# Differential Evolution Algorithm with Triangular Adaptive Control Parameter for SHEPWM Switching Pattern Optimization 

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#### Abstract

Differential evolution (DE) algorithm has been applied as a powerful tool to find optimum switching angles for selective harmonic elimination pulse width modulation (SHEPWM) inverters. However, the DE's performace is very dependent on its control parameters. Conventional DE generally uses either trial and error mechanism or tuning technique to determine appropriate values of the control paramaters. The disadvantage of this process is that it is very time comsuming. In this paper, an adaptive control parameter is proposed in order to speed up the DE algorithm in optimizing SHEPWM switching angles precisely. The proposed adaptive control parameter is proven to enhance the convergence process of the DE algorithm without requiring initial guesses. The results for both negative and positive modulation index $(M)$ also indicate that the proposed adaptive DE is superior to the conventional DE in generating SHEPWM switching patterns.


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## 1. INTRODUCTION

Selective harmonic elimination pulse width modulation (SHEPWM) is one of the methods to generate sinusoidal voltage output from single- or three-phase inverters. However, the SHEPWM often experiences with the complexity of calculation the switching angles which take quite long computing time to achieve the optimum values. That will be the disadvantage for on-line and real-time applications [1]-[3].

Evolutionary algorithms such as genetic algorithm (GA) [4]-[6] and differential evolution (DE) [7]-[10] have been used to reduce the complexity of the SHEPWM problems which initial values are usually not guessed properly. The evolutionary algorithms can generate a large jump step to find the most approximate optimal solutions. However, the runtime behavior of evolutionary algorithms associate with control parameters, i.e., crossover $(C F)$ and mutation factor $(F)$. The control parameters are very sensitive. Choosing the proper parameters is quite difficult and dependent on the problems [11]. Choosing inappropriate value of $C R$ and $F$ will make the search process suffer from slow convergence speed, premature convergence and stagnation problems [12], [13].

The objective of this paper is to propose an adaptive control parameter for DE in order to avoid or reduce requirements for tuning consumption time due to uncertainty of the $C R$ and $F$ values. As the adaptive function, a triangular probability distribution is employed to adjust the control operators of DE algorithm. The solution patterns for both positive and negative values of modulation index ( $M$ ) in the range of the SHEPWM capability are also investigated. Additionally, the performance of the proposed DE is compared to the conventional DE.

## 2. SELECTIVE HARMONIC ELIMINATION IN THREE-PHASE INVERTER

Figure 1 illustrates a generalized PWM pattern. There are only odd harmonic components exist due to the symmetries in the PWM waveform. The Fourier series for this output voltage waveform is defined as follows:

$$
\begin{equation*}
v(\omega t)=\sum_{n=1,5,7, \ldots}^{\infty} V_{n} \sin (n \omega t) \tag{1}
\end{equation*}
$$

The Fourier coefficients $\left(V_{n}\right)$ are then given by

$$
\begin{equation*}
V_{n}=\frac{4 V_{d c}}{n \pi}\left[-1-2 \sum_{j=1}^{N}(-1)^{j} \cos \left(n \alpha_{j}\right)\right] \tag{2}
\end{equation*}
$$



Figure 1. Generalized PWM waveform

Equation 2 can be solved in order to obtain $N$ switching angles, i.e., $\alpha_{l}$ to $\alpha_{N}$ by equating $N-1$ harmonics to zero. In three-phase inverters the modulation index is set as $M=V_{l} / V_{d c}$ and the normalized magnitudes of harmonics need to be eliminated are as follows [14]:

$$
\begin{align*}
& f_{1}(\alpha)=\frac{4}{\pi M}\left[-1-2 \sum_{j=1}^{N}(-1)^{j} \cos \left(\alpha_{j}\right)\right]-1=\varepsilon_{1} \\
& f_{5}(\alpha)=\frac{4}{5 \pi M}\left[-1-2 \sum_{j=1}^{N}(-1)^{j} \cos \left(\alpha_{j}\right)\right]=\varepsilon_{5} \\
& f_{7}(\alpha)=\frac{4}{7 \pi M}\left[-1-2 \sum_{j=1}^{N}(-1)^{j} \cos \left(7 \alpha_{j}\right)\right]=\varepsilon_{7} \\
& \left.f_{(3 N-2)}(\alpha)=\frac{4}{(3 N-2) \pi M}\left[-1-2 \sum_{j=1}^{N}(-1)^{j} \cos (\beta N-2) \alpha_{j}\right)\right]=\varepsilon_{(3 N-2)} \tag{3}
\end{align*}
$$

