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Stability and Robust Stabilization of 2-D Continuous Systems in Roesser Model Based on GKYP Lemma

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ABSTRACT

This paper is concerned with the stability and Robust stabilization problem for 2-D continuous systems in Roesser model, based on Generalized Kalman−Yakubovich−Popov lemma in combination with frequency-partitioning approach. Sufficient conditions of stability of the systems are formulated via linear matrix inequality technique. Finally, numerical examples are given to illustrate the effectiveness of the proposed method.

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1. INTRODUCTION

Stability of 2-D continuous systems is the major aim in all researches, in order to guarantee the normal operation of systems. In relation with these researches, there are various results in the past decades. For example, the stability of 2-D continuous systems has been solved lately in [1], the stability margin of 2-D continuous systems have been computed with a new method in [2], LMI based stability analysis for 2-D continuous systems was considered in [3], Robust stability analysis for 2-D continuous-time systems was obtained in [4], the stability analysis based on the quadratic Lyapunov function was obtained in [5], H_∞ filtering of uncertain 2-D continuous systems with time-varying delays was considered in [6]. In addition, the Robust stabilization and control design have been studied in some papers as well, to list some of these, authors in [7] proposed the robust state feedback H_∞ control for uncertain 2-D continuous state delayed Roesser systems. H_∞ control of 2-D continuous switched systems have been investigated in [8], multiobjective H_2/H_∞ control design was considered in [9], LMI based robust PID control has been solved in [10], and stabilization of two-dimensional continuous systems have been investigated in [11].

Recently, attention has been devoted towards the Kalman−Yakubovich−Popov (KYP) lemma in [12], this lemma makes equivalence between frequency domain inequality (FDI) characterizing a class of properties of a transfer function, and a linear matrix inequality (LMI) in [13], for its state space realization. Therefore, authors in [14] has proposed an extension of the KYP lemma, which is known as Generalized KYP (GKYP) lemma for the case of finite frequency domain. The 2-D GKYP lemma is obtained for Roesser model of 2-D continuous systems in [15], and for 2-D discrete systems, for both cases: Fornasini-Marchesini (FM) and for Roesser models, in [16] and [17], respectively. The GKYP combined with the frequency-partitioning approach to stability analysis, were obtained in [18] for 2-D discrete system, and for hybrid systems in [19, 20].

Motivated by the Previous research, in this paper, we suggest a sufficient conditions of stability of 2-D continuous Roesser systems, via GKYP lemma and frequency-partitioning approach, in order to reduce the conservativeness of the existing simple 2-D continuous Lyapunov inequality. Generally, in order to realize

a series of novel stability conditions for our system, the GKYP lemma is applied on each one of the N intervals of the entire frequency domain. Moreover, robust stabilization is also considered based on the proposed stability conditions. Finally, numerical examples are given to demonstrate the effectiveness of the proposed method.

Notation: we use the following notation throughout this paper. The superscript T, *, -1 stand for matrix transpose, matrix complex conjugate transpose, and matrix inverse, respectively. I denotes an identity matrix with appropriate dimension. The notation $P > 0$ ($P < 0$) means that matrix P is positive (negative) definite. $diag\{.\}$ stands for the block diagonal matrix, $Re\lambda(.)$ is the real of eigenvalue of a square matrix. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. PROBLEM FORMULATION AND PRELIMINARIES

Consider the following Roesser model for 2-D continuous systems :

$$\begin{bmatrix} \frac{\partial x^h(t_1,t_2)}{\partial t_1} \\ \frac{\partial x^v(t_1,t_2)}{\partial t_2} \end{bmatrix} = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \begin{bmatrix} x^h(t_1,t_2) \\ x^v(t_1,t_2) \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u(t_1,t_2) \tag{1}$$

where $x^h(t_1,t_2) \in \mathbb{R}^{nh}$, $x^v(t_1,t_2) \in \mathbb{R}^{nv}$ and $u(t_1,t_2) \in \mathbb{R}^m$ are the horizontal state, vertical state and input of system, respectively, and A_1, A_2, A_3, A_4, B_1 and B_2, are real matrices with appropriate dimensions.
For real numbers t_1 and t_2, we introduce notations

$$X^h = \sup_{t_2} \| x^h(0,t_2) \|, \quad X^v = \sup_{t_1} \| x^v(t_1,0) \| .$$

Assumption 1

$$\lim_{t_1 \longrightarrow \infty} \| x(t_1,0) \| = 0 \quad and \quad \lim_{t_2 \longrightarrow \infty} \| x(0,t_2) \| = 0.$$

They are inferred to the initial condition for the system (1).

In the stability analysis of 2-D continuous system (1), it is required to consider the zeros of the 2-D characteristic polynomial given by

$$C(s_1,s_2) = \det \begin{bmatrix} s_1 I_{nh} - A_1 & -A_2 \\ -A_3 & s_2 I_{nv} - A_4 \end{bmatrix} \tag{2}$$

It is known in the literature that the 2-D continuous system is asymptotically stable if and only if $C(s_1,s_2) \neq 0$ $\forall (s_1,s_2)$: $Re(s_1) \geq 0$ and $Re(s_2) \geq 0$.
In general, this condition is difficult to use in practice to verify the stability, therefore, another method will be used via LMI.

***Lemma* 1** *Simple necessary conditions for asymptotic stability of the 2-D continuous system (1) are as follows:*
i) A_1 is Hurwitz (i.e. all its eigenvalues have negative real parts, $Re\lambda_i(A_1) < 0$; $i = 1,..,n_h$).
ii) A_4 is Hurwitz.

Proof From (1) for $A2 = A3 = A4 = 0$, we obtain the state equation of the continuous system (for the fixed $0 \leq t_2 \in R$)

$$\frac{\partial x^h(t_1,t_2)}{\partial t_1} = A_1 x^h(t_1,t_2) \tag{3}$$

The system (3) is asymptotically stable if the matrix A_1 is Hurwitz.
Similarly, we can proof ii). □

Therefore, we assume the following throughout the paper.

Assumption 2 *The matrices A_1 and A_4 are Hurwitz.*

***Lemma* 2** *Let the assumption 2 be satisfied, the 2-D continuous system (1) is asymptotically stable if and only if*

$$S(s) = A_3(sI - A_1)^{-1} A_2 + A_4, \quad s = j\omega \tag{4}$$

is Hurwitz matrix for $\omega \in \mathbb{R}$.

