A nonlinear model for a three-phase 12/8 switched reluctance machine

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Article Info

ABSTRACT
The paper presents a methodology for the synthesis of a nonlinear model for a three-phase 12/8 switched reluctance machine (SRM) in a MATLAB/Simulink environment. The required switched reluctance motor characteristics are derived using the finite element analysis (FEA). Graphical and tabular results of the relations flux/current/position, current/torque/position, and inductance/current/position used to create the SRM model are presented. The structure of the nonlinear three-phase 12/8 SRM model is discussed and its implementation in Simulink is presented. A series of simulation and experimental results are performed to verify the accuracy of the built three-phase, 12/8 SRM (H55PW/BKM-1844) nonlinear model. A very good alignment between simulation and experimental results is observed.

Keywords:
Generating mode
Modeling
Motoring mode
SRM nonlinear model
Switched reluctance motor

1. INTRODUCTION
Switched reluctance machines (SRMs) are characterized by many advantages such as: a simple design, a wide range of possibilities for optimizing the operating modes when changing the speed and load (a possibility to adjust the output parameters of the electric drive), the overload torque capability, a high energy efficiency, reliability and low-cost, which are the reasons that make them considered as promising electromechanical devices [1], [2]. A complete summary of their performances, control methods and other capabilities can be found in [3]. The complexity of modern electric drive systems increases the need to use simulation methods for analysis and optimization based on various computer-aided design systems. Their use allows for determining the permissible deviations of the values of various parameters and control signals in electric drive systems from their calculated values so that the electric drive maintains its efficiency [4].

A powerful tool for an effective design, study and control of an SRM is modelling with a MATLAB/Simulink environment. SRM models are divided into two main categories-linear and nonlinear models [5]. Linear models are applicable when the motor assumes to remain magnetically unsaturated during operation. Furthermore, the phase inductances change linearly from the rotor position. Linear models of three-phase 6/4 SRM and three-phase 12/8 SRM are demonstrated in [5]-[7]. These models combine the ability to obtain acceptable results with minimal simulation time. However, the accurate analysis of SRM performance requires precision modelling of the behavior of this motor with its nonlinear magnetic characteristics. The built-in nonlinear models in the MATLAB/Simulink environment are types 6/4, 8/6 and 10/8 SRM [8]. The nonlinear models of other types of SRMs are not included in the Simulink libraries.

The three-phase 6/4 SRM has two magnetic poles per phase, while the three-phase 12/8 SRM has four. The electrical cycle of 6/4 SRM corresponds to a 90° rotation of the rotor position, while the electrical
cycle of 12/8 SRM corresponds to a 45° rotation. Besides, when the speed of 12/8 SRM is equal to half of 6/4 SRM speed, the inductance cycles of both machines are equal. The turn-on and turn-off angles of the 12/8 SRM are half of the same angles at 6/4 SRM. In addition, [9] discuss the possibility of modelling in Simulink the three-phase 12/8 SRM by modifying the existing model of the three-phase 6/4 SRM.

Furthermore, the existing SRM model can be modified directly, or a new model can be added to the model library [9]. However, this approach does not allow to precisely model the behaviour of a specific type of 12/8 SRM. It should be noted that the flux linkage profile and the torque profile are changed with current and position. Besides, when operating at higher phase currents, the change with current is nonlinear.

The method for structuring a nonlinear model of SRM considered in [10] allows the user to create their model with the ability to set multiple input parameters and implement nonlinear static characteristics. The initial data for the proposed model are the relationship of flux linkage as a function of position and the phase current. The necessary values of the current depending on the position and the flux linkage and the torque depending on position and current are received, solving the corresponding differential equations. The use of partial derivatives should be avoided because they can become a source of error [5]. The waveforms obtained during simulations are not verified experimentally.

Saidani and Ghariani [11], presented the results from the modelling of 12/8 SRM. The flux linkage curve is prepared through coupled to MATLAB FEMM simulation for different rotor positions and then is used to extract the other performance as static torque, inductance and co-energy. The look-up table, developed in the form of a block in the Simulink model, is obtained through MATLAB/Simulink. The simulation results are not confirmed with an experiment. The presented nonlinear models in [12]-[14] describe methods for structuring an SRM nonlinear model. These models use extensive analytical calculations and partial derivative calculations. Polynomial interpolation is also used. Thus, they emphasize the computational time of SRM simulations and the accuracy of the flux linkage partial derivative. These methods do not allow easy synthesis of an SRM model.