All triple harmonics are absent from Equation 3, e.g., for $N=9$, there are 8 harmonics to be eliminated, i.e, $f_{5}$, $f_{7}, f_{11}, f_{13}, f_{17}, f_{19}, f_{23}$, and $f_{25}$. The objective function of the problem is

$$
\begin{equation*}
f(\alpha)=\varepsilon_{1}+\varepsilon_{5}+\varepsilon_{7}+\ldots+\varepsilon_{(3 N-2)} \tag{4}
\end{equation*}
$$

## 3. ADAPTIVE DIFFERENTIAL EVOLUTION ALGORITHM

Differential evolution algorithm utilizies control parameters, i.e., mution factor ( $F$ ) and crossover rate $(C R)$ to control perturbance and improve convergence of the optimization process. Both $F$ and $C R$ have values in the range of $[0,1]$. The flowchart of proposed adaptive DE to optimize $N$ switching angles is presented as in Figure 2. The first step of the flowchart is generating the first generation of population.

$$
\begin{equation*}
\alpha_{i, j}^{(1)}=\alpha_{\operatorname{minj}}+\operatorname{rand}_{j}\left(\alpha_{\operatorname{maxj}}-\alpha_{\operatorname{minj}}\right) \tag{5}
\end{equation*}
$$

where $\alpha_{\operatorname{minj}}$ and $\alpha_{\operatorname{maxj}}$ are lower and upper bounds of the swithing angles, $\operatorname{rand}_{j} \in[0,1], j=1,2, \ldots, N ; i=1$, $2, \ldots, N P$, and $N P$ is the population size. Evaluating the population is conducted to determine which switching angles of the current generation will be the best candidates in order to satisfy the constraints of the objective function.

$$
\begin{equation*}
f_{\text {best } j}{ }^{(1)}=f\left(\alpha_{\text {best } j}{ }^{(1)}\right), \alpha_{\text {best } j}{ }^{(1)} \in \alpha_{i, j}^{(1)} \tag{6}
\end{equation*}
$$

New individuals are evolved by using mutation and crossover operators. The operators are employed to increase the population diversity and promote faster convergence. Mutation operation has a responsibility to create mutant individuals, while crossover operation is to create trial individuals. The individuals in this case are the switching angles. Both mutation and crossover processes are shown as in Figure 3. In the mutation process, the indices $r a, r b, r c, r d$, and $r e$ are five mutually distinct integers taken randomly from $\{1,2,3, \ldots$, $N P\}$. Further, the integers $i, r a, r b, r c, r d$, and $r e$ must be different. In selection process as seen in Figure 4, the trial individuals $\left(t_{i, j}\right)$ become the best next generation individuals if they offer equal or lower values of the objective function than those of their parent.


Figure 2. The flowchart of adaptive DE


Figure 3. Mutation and crossover operations


Figure 4. Selection operation

### 3.1. Adaptations of Mutation and Crossover

The adaptations of mutation and crossover are employed to avoid ineffective either trial and error or tuning processes in order to meet the best values of $F$ and $C R$. The control paramaters at the $G$ th-generation, i.e., $F_{i}^{(G)}$ and $C R_{i}^{(G)}$ are adapted by using triangular probability distribution as follows:

$$
\begin{align*}
& X_{i}^{(G)}= \begin{cases}X_{\min }+\sqrt{\operatorname{rand}_{j}\left(X_{\max }-X_{\min }\right)\left(X_{\bmod }-X_{\min }\right)} & \text { if } \text { rand }_{j}<\tau \\
\left.X_{\max }-\sqrt{\left(1-\operatorname{rand}_{j}\right)\left(X_{\max }-X_{\min }\right)\left(X_{\max }-X_{\bmod }\right.}\right) & \text { otherwise }\end{cases}  \tag{6}\\
& \tau=\left(X_{\bmod }-X_{\min }\right) /\left(X_{\max }-X_{\min }\right) \tag{7}
\end{align*}
$$

The triangular probability distribution on $[0,1]$ is defined by three values, i.e., lower limit $X_{\text {min }}$, upper limit $X_{\max }$, and mode $X_{\text {mod }}$, where $X_{\min }<X_{\max }$ and $X_{\text {min }} \leq X_{\text {mod }} \leq X_{\max }$. There are three possible cases [15] as shown in Figure 5, i.e., (i) right triangular ( $X_{\text {mod }}=X_{\text {min }}$ ), (ii) middle triangular ( $X_{\text {min }}<X_{\text {mod }}<X_{\text {max }}$ ), and (iii) left triangular $\left(X_{\text {mod }}=X_{\text {max }}\right)$.