Proof Let $u(t_1, t_2) = 0$, and taking the Laplace transformation of system (1) for t_1 only, and under zero initial condition, we get

$$\begin{bmatrix} sX^h(s,t_2) \\ \frac{\partial X^v(s,t_2)}{\partial t_2} \end{bmatrix} = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \begin{bmatrix} X^h(s,t_2) \\ X^v(s,t_2) \end{bmatrix} \tag{5}$$

Solving (5), we obtain

$$\frac{\partial X^v(s,t_2)}{\partial t_2} = [A_3(sI - A_1)^{-1}A_2 + A_4]X^v(s,t_2) \tag{6}$$

System (6) can be regarded as a 1-D continuous system with complex variable s, and we notice that the variable t_2 of the system doesn't depend on the variable s. So the 1-D continuous system (6) is asymptotically stable if and only if $[A_3(sI - A_1)^{-1}A_2 + A_4]$ is Hurwitz matrix for $Re(s) = 0$. Hence, the 2-D continuous system (1) is asymptotically stable if and only if $[A_3(j\omega I - A_1)^{-1}A_2 + A_4]$ is Hurwitz matrix $\forall w \in \mathbb{R}$. □

Remark 1 *Notice that when interchanging A_1 with A_4, and A_2 with A_3, the 2-D continuous system (1) is asymptotically stable if $A_2(j\omega I - A_4)^{-1}A_3 + A_1$ is Hurwitz matrix $\forall\omega \in \mathbb{R}$.*

We will use the following lemmas, known as the KYP lemma, the GKYP, and the Projection Lemma, respectively.

Lemma 3 *[12] Let matrices A, B, and $\Theta = \Theta^T$ be given, if $det(jwI - A) \neq 0\, \forall\omega \in \mathbb{R}$. Then the following two statements are equivalent.*
(i) For any $\omega \in \mathbb{R} \cup \infty$,

$$\begin{bmatrix} (jwI - A)^{-1}B \\ I \end{bmatrix}^* \Theta \begin{bmatrix} (jwI - A)^{-1}B \\ I \end{bmatrix} < 0 \tag{7}$$

(ii) There exists a symmetric matrix P such that

$$\begin{bmatrix} A & B \\ I & 0 \end{bmatrix}^* \begin{bmatrix} 0 & P \\ P & 0 \end{bmatrix} \begin{bmatrix} A & B \\ I & 0 \end{bmatrix} + \Theta < 0 \tag{8}$$

Lemma 4 *[14] Let the matrices Θ, F, Φ and Ψ be given, and denote N_ω is the null space of $T_\omega F$, where $T_\omega = \begin{bmatrix} I & -j\omega I \end{bmatrix}$. The inequality*

$$N_\omega^* \Theta N_\omega < 0, \quad with \quad \omega \in [\omega_1, \omega_2], \tag{9}$$

holds if and only if there exist $Q > 0$ and a symmetric matrix P, such that

$$F^*(\Phi \otimes P + \Psi \otimes Q)F + \Theta < 0 \tag{10}$$

where $\Phi = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $\Psi = \begin{bmatrix} -1 & j\omega_c \\ -j\omega_c & -\omega_1\omega_2 \end{bmatrix}$, $w_c = \frac{(\omega_1+\omega_2)}{2}$.

Lemma 5 *[13] Given a symmetric matrix $\Sigma \in \mathbb{R}^{p\times p}$ and two matrices X, Z of column dimension p, there exists a matrix Y such that the LMI*

$$\Sigma + symX^TYZ < 0 \tag{11}$$

holds if and only if the following two projection inequalities with respect to Y are satisfied:

$$X^{\perp^T}\Sigma X^\perp < 0, \quad Z^{\perp^T}\Sigma Z^\perp < 0. \tag{12}$$

where $X^\perp$ and $Z^\perp$ are arbitrary matrices whose columns form a basis of the null spaces of X and Z, respectively.

3. STABILITY ANALYSIS

we are now in a position to present a new condition for checking the stability of 2-D continuous systems of Roesser model.

***Lemma* 6** *The 2-D continuous system (1) is asymptotically stable if there exist $P_1 > 0$ and $P_2 > 0$ such that the LMI*

$$A^T P + PA < 0 \tag{13}$$

is feasible. Where $P = diag\{P_1, P_2\}$ *and* $A = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}$.

Proof LMI (13) can be rewritten as

$$\begin{bmatrix} P_1A_1 + A_1^T P_1 & A_3^T P_2 + P_1 A_2 \\ * & P_2 A_4 + A_4^T P_2 \end{bmatrix} < 0 \tag{14}$$

and this latter LMI can be rewritten as

$$\begin{bmatrix} A_1 & A_2 \\ I & 0 \end{bmatrix}^T \begin{bmatrix} 0 & P_1 \\ P_1 & 0 \end{bmatrix} \begin{bmatrix} A_1 & A_2 \\ I & 0 \end{bmatrix} + \Theta < 0 \tag{15}$$

where

$$\Theta = \begin{bmatrix} A_3 & A_4 \\ 0 & I \end{bmatrix}^T \begin{bmatrix} 0 & P_2 \\ P_2 & 0 \end{bmatrix} \begin{bmatrix} A_3 & A_4 \\ 0 & I \end{bmatrix} \tag{16}$$

by Lemma 3, (15) is equivalent to

$$\begin{bmatrix} (jwI - A_1)^{-1} A_2 \\ I \end{bmatrix}^* \Theta \begin{bmatrix} (jwI - A_1)^{-1} A_2 \\ I \end{bmatrix} < 0,$$

or

$$\begin{bmatrix} S(j\omega) \\ I \end{bmatrix}^* \begin{bmatrix} 0 & P2 \\ P2 & 0 \end{bmatrix} \begin{bmatrix} S(j\omega) \\ I \end{bmatrix} < 0 \tag{17}$$

where $S(jw)$ is the frequency response matrix obtained from $S(s)$ of Lemma 4, moreover, (17) can be written as

$$S(jw)^* P_2 + P_2 S(jw) < 0. \tag{18}$$

and the existence of a $P_2 > 0$ satisfying this last condition immediately implies that all eigenvalues of $S(j\omega)$ must have strictly negative real parts, $\forall \omega \in \mathbb{R} \cup \infty$, that is, feasibility of (13) guarantees that condition of Lemma 2 holds. Moreover, feasibility of (13) implies that

$$P_1 A_1 + A_1^T P_1 < 0, \quad P_2 A_4 + A_4^T P_2 < 0,$$

and, since $P_1 > 0$ and $P_2 > 0$, all eigenvalues of the matrices A_1 and A_4 must have strictly negative real parts, and feasibility of (13) guarantees that Lemma 1 and Lemma 2 are satisfied. □