The nonlinear model presented in [15] uses the data obtained from measurements of the static characteristics of the motor. They were implemented in the form of 2D tables in the MATLAB/Simulink environment. Disadvantages of the considered model are: a missing detailed description for structuring the model; a menu for setting the basic parameters of the studied motor; a method by which the data obtained from measurements are introduced into the model. The main disadvantages of the experimentally obtained static characteristics for magnetization and torque of the motor are the lack of precision in the measurement and the inability to stop the rotor at high currents without minimal slippage. To perform a simulation study to another type of SRM, it is necessary to provide the user with the capability to create their specific model based on catalogue data and the same time, save the computational time of SRM simulations.

The present study presents a methodology for synthesising a custom nonlinear model of three-phase 12/8 SRM in the Simulink environment. The procedure we used with MATLAB/Simulink is based on creating look-up tables that approximate the necessary relations. For this purpose, the approach given in [5] is applied. The required SRM characteristics of the tested 12/8 SRM are modelled by a FEM, as demonstrated in [16], providing us with the different magnetic data to realise the look-up tables. Thus, the used procedure allows avoiding all partial derivatives. The data for the look-up tables in excel files are entered using the source code for reading data in the Simulink software simulator. The individual blocks for structuring a proposed nonlinear model in MATLAB/Simulink are described step by step.

2. MATHEMATICAL DESCRIPTION OF THE SRM NONLINEAR MODEL

The expression of the phase voltage for a one phase of the SRM, describing the dynamic behaviour of the SRM, while assuming that there is no mutual coupling to other phases, is given by [1]:

\[ V_{ph}(t) = i_{ph}(t).R_{ph} + \frac{d\lambda_{ph}(i_{ph},\theta_{ph})}{dt}, \]  

(1)

where \( V_{ph} \) is the voltage applied to the terminals of one phase, \( i_{ph} \) is the phase current, \( R_{ph} \) is the resistance per phase, \( \lambda_{ph}(i_{ph},\theta_{ph}) \) is the flux linkage per phase at different excitation currents and rotor positions \( \theta_{ph} \). The flux linkage in an SRM phase is given by:

\[ \lambda_{ph} = L_{ph}(i_{ph},\theta_{ph}).i_{ph}, \]  

(2)

where \( L_{ph}(i_{ph},\theta_{ph}) \) shows the dependence of the phase inductance on the rotor position and the excitation current. The torque produced by one phase \( T_{ph} \) can be approximated by:

\[ T_{ph}(i_{ph},\theta_{ph}) = \int_{0}^{i_{ph}} \frac{\partial \lambda_{ph}(i_{ph},\theta_{ph})}{\partial \theta_{ph}} d i_{ph}, \]  

(3)
the total electromagnetic torque for the SRM with 3-phases is given by:

$$T_e = \sum_{j=1}^{3} T_{phj}(i_{phj}, \theta_{phj}),$$

where \(j\) is the phase number. The following mechanical equations were used to obtain the output speed of an SRM:

$$J \frac{da}{dt} = T_e - T_{load} - F \omega,$$

$$\omega = \frac{d\theta}{dt},$$

where \(J\) is the machine inertia, \(T_{load}\) is the load torque, \(F\) is the friction coefficient and \(\omega\) is the rotor speed of the SRM.

### 2.1. The nonlinear SRM model

This section presents the model synthesis using some of the techniques described in [10] and the embedded models in Simulink. The required static characteristics are obtained by the finite element method (FEM) [16]-[20]. The mathematical model of the SRM is divided into electrical and mechanical parts. The electrical part is represented by in (1) and (2), while the mechanical part is represented by in (3)-(6).

Creating an adequate nonlinear mathematical model requires considering the highly nonlinear characteristics of the motor such as the inductance, the flux linkage and the torque. For this reason, these dependencies are presented in the two-dimensional tables (look-up tables) at different rotor positions and discrete values of the currents through the phases. The required SRM characteristics are built with the Infolytica MotorSolve software. The modern software for the finite element analysis FEM analysis is characterized with high accuracy and also includes an extensive database of materials from magnets, steels and conductors with measured values of properties provided by their manufacturers. The MotorSolve software simulates machine performance using equivalent circuit calculations and a unique automated finite element analysis machine [21]. Several analysis methods are included in this platform, which give the user control of the accuracy versus time.