Figure 5. Tringular probability distributions

## 4. RESULTS AND ANALYSIS

The adaptive DE is applied to minimize the objective function. The proposed DE is implemented by using MATLAB software package in order to generate SHEPWM switching patterns. There are three cases of triangular (as in Figure 5), which are investigated to adapt both $F$ and $C R$.

### 4.1. Triangular Adaptive DE

There are 9 possible combinations for $F$ and $C R$. i.e., ( $F=$ right triangular, $C R=$ right triangular), ( $F=$ right triangular, $C R=$ middle triangular $),(F=$ right triangular, $C R=$ left triangular $),(F=$ middle triangular, $C R=$ right triangular $),(F=$ middle triangular, $C R=$ middle triangular $),(F=$ middle triangular, $C R=$ left triangular $),(F=$ left triangular, $C R=$ right triangular $),(F=$ left triangular, $C R=$ middle triangular $)$, and ( $F=$ left triangular, $C R=$ left triangular). From the aforementioned combinations, there is only one combination is applicable to determine SHEPWM switching patterns, that is ( $F=$ left triangular, $C R=$ right triangular). The other combinations will result in the non-corvergence DE .

Figure 6 shows $F$ and $C R$ tracking of the triangular adaptive DE in order to satisfy the best switching angles for the SHEPWM with $N=9, M=0.05$, and $f(\alpha)<0.0001$. It needs 25 generations to meet the optimum switching angles as shown in Figure 7. The triangular adaptive DE is faster than the conventional DE using fixed $F$ and $C R$ due to it has a large jump to the optimum solutions. According to the report in [14], the best fixed $F$ and $C R$ values are 0.26 and 1.00 , respectively. However, it needs 45 generations to solve this switching angles problem, as shown in Figure 8. The final best switching angles to be found by both DE are $\alpha_{1}=11.7423^{\circ}, \alpha_{2}=12.0905^{\circ}, \alpha_{3}=23.7342^{\circ}, \alpha_{4}=24.1551^{\circ}, \alpha_{5}=35.7282^{\circ}$, $\alpha_{6}=36.2035^{\circ}, \alpha_{7}=47.7291^{\circ}, \alpha_{8}=48.2380^{\circ}$, and $\alpha_{9}=59.7398^{\circ}$.


Figure 6. F and CR adaptations


Figure 7. Triangular adaptive DE tracking the best switching angles


Figure 8. Conventional DE tracking the best switching angles

### 4.2. SHEPWM Switching Patterns

There are 4 types of possible switching patterns for $N=9$ with both negative and positive values of $M$, as presented in Figure 9. The upper panels correspond with the type-1 of both negative and positive switching patterns, and other panels correspond with type-2, type-3 and type-4, respectively. The negative and positive values of $M$ are ranged between $-1.5 \leq M \leq 0.0$ and $0.0 \leq M \leq 1.5$, respectively. The computation process is employed with a step of 0.05 . The number of generations to generate every type of patterns is summarized as in Table 1. The number of generations is the averaged values over 100 runs and with $100 \%$ success rate.


Figure 9. SHEPWM swithing patterns for $N=9$ with $M$ negative and positive

As shown in Table 1, the adaptive DE is faster than the conventional DE in generating switching patterns of SHEPWM. Note that the conventional DE used fixed $F$ and $C R$ of 0.26 and 1.00 , respectively. It is discovered that the triangular adaptive $F$ and $C R$ improve the speed of $D E$ to reach the optimum values of
switching angles of the SHEPWM problems. The adaptive DE requires less number of generations when compared to the conventional DE in generating SHEPWM switching patterns.

Table 1. Number of Generations of DE

| Table 1. Number of Generations of DE |  |  |  |
| :---: | :---: | :---: | :---: |
|  | Patterns | Number of generations <br> Adaptive <br> Conventional |  |
|  | DE | DE |  |
| M | Type-1 | 44 | 82 |
|  | Type-2 | 45 | 77 |
|  | Type-3 | 47 | 74 |
|  | Type-4 | 45 | 76 |
| M | Type-1 | 91 | 182 |
| positive | Type-2 | 91 | 197 |
|  | Type-3 | 97 | 160 |
|  | Type-4 | 49 | 77 |

## 5. CONCLUSION

The application of triangular adaptive control parameter has an advantage in increasing the DE's performance to solve the SHEPWM switching angles problem. It reduces half of the number of generations required to meet the optimum values of switching angles in comparison to the conventional DE. It works relative fast with $100 \%$ success rate for both negative and positif values of modulation index. These results provide usefull information to help designers to make a decision in their SHEPWM inverter for real-time applications

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