Lemma 6 proposes an LMI condition for the asymptotical stability of the system in (1), there exists some conservativeness due to the requirement of a constant matrix P_2 for all $\omega \in \mathbb{R} \cup \infty$, though. Following the similar line of [18, 19, 20], the existence of $P_2(j\omega) > 0$ such that

$$S(jw)^* P_2(jw) + P_2(jw) S(jw) < 0, \quad \forall \omega \in \mathbb{R} \cup \infty,$$

is a sufficient condition for asymptotical stability of the system (1). Based on this result, and in order to reduce the conservativeness of Lemma 6, we attempt to obtain a piecewise constant matrices $P_2(j\omega)$ via a frequency-partitioning appoach, over the entire frequency field.
Denote $\Omega = \mathbb{R} \cup \infty$, and due to $S(-j\omega) = S(j\omega)^*$, the following identities hold:

$$\sup_{\omega \in \Omega} Re\lambda(S(jw)) = \sup_{\omega \in \Omega^+} Re\lambda(S(jw)) = \sup_{\omega \in \Omega^-} Re\lambda(S(jw)) \tag{19}$$

where $\Omega^+ = [0, \infty]$, $\Omega^- = [\infty, 0]$. Therefore, it suffices to consider the half frequency field Ω^+. Now, for a given positive integer N, dividing the frequency domain Ω^+ into N intervals, such that

$$\Omega^+ = \bigcup_{i=1}^{N} \Omega_i; \quad \Omega_i = [w_{i-1}, w_i], \quad \omega_0 = 0, \quad \omega_N = \infty, \tag{20}$$

then applying the result of Lemma 4 on each interval, we obtain the following theorem.

Theorem 1 *For a given positive integer N, define frequency intervals Ω^+ as in (20). System (1) is asymptotically stable if there exist $P_{sj} > 0$, j=1,2. P_{1i}, P_{2i}, $Q_i > 0$, $i = 1, 2, ..., N$*

$$A^T P_i + P_i A + F^T (\Psi_i \otimes Q_i) F < 0 \tag{21}$$

$$A_{j\times j}^T P_{sj} + P_{sj} A_{j\times j} < 0, \quad j = 1, 2. \tag{22}$$

where $A = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}$, $F = \begin{bmatrix} A_1 & A_2 \\ I & 0 \end{bmatrix}$, *and* $P_i = diag\{P_{1i}, P_{2i}\}$.

For $i = 2, 3, ..., N-1$,

$$\Psi_i = \begin{bmatrix} -1 & jw_{ci} \\ -jw_{ci} & -w_{i-1}w_i \end{bmatrix}, \quad with \quad w_{ci} = \frac{(w_{i-1} + w_i)}{2}. \tag{23}$$

For $i = 1$ and $i = N$,

$$\Psi_1 = \begin{bmatrix} -1 & 0 \\ 0 & w_1^2 \end{bmatrix} \quad and \quad \Psi_N = \begin{bmatrix} 1 & 0 \\ 0 & -w_{N-1}^2 \end{bmatrix} \tag{24}$$

respectively.

Proof By Assumption 2, we have $Re\lambda(A_1) < 0$ and $Re\lambda(A_4) < 0$ if and only if there exist $P_{s1} > 0$ and $P_{s1} > 0$ such that $A_1^T P_{s1} + P_{s1} A_1 < 0$, $A_4^T P_{s2} + P_{s2} A_4 < 0$, LMIs in (22) are satisfied.
For $i = 2, ..., N-1$, and $\{i = 1, i = N\}$, according to [14] the matrix Ψ_i should be taking as (23) and (24), respectively. The condition in (21), can be written as

$$F^T (\Phi \otimes P_{1i} + \Psi_i \otimes Q_i) F + \Theta_i < 0 \tag{25}$$

where

$$\Phi = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{26}$$

and

$$\Theta_i = \begin{bmatrix} A_3 & A_4 \\ 0 & I \end{bmatrix}^T (\Phi \otimes P_{2i}) \begin{bmatrix} A_3 & A_4 \\ 0 & I \end{bmatrix} \tag{27}$$

Denote $G(j\omega) = (j\omega I - A_1)^{-1} A_2$, and $S(j\omega) = A_4 + A_3 (j\omega I - A_1)^{-1} A_2 = A_3 G(j\omega) + A_4$, by lemma 4, the following inequality follows:

$$\begin{bmatrix} G(j\omega) \\ I \end{bmatrix}^* \Theta_i \begin{bmatrix} G(j\omega) \\ I \end{bmatrix} < 0, \forall \omega \in \Omega^+ \tag{28}$$

or

$$\begin{bmatrix} S(j\omega) \\ I \end{bmatrix}^* (\Phi \otimes P_2^{(l)}) \begin{bmatrix} S(j\omega) \\ I \end{bmatrix} < 0, \quad \forall \omega \in \Omega^+ \tag{29}$$

or in a more compact form

$$S(j\omega)^* P_{2i} + P_{2i} S(j\omega) < 0, \quad P_{2i} > 0, \quad \forall \omega \in \Omega^+ \tag{30}$$

So $Re\lambda(S(j\omega)) < 0$ is finally guaranteed for all $\omega \in \Omega^+$. Combining $Re\lambda(A_1) < 0$, $Re\lambda(A_4) < 0$ and $Re\lambda(S(j\omega)) < 0$, we conclude that system (1) is asymptotically stable based on Lemma 2. The proof is completed. □

Remark 2 *When $N = 1$, and if letting $Q_i = 0$, $P_{1i} > 0$ and $P_{2i} > 0$ be real, then (21) reduces to (13), that is, Lemma 6 is a special case of Theorem 1.*

Theorem 2 *For a given positive integer N, define frequency intervals Ω^+ as in (20). System (1) is asymptotically stable if there exist $P_{sj} > 0$, j=1,2. P_{1i}, P_{2i}, $Q_i > 0$, W_{1i}, W_{2i}, $i = 2, 3, ..., N-1$, such that*

$$\Xi_i = \begin{bmatrix} \Lambda_{11i} & \Lambda_{12i} & 0 & \Lambda_{14i} \\ * & \Lambda_{22i} & \Lambda_{23i} & \Lambda_{24i} \\ * & * & \Lambda_{33i} & \Lambda_{34i} \\ * & * & * & \Lambda_{44i} \end{bmatrix} < 0 \tag{31}$$

$$\Xi_{sji} = \begin{bmatrix} \Lambda_{sj1i} & \Lambda_{sj2i} \\ * & \Lambda_{sj3i} \end{bmatrix} < 0, \quad j = 1, 2. \tag{32}$$