#### 2.1.1. The SRM operational characteristics and inductance profile

As the first step, a set of relations flux/current/position, current/torque/position, and inductance/current/position are necessary to create the model. The graphical representation of the results for the flux linkages dependence \(\lambda_{ph}(i_{ph}, \theta_{ph})\), obtained after the FEM analysis for the tested three-phase 12/8 SRM, is shown in Figure 1. The flux linkage values obtained by the FEM correspond to 90 rotor positions from 0° to 45° (at resolution of 0.5°) and 100 different values of the current from 0 to 20 A (current resolution of 0.2 A).

The next characteristic is the inductance profile \(L_{ph}(i_{ph}, \theta_{ph})\). Using FEM tools, the dependence of the inductance versus rotor position of the used three-phase 12/8 SRM for different excitation currents is derived. The obtained results for the phase inductance, displayed in Figure 2, correspond to 90 rotor positions from 0° to 45° (at resolution of 0.5°) and 99 different values of the current from 0.2 to 20 A (current resolution of 0.2 A).

The most challenging task is the construction of the flux data \(\lambda_{ph,FEM}(i_{ph}, \theta_{ph})\) since the model calculates the current as a function of flux and position \(i_{ph}(\lambda_{ph,FEM}, \theta_{ph})\). Hence, it is necessary to process and rearrange the data \(\lambda_{ph,FEM}(i_{ph}, \theta_{ph})\) to produce the required current \(i_{ph}\). The graphical representation for the set of curves \(i_{ph}(\lambda_{ph,FEM}, \theta_{ph})\) in the Infolytica MotorSolve is presented in Figure 3. The obtained values correspond to 90 rotor positions from 0° to 45° (at resolution of 0.5°). It should be noted that the data for the flux linkages dependence \(\lambda_{ph,FEM}(i_{ph}, \theta_{ph})\) and the inductance profile \(L_{ph,FEM}(i_{ph}, \theta_{ph})\) corresponds to the values for one phase. Because the study assumes there is no mutual coupling between the phases, the look-up tables used to for one phase are also used to model all phases.

The dependencies of the torque produced by one phase as a function of rotor position and phase current values \(T_{ph}(i_{ph}, \theta_{ph})\) are shown in Figure 4. The obtained results correspond to 90 rotor positions from 0° to 45° (at resolution of 0.5°) and 100 different values of the current from 0 to 20 A (current resolution of 0.2 A). The obtained graphs and numerical results correspond to the operation of the SRM in motoring and generating mode. In addition, the results show the influence of the nonlinearity of the magnetization characteristic on the phase torque.
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Figure 6. The block source schematic diagram

The data for the two-dimensional tables (2-D lookup tables) in excel files are entered using the source code for reading data in the simulink software simulator in the PostLoadFcn menu. Snapshots of the PostLoadFcn menu with part of the code and the user interface, where the values for the current are entered, for the creation of the respective block 2D-Lookup table are shown in Figure 7 and Figure 8.

The block denoted as Torque implements the mechanical in (4)-(6). Figure 9 illustrates their internal structures. The input data are the signals of the load torque TL and the total torque Te, which determine the rotor speed \( \omega \) and the magnitude of the rotor position \( \Theta \). The internal structure of the rotor position block is shown in Figure 10. In this block, the rotating speed is integrated, and as a result, the angular position of the rotor for three phases is obtained. The block named phase A (phase B, phase C) includes a current source and a resistor connected in parallel, modelling the phase active resistance. The phase current, derived from the differential equation, is fed to input I of block phase A. Its output signal produces the voltage applied to the phase.

Figure 7. PostLoadFcn menu with source code part for reading data received from FEM analysis

Figure 8. The user interface for entering FEM simulation results on the corresponding table

The tested SRM is fed from the asymmetric bridge converter. This topology is the most widely used in SRM drives in terms of converter losses and cost, control performance, the independent phase windings,
allowing a fault-tolerant operation capability [7]. Hence, for example, the terminals named "A+" and "A-" (block phase A) connect the motor phase A to the corresponding bridge of the asymmetric converter circuit.

The final synthesis of the three-phase 12/8 SRM nonlinear model includes the creation of a subsystem (block subsystem), shown in Figure 11, where the user interface (mask) sets the necessary input parameters such as a phase resistance (Ω), a friction coefficient (N.m.s) and inertial value (kg.m.m). In addition, by checking the plot magnetization curves field of the subsystem, the flux linkages dependencies are displayed as shown in Figure 12.

