol and Good gain PID controllers takes 4 msec. and 5 msec. respectively indicating their slow transient. The controller's autotunig parametric response to setpoint change and disturbance rejection shows that both Ziegler-Nichol frequency domain and damped oscillation PID controller offer the best transient in terms of rise and settling time. The damped oscillation has a distinct advantage of lower overshoot as compared to Ziegler-Nichol frequency domain which will reduce voltage stress on field effect transitors (FET) device like MOSFET during practical implementation.

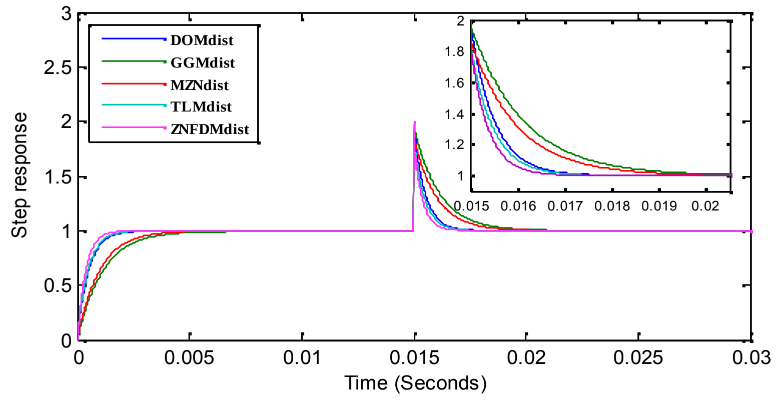

Figure 6. Tuned PID controller response to unit step disturbance

5. CONCLUSION

PID controllers have been designed using different tuning methods and their performance have been analysed on a DC-DC converter in Matlab-Simulink environment.The transient response in terms of rise time and settling time of the controllers to step input change and disturbance rejection capability have been investigated. The Ziegler-Nichol PID controller demonstrated fastest transient response followed by damped oscillation and Tyreus-Luyben controller respectively. Contrary to fastest transient associated with Ziegler-Nichol controller as compared to the second fastest damped oscillation PID controller, it has higher overshoot. The lower overshoot with damped oscillation PID controller makes it a better choice in a sutuition where there is need to reduce voltage stress on circuit elements like MOSFET during practical implementation of power converters.

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