$\Lambda_{11i} = -Q_i - W_{1i} - W_{1i}^T$,
$\Lambda_{12i} = P_{1i} + jw_{ci}Q_i - W_{1i}^T + W_{1i}A_1$,
$\Lambda_{14i} = W_{1i}A_2$,
$\Lambda_{22i} = -w_{i-1}w_iQ_i + A_1^TW_{1i}^T + W_{1i}A_1$,
$\Lambda_{23i} = A_3^TW_{2i}^T$,
$\Lambda_{24i} = A_3^TW_{2i}^T + W_{1i}A_2$,
$\Lambda_{33i} = -W_{2i} - W_{2i}^T$,
$\Lambda_{34i} = P_{2i} - W_{2i}^T + W_{2i}A_4$,
$\Lambda_{44i} = W_{2i}A_4 + A_4^TW_{2i}^T$.
$\Lambda_{sj1i} = -W_{ji} - W_{ji}^T$,
$\Lambda_{sj2i} = P_{sj} - W_{1i}^T + W_{ji}A_{j\times j}$,
$\Lambda_{sj3i} = A_{j\times j}^TW_{ji}^T + W_{ji}A_{j\times j}$, *j=1,2.*

For i=1, we replace Λ_{11i}, and Λ_{12i} in (31) by
$\Lambda_{121} = P_{11} - W_{11}^T + W_{11}A_1$,
$\Lambda_{221} = w_1^2Q_1 + A_1^TW_{11}^T + W_{11}A_1$, *respectively.*

For i=N, we replace Λ_{11i}, Λ_{12i} and Λ_{22i} in (31) by
$\Lambda_{11N} = Q_N - W_{1N} - W_{1N}^T$,
$\Lambda_{12N} = P_{1N} - W_{1N}^T + W_{1N}A_1$,
$\Lambda_{22N} = -w_{N-1}^2Q_N + A_1^TW_{1N}^T + W_{1N}A_1$, *respectively.*

Proof From Theorem 1, let

$$\Sigma = \begin{bmatrix} \Phi \otimes P_{1i} + \Psi_i \otimes Q_i & 0 \\ * & \Phi \otimes P_{2i} \end{bmatrix}, \tag{33}$$

According to [14], for $i = 2, ..., N-1$, $i = 1$ and $i = N$, Ψ_i as in (23), (24), and Φ as in (26).
Let $Y = \begin{bmatrix} W_{1i} & 0 \\ W_{1i} & 0 \\ 0 & W_{2i} \\ 0 & W_{2i} \end{bmatrix}$, $Z = \begin{bmatrix} -I & A_1 & 0 & A_2 \\ 0 & A_3 & -I & A_4 \end{bmatrix}$, $X = I$, (31) is equivalent to

$$sym(X^TYZ) + \Sigma < 0 \tag{34}$$

since one can choose $X^\perp = 0$, the first inequality in (12) vanishes, and then by lemma 5, (34) hold for some Y if and only if ${Z^\perp}^T\Sigma Z^\perp < 0$. Note that $Z^\perp$ can be selected as $Z^\perp = \begin{bmatrix} A_1 & A_2 \\ I & 0 \\ A_3 & A_4 \\ 0 & I \end{bmatrix}$, and then by calculation, we can obtain the equivalence between ${Z^\perp}^T\Sigma Z^\perp < 0$ and (21). Consequently (21) is equivalent to (31).
In addition from (22), we get

$$\begin{bmatrix} A_{j\times j} \\ I \end{bmatrix}^T \begin{bmatrix} 0 & P_{sj} \\ P_{sj} & 0 \end{bmatrix} \begin{bmatrix} A_{j\times j} \\ I \end{bmatrix} < 0, \quad j = 1, 2. \tag{35}$$

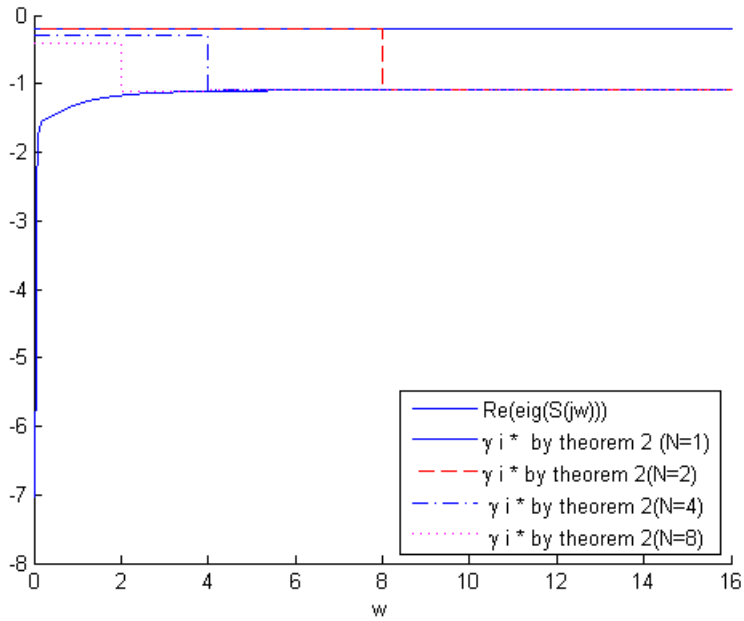

Figure 1: $Re\lambda_{max}(S(j\omega))$ and γ_i^*

The equivalence between (35) and (32) can be similarly found by re-introducing $\Sigma = \begin{bmatrix} 0 & P_{sj} \\ P_{sj} & 0 \end{bmatrix}$, $Y = \begin{bmatrix} W_{ji} \\ W_{ji} \end{bmatrix}$, $Z = \begin{bmatrix} -I & A_{j\times j} \end{bmatrix}$, $X = I$, j=1,2. Thus, Theorem 2 is equivalent to Theorem 1. □

Example 1 *In this part, we provide an example to illustrate the application of the proposed method. consider system in (1), where the matrices in the system are obtained by a suitable transformation from the original system matrices [21]:*

$$A_c = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} = (A_d - 1)(A_d + 1)^{-1}$$