Figure 13 presents the schematic diagram used to simulate in MATLAB/Simulink the proposed nonlinear three-phase 12/8 SRM model. It consists of the following main blocks: block SRM 12/8 (nonlinear SRM model), block converter, block position_sensor, monitoring the position of the rotor and switching the phase transistors. The internal circuit of the phase bridge A of the asymmetric converter is shown in Figure 14.
The input parameters are as follows: input voltage for motoring \(V=45\) V and generating mode \(V=8\) V; phase resistance \(R_{ph}=1.795\) ohms, inertial value \(J=0.00057\) [kg.m.m], friction coefficient \(F=0.0013\) [N.m.s], the used MOSFETs are IRFP460 with the following parameters: drain-source on-resistance \(R_{DS(on)}=0.24\) m\(\Omega\), body diode on-resistance \(R_d=0.01\) \(\Omega\), body diode voltage \(V_{SD}=1.8\) V; the asymmetric bridge diodes are FR604-max. forward voltage \(V_F=1.3\) V. Table 1 shows the specification of the tested three-phase switched reluctance motor type 12/8 model - H55PWBKM-1844.

3. RESULTS AND DISCUSSION

In this section we proposed the performed simulation and experimental results. To verify the built nonlinear model accuracy, the series of simulation and experimental results are accomplished. The tested SRM is fed from the asymmetric bridge converter. Primarily, the results of the FEM analysis for the inductance profile are compared with those obtained experimentally. For this purpose, the current saturation method for SRM inductance measurement proposed in [22] is applied. The method uses low frequency and low voltage to minimize iron losses; no additional specific devices are needed; the setup is not complicated and compared to classic methods [23]-[27], a higher accuracy of the inductance profile measurement is achieved. Figure 15 shows the simulation and experimental verification of the inductance profile at various excitation currents for the tested three-phase 12/8 SRM.
A very good alignment between simulation and experimental results is observed. In the present study, to confirm the accuracy of the built three-phase 12/8 SRM nonlinear model, the machine is energized using a voltage source control method. The operation of the motor is examined in quadrants I and II at motor speed \( n=1500 \) rpm. At the moment when the positive voltage \( (V+) \) feeds to one phase and the motor starts to rotate, the two MOSFETs (Q1, Q2) shown in Figure 14 must be on. Conversely, when the applied voltage has a negative value \( (V-) \), the two diodes (D1, D2) are used to ensure a continuous current mode [1]. The proposed nonlinear model is verified experimentally. Figure 16(a) presents a built three-phase asymmetric bridge converter with the controller. The measuring stand for experimental study of the operation of the three-phase 12/8 SRM with position and torque sensors is shown in Figure 16(b).

Figure 15. Simulation and experimental verification of the inductance profile \( L_{ph} \) at various excitation currents for the tested three-phase 12/8 SRM (H55PWBKM-1844)

![Figure 15](image)

Figure 16. Experimental study: (a) three-phase asymmetric bridge converter with the controller and (b) the measuring stand with position and torque sensors

![Figure 16](image)

Depending on the operating mode and the direction of the motor rotation, there are different turn-on and turn-off angles of the motor phases. The oscillograms shown in the following figures correspond to motoring and generating mode, respectively, i.e. the operation in the first and second quadrants. Hence, the turn-on and turn-off angles must be selected in the rising and falling phase inductance for motoring and generating mode. Figure 2 shows that these areas are between 22.5° and 45° for first quadrant and 0° and 22.5° for second quadrant. Therefore, to ensure a rapid increase and decrease of the phase current, it is necessary switching the applied phase voltage in these areas, leading angles corresponding to the minimum and maximum phase inductance (or 22.5° and 45°) for motoring mode and maximum and minimum phase inductance (or 0° and 22.5°) for generating mode. After comparing the voltages and currents of the tested SRM during operation in the I quadrant (motoring mode), the operation of the SRM in the II quadrant (generating mode) was checked by using a collector machine for the motor. During the simulation, load torque \( TL=0.65 \) Nm and the motor speed \( n=1500 \) rpm were specified.
Figure 17(a) illustrates from top to bottom the simulation waveforms of the one-phase voltage and the one-phase current at $\theta_{on} = 25.3^\circ$, $\theta_{off} = 43.44^\circ$, torque load $TL=0$ Nm and speed $n=1500$ rpm for the motoring mode (I quadrant). Figure 17(b) illustrates the simulation waveforms of the same quantities - from top to bottom waveforms of the one-phase voltage and the one-phase current at $\theta_{on} = 3.8^\circ$, $\theta_{off} = 20.944^\circ$, torque load $TL=0.65$ Nm and speed $n=1500$ rpm for the generating mode (II quadrant).