The matrices in the original problem are as the following form [18]:

$$A_d = \begin{bmatrix} A_{d1} & A_{d2} \\ A_{d3} & A_{d4} \end{bmatrix}, \quad A_{d1} = \begin{bmatrix} 0.5 & 0.5 \\ 0.1 & -0.1 \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} 0.4 & 1.1 \\ 0.6 & 0.1 \end{bmatrix}, \quad A_{d3} = \begin{bmatrix} -0.1 & -0.1 \\ -0.2 & 0.6 \end{bmatrix},$$

$$A_{d4} = \begin{bmatrix} -0.5 & -0.5 \\ -0.1 & -0.7 \end{bmatrix}.$$

we obtain

$$A_1 = \begin{bmatrix} 5.2979 & -16.0426 \\ 4.2128 & -12.7447 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 20.5957 & 23.9149 \\ 16.4255 & 16.5106 \end{bmatrix}, \quad A_3 = \begin{bmatrix} -5.7872 & 16.2553 \\ -7.4894 & 22.2128 \end{bmatrix},$$

$$A_4 = \begin{bmatrix} -22.5745 & -23.4894 \\ -26.9787 & -30.5745 \end{bmatrix}.$$

Denote γ_i^* as the minimum value of $-\gamma_i$ that satisfies

$$\sup_{\omega\in\Omega_i} Re\lambda_{max}(S(j\omega)) < -\gamma_i < 0$$

γ_i^* could be computed from (31) by replacing $\Phi \otimes P_{2i}$ in (33) by $\begin{bmatrix} 0 & P_{2i} \\ P_{2i} & \gamma_i \end{bmatrix}$ and minimizing $-\gamma_i$. Figure 1 shows $Re\lambda_{max}(S(j\omega))$ and the executed γ_i^* by Theorem 2 with $N = 1, 2, 4, 8$. The stability of the above system is verified, since $Re(\lambda_{max}(S(j\omega))) < 0$ is evident. With N growing, It is further shown that $-\gamma_i$ tends to the value of $Re\lambda_{max}(S(j\omega))$ over Ω^+.

Theorem 2 with $N = 1$ fails to decide the stability of the above system. By increasing N, it is found that Theorem 2 with $N = 2, 4, 8$ succeeds, note that, the above system is asymptotically stable only for $i = 2, ..., N$. But for $i = 1$, whatever N, and whatever the way of partitioning the entire interval, always system is not stable. This is due to the rapid

variation of the curve of $Re\lambda_{max}(S(j\omega))$ in the vicinity of $\omega_0 = 0$.

Remark 3 *In the following domain [16,+∞], $Re\lambda_{max}(S(j\omega))$ remains relatively stationary to the value $Re\lambda_{max}(S(j\infty)) = Re\lambda_{max}(A_4) = -1.0850$, then γ_i^* also tends to this value throughout the domain. Even if it decomposed, we find very similar values to -1.0850. That's why we worked on just the domain $[0, 16]$ (Figure 1).*

4. CONTROL LAW DESIGN

In this section, Theorem 2 is further developed for state-feedback control of the uncertain 2-D continuous systems.

Consider a 2-D continuous system of Roesser model with norm-bounded uncertainty:

$$\begin{bmatrix} \frac{\partial x^h(t_1,t_2)}{\partial t_1} \\ \frac{\partial x^v(t_1,t_2)}{\partial t_2} \end{bmatrix} = \begin{bmatrix} A_1 + \Delta A_1 & A_2 + \Delta A_2 \\ A_3 + \Delta A_3 & A_4 + \Delta A_4 \end{bmatrix} \begin{bmatrix} x^h(t_1,t_2) \\ x^v(t_1,t_2) \end{bmatrix} + \begin{bmatrix} B_1 + \Delta B_1 \\ B_2 + \Delta B_2 \end{bmatrix} u(t_1,t_2) \tag{36}$$

where the uncertain matrices ΔA_q, $q = 1, 2, 3, 4$ and ΔB_p, $p = 1, 2$ formed as

$$\begin{aligned} \begin{bmatrix} \Delta A_1 & \Delta A_2 & \Delta B_1 \end{bmatrix} &= H_1 \Delta \begin{bmatrix} E_1 & E_2 & L_1 \end{bmatrix} \\ \begin{bmatrix} \Delta A_3 & \Delta A_4 & \Delta B_2 \end{bmatrix} &= H_2 \Delta \begin{bmatrix} E_3 & E_4 & L_2 \end{bmatrix} \end{aligned} \tag{37}$$

where H_1, H_2, E_1, E_2, E_3, E_4, L_1 and L_2 are known constant matrices, Δ is norm-bounded parameter uncertainty satisfying $\Delta^T \Delta \leq I$. Suppose the system (36) is controlled by a state-feedback controller:

$$u(t_1,t_2) = \begin{bmatrix} K_1 & K_2 \end{bmatrix} \begin{bmatrix} x^h(t_1,t_2) \\ x^v(t_1,t_2) \end{bmatrix} \tag{38}$$

where K_1 and K_2 are the controller gains to be found, then the closed-loop system is given by:

$$\begin{bmatrix} \frac{\partial x^h(t_1,t_2)}{\partial t_1} \\ \frac{\partial x^v(t_1,t_2)}{\partial t_2} \end{bmatrix} = \begin{bmatrix} A_{c1} + \Delta A_{c1} & A_{c2} + \Delta A_{c2} \\ A_{c3} + \Delta A_{c3} & A_{c4} + \Delta A_{c4} \end{bmatrix} \begin{bmatrix} x^h(t_1,t_2) \\ x^v(t_1,t_2) \end{bmatrix} \tag{39}$$

where
$A_{c1} = A_1 + B_1K_1, \quad A_{c2} = A_2 + B_1K_2, \quad A_{c3} = A_3 + B_2K_1, \quad A_{c4} = A_4 + B_2K_2,$
$\Delta A_{c1} = \Delta A_1 + \Delta B_1K_1, \quad \Delta A_{c2} = \Delta A_2 + \Delta B_1K_2, \quad \Delta A_{c3} = \Delta A_3 + \Delta B_2K_1, \quad \Delta A_{c4} = \Delta A_4 + \Delta B_2K_2.$
Our objective is to find a state-feedback controller in (38) for the system (36) such that the closed-loop system (39) is asymptotically stable for all possible uncertainties. Before we proceed, the following lemma which is usually used in the robust control of systems will be given first.

Lemma 7 *[22] Let Σ_1,Σ_2 and Δ be real matrices with appropriate dimensions such that $\Delta^T\Delta \leq I$. Then, for any scalar $\varepsilon > 0$ the following inequality holds:*

$$\Sigma_1 \Delta \Sigma_2 + \Sigma_2^T \Delta^T \Sigma_1^T \leq \varepsilon^{-1} \Sigma_1 \Sigma_1^T + \varepsilon \Sigma_2^T \Sigma_2 \tag{40}$$

Now, based on Theorem 2, we have the following analysis result on robust stabilization of the 2-D continuous system (39).

Proposition 1 *For a given positive integer N, define frequency intervals Ω^+ as in (20). System (39) is asymptotically stable for all Δ satisfying $\Delta^T\Delta \leq I$, if there exist matrices $P_{sj} > 0$, $j = 1, 2$. $P_{s2} > 0$, P_{1i}, P_{2i}, $Q_i > 0$, W_1, W_2, and scalars $\varepsilon_i > 0$, $i = 1, 2, ..., N$, such that*

$$\overline{\Xi} = \begin{bmatrix} \overline{\Xi}_1 & \overline{\Xi}_2 \\ * & \overline{\Xi}_3 \end{bmatrix} < 0 \tag{41}$$

$$\overline{\Xi}_{sj} = \begin{bmatrix} \overline{\Xi}_{sj1} & \overline{\Xi}_{sj2} \\ * & \overline{\Xi}_{sj3} \end{bmatrix} < 0, \quad j = 1, 2. \tag{42}$$

$$\overline{\Xi}_1 = \begin{bmatrix} \overline{\Lambda}_{11} & \overline{\Lambda}_{12} & 0 & \overline{\Lambda}_{14} \\ * & \overline{\Lambda}_{22} & \overline{\Lambda}_{23} & \overline{\Lambda}_{24} \\ * & * & \overline{\Lambda}_{33} & \overline{\Lambda}_{34} \\ * & * & * & \overline{\Lambda}_{44} \end{bmatrix}, \quad \overline{\Xi}_2 = \begin{bmatrix} \Upsilon_1 & \varepsilon_i \Upsilon_2 \end{bmatrix}, \quad \overline{\Xi}_3 = diag\{-\varepsilon_i I, -\varepsilon_i I, -\varepsilon_i I, -\varepsilon_i I\}.$$

$\overline{\Lambda}_{11} = -Q_i - W_1 - W_1^T,$
$\overline{\Lambda}_{12} = P_{1i} + j\omega_{ci}Q_i - W_1^T + W_1A_{c1},$
$\overline{\Lambda}_{14} = W_1A_{c2},$
$\overline{\Lambda}_{22} = -\omega_{i-1}\omega_iQ_i + A_{c1}^TW_1^T + W_1A_{c1},$

$\overline{\Lambda}_{23} = A_{c3}^T W_2^T,$
$\overline{\Lambda}_{24} = A_{c3}^T W_2^T + W_1 A_{c2},$
$\overline{\Lambda}_{33} = -W_2 - W_2^T,$
$\overline{\Lambda}_{34} = P_{2i} - W_2^T + W_2 A_{c4},$
$\overline{\Lambda}_{44} = W_2 A_{c4} + A_{c4}^T W_2^T.$

$$\Upsilon_1 = \begin{bmatrix} W_1 H_1 & 0 \\ W_1 H_1 & 0 \\ 0 & W_2 H_2 \\ 0 & W_2 H_2 \end{bmatrix}, \quad \Upsilon_2 = \begin{bmatrix} 0 & 0 \\ (E_1 + L_1 K_1)^T & (E_3 + L_2 K_1)^T \\ 0 & 0 \\ (E_2 + L_1 K_2)^T & (E_4 + L_2 K_2)^T \end{bmatrix},$$

$$\overline{\Xi}_{sj1} = \begin{bmatrix} \overline{\Lambda}_{sj1} & \overline{\Lambda}_{sj2} \\ * & \overline{\Lambda}_{sj3} \end{bmatrix}, \quad \overline{\Xi}_{sj2} = \begin{bmatrix} \Upsilon_{sj1} & \varepsilon_i \Upsilon_{sj2} \end{bmatrix}, \quad \overline{\Xi}_{sj3} = diag\{-\varepsilon_i I, -\varepsilon_i I\}.$$

$\overline{\Lambda}_{sj1} = -W_j - W_j^T,$
$\overline{\Lambda}_{sj2} = P_{sj} - W_j^T + W_j A_{cj\times j},$
$\overline{\Lambda}_{sj3} = A_{cj\times j}^T W_j^T + W_j A_{cj\times j},$

$$\Upsilon_{sj1} = \begin{bmatrix} W_j H_j \\ W_j H_j \end{bmatrix}, \quad \Upsilon_{sj2} = \begin{bmatrix} 0 \\ (E_{j\times j} + L_j K_j)^T \end{bmatrix}, \quad j = 1, 2.$$

Proof From Theorem 2, by replacing W_{1i} and W_{2i} with W_1 and W_2 respectively. System (39) is asymptotically stable if there exist $P_{sj} > 0$, $j = 1, 2$. P_{1i}, P_{2i}, $Q_i > 0$ such that inequalities in (31) and (32) satisfied, in which A_q should be $A_{cq} + \Delta A_{cq}$ for $q = 1, 2, 3, 4$. The above LMIs can be re-written into the following form:

$$\overline{\Xi}_1 + \Upsilon_1 \Delta \Upsilon_2^T + \Upsilon_2 \Delta^T \Upsilon_1^T < 0. \tag{43}$$

$$\overline{\Xi}_{sj1} + \Upsilon_{sj1} \Delta \Upsilon_{sj2}^T + \Upsilon_{sj2} \Delta^T \Upsilon_{sj1}^T < 0. \quad j = 1, 2. \tag{44}$$

According to Lemma 7, the above inequalities holds for all Δ if and only if there exist some scalars $\varepsilon_i > 0$ such that

$$\overline{\Xi}_1 + \varepsilon_i^{-1} \Upsilon_1 \Upsilon_1^T + \varepsilon_i \Upsilon_2 \Upsilon_2^T < 0 \tag{45}$$

$$\overline{\Xi}_{sj1} + \varepsilon_i^{-1} \Upsilon_{sj1} \Upsilon_{sj1}^T + \varepsilon_i \Upsilon_{sj2} \Upsilon_{sj2}^T < 0 \tag{46}$$

which, by the Schur complement in [23], (45) and (46) give rise to (41) and (42). □

Remark 4 *It is interesting to note that, as the LMIs including their proofs for $i = 1$ and $i = N$ are similar to those for cases of $i = 2, ..., N-1$, we give these Proposition 1 in one unified form for all possible value of i for reason of space. The same expression applies to the following Theorem 3.*

Now, based on Proposition 1, we are in a position to give a new method of state-feedback stabilization controller design for the Reosser model.

Theorem 3 *For a given positive integer N, define frequency intervals Ω^+ as in (20). System (39) is asymptotically stable for all Δ satisfying $\Delta^T \Delta \leq I$, by a state feedback controller in (38), if there exist matrices $\widetilde{P}_{sj} > 0$, $j = 1, 2$. $\widetilde{P}_{1i}$, $\widetilde{P}_{2i}$, $\widetilde{Q}_i > 0$, $\widetilde{W}_1$, $\widetilde{W}_2$, N_1, N_2 and scalars $\delta_i > 0$, $i = 1, 2, ..., N$, such that*

$$\widetilde{\Xi} = \begin{bmatrix} \widetilde{\Xi}_1 & \widetilde{\Xi}_2 \\ * & \widetilde{\Xi}_3 \end{bmatrix} < 0 \tag{47}$$

$$\widetilde{\Xi}_{sj} = \begin{bmatrix} \widetilde{\Xi}_{sj1} & \widetilde{\Xi}_{sj2} \\ * & \widetilde{\Xi}_{sj3} \end{bmatrix} < 0 \tag{48}$$

$$\widetilde{\Xi}_1 = \begin{bmatrix} \widetilde{\Lambda}_{11} & \widetilde{\Lambda}_{12} & 0 & \widetilde{\Lambda}_{14} \\ * & \widetilde{\Lambda}_{22} & \widetilde{\Lambda}_{23} & \widetilde{\Lambda}_{24} \\ * & * & \widetilde{\Lambda}_{33} & \widetilde{\Lambda}_{34} \\ * & * & * & \widetilde{\Lambda}_{44} \end{bmatrix}, \quad \widetilde{\Xi}_2 = \begin{bmatrix} \delta_i \widetilde{\Upsilon}_1 & \widetilde{\Upsilon}_2 \end{bmatrix}, \quad \widetilde{\Xi}_3 = diag\{-\delta_i I, -\delta_i I, -\delta_i I, -\delta_i I\}$$

$\widetilde{\Lambda}_{11} = -\widetilde{Q}_i - \widetilde{W}_1 - \widetilde{W}_1^T,$
$\widetilde{\Lambda}_{12} = \widetilde{P}_{1i} + j\omega_c \widetilde{Q}_i - \widetilde{W}_1 + A_1 \widetilde{W}_1^T + B_1 N_1,$
$\widetilde{\Lambda}_{14} = A_2 \widetilde{W}_2^T + B_1 N_2,$
$\widetilde{\Lambda}_{22} = -\omega_{i-1}\omega_i \widetilde{Q}_i + \widetilde{W}_1 A_1^T + A_1 \widetilde{W}_1^T + B_1 N_1 + N_1^T B_1^T,$
$\widetilde{\Lambda}_{23} = \widetilde{W}_1 A_3^T + N_1^T B_2^T,$
$\widetilde{\Lambda}_{24} = A_2 \widetilde{W}_2^T + \widetilde{W}_1 A_3^T + N_1^T B_2^T + B_1 N_2,$
$\widetilde{\Lambda}_{33} = -\widetilde{W}_2 - \widetilde{W}_2^T,$
$\widetilde{\Lambda}_{34} = \widetilde{P}_{2i} - \widetilde{W}_2 + A_4 \widetilde{W}_2^T + B_2 N_2,$

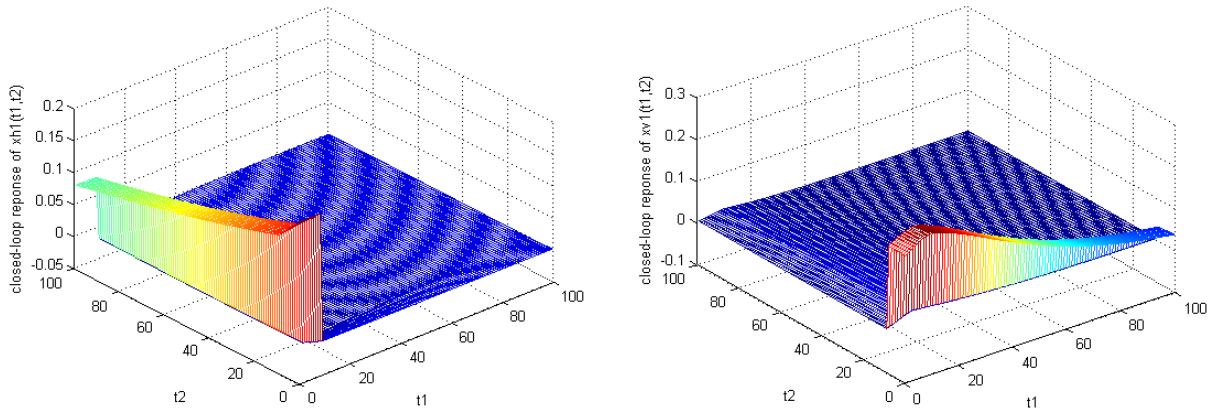

Figure 2: Closed-loop responses of $x_1^h(t_1,t_2)$ and $x_1^v(t_1,t_2)$.

$\widetilde{\Lambda}_{44} = A_4\widetilde{W}_2^T + B_2N_2 + \widetilde{W}_2A_4^T + N_2^TB_2.$

$$\widetilde{\Upsilon}_1 = \begin{bmatrix} H_1 & 0 \\ H_1 & 0 \\ 0 & H_2 \\ 0 & H_2 \end{bmatrix}, \quad \widetilde{\Upsilon}_2 = \begin{bmatrix} 0 & 0 \\ \widetilde{W}_1E_1^T + N_1^TL_1^T & \widetilde{W}_1E_3^T + N_1^TL_1^T \\ 0 & 0 \\ \widetilde{W}_2E_2^T + N_2^TL_1^T & \widetilde{W}_2E_4^T + N_2^TL_2^T\} \end{bmatrix}.$$

$$\widetilde{\Xi}_{sj1} = \begin{bmatrix} \widetilde{\Lambda}_{sj1} & \widetilde{\Lambda}_{sj2} \\ * & \widetilde{\Lambda}_{sj3} \end{bmatrix}, \quad \widetilde{\Xi}_{sj2} = \begin{bmatrix} \delta_i\widetilde{\Upsilon}_{sj1} & \widetilde{\Upsilon}_{sj2} \end{bmatrix}, \quad \widetilde{\Xi}_{sj3} = diag\{-\delta_iI, -\delta_iI\},$$

$\widetilde{\Lambda}_{sj1} = -\widetilde{W}_j - \widetilde{W}_j^T,$

$\widetilde{\Lambda}_{sj2} = \widetilde{P}_{sj} - \widetilde{W}_j + A_{j\times j}\widetilde{W}_j^T + B_jN_j,$

$\widetilde{\Lambda}_{sj3} = \widetilde{W}_jA_{j\times j}^T + A_{j\times j}\widetilde{W}_j^T + B_jN_j + N_j^TB_j^T,$

$$\widetilde{\Upsilon}_{sj1} = \begin{bmatrix} H_j \\ H_j \end{bmatrix}, \quad \widetilde{\Upsilon}_{sj2} = \begin{bmatrix} 0 \\ \widetilde{W}_jE_{j\times j}^T + N_j^TL_j^T \end{bmatrix}, \quad j = 1, 2.$$

If the above conditions are satisfied, a stabilizing control law $\begin{bmatrix} K_1 & K_2 \end{bmatrix}$ *is given by*
$K_1 = N_1\widetilde{W}_1^{-T}, \quad K_2 = N_2\widetilde{W}_2^{-T}.$

Proof If (42) holds, W_1 and W_2 are nonsingular. Pre-and post-multiplying (41) by nonsingular matrices: $diag\{W_1^{-1}, W_1^{-1}, W_2^{-1}, W_2^{-1}, \varepsilon_i^{-1}I, \varepsilon_i^{-1}I, \varepsilon_i^{-1}I, \varepsilon_i^{-1}I\}$ and $diag\{W_1^{-T}, W_1^{-T}, W_2^{-T}, W_2^{-T}, \varepsilon_i^{-T}I, \varepsilon_i^{-T}I, \varepsilon_i^{-T}I, \varepsilon_i^{-T}I\}$, and Pre-and post-multiplying (42) by nonsingular matrices:
$diag\{W_j^{-1}, W_j^{-1}, \varepsilon_i^{-1}I, \varepsilon_i^{-1}I\}$ and $diag\{W_j^{-T}, W_j^{-T}, \varepsilon_i^{-T}I, \varepsilon_i^{-T}I\}, \quad j = 1, 2.$
Making change of variables as follows:
$\widetilde{W}_1 = W_1^{-1}, \quad \widetilde{W}_2 = W_2^{-1}, \quad \delta_i = \varepsilon_i^{-1}, \quad \widetilde{Q}_i = \widetilde{W}_1Q_i\widetilde{W}_1^T, \quad \widetilde{P}_{1i} = \widetilde{W}_1P_{1i}\widetilde{W}_1^T, \quad \widetilde{P}_{2i} = \widetilde{W}_2P_{2i}\widetilde{W}_2^T,$
$\widetilde{P}_{s1} = \widetilde{W}_1P_{s1}\widetilde{W}_1^T, \quad \widetilde{P}_{s2} = \widetilde{W}_2P_{s2}\widetilde{W}_2^T.$
We can obtain the equivalence between Theorem 3 and Proposition 1, where
$N_1 = K_1\widetilde{W}_1^T, \quad N_2 = K_2\widetilde{W}_2^T.$ □

In the following, we provide an example to demonstrate the effectiveness of the proposed method in this section.

Example 2 *Consider the uncertain 2-D continuous system in (39) with parameters given by: [11]*

$$A_1 = \begin{bmatrix} -1.2 & 0.3 & -0.7 \\ -1 & 0.5 & 0.6 \\ 0 & 0.2 & -1.8 \end{bmatrix}, A_2 = \begin{bmatrix} -0.7 & 0.2 \\ 0.5 & 1 \\ -0.5 & 0 \end{bmatrix}, A_3 = \begin{bmatrix} 0.9 & 0 & -1.5 \\ 0 & 0.2 & 0.1 \end{bmatrix}, A_4 = \begin{bmatrix} -0.8 & 0.2 \\ 0.1 & 0.6 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} -0.3 & -0.1 & 0.5 \\ 1 & 0.5 & 0 \\ 1 & 0.2 & 0.6 \end{bmatrix}, B_2 = \begin{bmatrix} -1 & 0.3 & 0.2 \\ 1 & -0.6 & 0.5 \end{bmatrix}, H_1 = \begin{bmatrix} 0.1 \\ 0 \\ -0.2 \end{bmatrix}, H_2 = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix},$$

$E_1 = \begin{bmatrix} 0.1 & 0 & 0.2 \end{bmatrix}, E_2 = \begin{bmatrix} 0.1 & -0.2 \end{bmatrix}, L_1 = \begin{bmatrix} 0.1 & -0.2 & 0 \end{bmatrix}, E_3 = E_1, E_4 = E_2, L_2 = L_1.$

Because the eigenvalues of matrices A_1 *and* A_4 *contain positive eigenvalues given by* 0.4072 *and* 0.6141*, respectively. Therefore, the nominal 2D continuous system under consideration is not asymptotically stable. The aim of this example is to design a frequency-partitioning state feed-back controller such that the resulting closed-loop system is*

asymptotically stable for all admissible uncertainties. By application of Theorem 3 on 2-D contunuous Roesser model, we obtain the state feedback controller parameters as results:

$$K_1 = \begin{bmatrix} 0.3643 & -0.2418 & -0.5541 \\ 0.1982 & -0.2928 & 1.0384 \\ 0.2830 & -0.6519 & 2.4311 \end{bmatrix}, \quad K_2 = \begin{bmatrix} -0.1881 & -0.5412 \\ 0.3719 & 1.3484 \\ 0.1270 & -1.8949 \end{bmatrix} \tag{49}$$

To show the asymptotic stability via the state-feedback control in (38) with (47,48), the state evolution of $x^h(t_1, t_2)$ and $x^v(t_1, t_2)$ of the closed-loop system in (39) are depicted in Figure 2. For simulations, assume that $\omega(t_1, t_2) = 0$ and let the initial and boundary conditions to be:

$$\begin{aligned} x^h(0, t_2) &= 0.2; & 0 \le t_2 \le 10 \\ x_v(t_1, 0) &= 0.2; & 0 \le t_1 \le 10 \\ x^h(0, t_2) = x_v(t_1, 0) &= 0; & t_1, t_2 > 10. \end{aligned}$$

The simulation results show that the closed-loop system in (39) is asymptotically stable.

5. CONCLUSIONS

This paper has been tackled the stability and robust stabilization problem of 2-D continuous Roesser systems. The proposed conditions of the system's stability has been provided in terms of LMIs. Furthermore, GKYP lemma combined with frequency-partitioning approach is used to reduce the conservativeness. Robust stabilization using static state feedback has been studied as well, and the stabilizing feedback gain matrices have been constructed based on the solutions of certain LMIs. Finally, numerical examples demonstrate that the proposed methods are effective.

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