![Simulation waveforms](image1)

Figure 17. Simulation waveforms of the phase voltage and the phase current: (a) motoring mode at the motor speed $n=1500$ rpm, $TL=0$ Nm; and (b) generating mode at the $n=1500$ rpm, $TL=0.65$ Nm

The following Figure 18(a) and Figure 18(b) present experimental waveforms of the one-phase voltage and the one-phase current in motoring mode and generating mode. Figure 18(a) illustrates for the motoring mode (I quadrant) from top to bottom the experimental waveforms of the one-phase voltage and the one-phase current at $\theta_{on} = 24.13^\circ$, $\theta_{off} = 42.274^\circ$, torque load $TL=0$Nm and speed $n=1500$ rpm. Figure 18(b) illustrates the experimental waveforms of the same quantities for the generating mode (II quadrant) - from top to bottom waveforms of the one-phase voltage and the one-phase current at $\theta_{on} = 3.63^\circ$, $\theta_{off} = 20.87^\circ$, torque load $TL=0.65$ Nm and speed $n=1500$ rpm.

The next Figure 19(a) and Figure 19(b) present simulation waveforms for the motor speed at the operation in the first and second quadrants respectively. Figure 20(a) and Figure 20(b) illustrate the simulation results of the total torque $Te$ at the operation of the SRM in motoring and generating mode respectively. The following Figure 21(a) and Figure 21(b) show the simulation results of the one-phase inductance at the operation of the SRM in the first and second quadrants respectively. Table 2 and Table 3 summarize the comparison between simulation and experimental results for the two operation modes. The relative error between simulation and experimental results is within 5-10%. The differences between them are mainly due to the inaccuracy in the measurement of the angular position, emerging electromagnetic processes, noise and vibration when the phase winding is excited.

![Experimental waveforms](image2)

Figure 18. Experimental waveforms of the phase voltage and the phase current, (a) motoring mode at the motor speed $n=1500$ rpm, the torque load $TL=0$ Nm and (b) generating mode at the $n=1500$ rpm, $TL=0.65$ Nm
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Figure 19. Simulation results of the motor speed at operation in: (a) the first quadrant and the torque load is TL=0 Nm; and (b) the second quadrant and the TL=0.65 Nm

Figure 20. Simulation results for the total torque $T_e$ at operation in: (a) motoring mode at the motor speed $n=1500$ rpm, the torque load TL=0 Nm; and (b) generating mode at the $n=1500$ rpm, TL=0.65 Nm

Figure 21. Simulation results of one-phase inductance: (a) first quadrant at the motor speed $n=1500$ rpm, the torque load TL=0 Nm; and (b) second quadrant at the $n=1500$ rpm, TL=0.65 Nm

Table 2. A comparison between simulation and experimental results in the motoring mode (I quadrant)

<table>
<thead>
<tr>
<th>Experimental results</th>
<th>Nonlinear model simulation results</th>
<th>Relative error, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turn-on angle $\theta_{on}$</td>
<td>24.13</td>
<td>25.3</td>
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<tr>
<td>Turn-off angle $\theta_{off}$</td>
<td>42.274</td>
<td>43.44</td>
</tr>
<tr>
<td>Speed, rpm</td>
<td>1500</td>
<td>1500</td>
</tr>
<tr>
<td>Phase voltage [V]</td>
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<td>45</td>
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<tr>
<td>Phase current [A]</td>
<td>2.05</td>
<td>2.02</td>
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<tr>
<td>Total torque $T_e$[Nm]</td>
<td>0.31</td>
<td>0.2965</td>
</tr>
</tbody>
</table>

Table 3. A comparison between simulation and experimental results in the generating mode (II quadrant)

<table>
<thead>
<tr>
<th>Experimental results</th>
<th>Nonlinear model simulation results</th>
<th>Relative error, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turn-on angle $\theta_{on}$</td>
<td>3.63</td>
<td>3.8</td>
</tr>
<tr>
<td>Turn-off angle $\theta_{off}$</td>
<td>20.87</td>
<td>20.944</td>
</tr>
<tr>
<td>Speed, rpm</td>
<td>1500</td>
<td>1500</td>
</tr>
<tr>
<td>Phase voltage [V]</td>
<td>10</td>
<td>9</td>
</tr>
<tr>
<td>Phase current [A]</td>
<td>2</td>
<td>2.2</td>
</tr>
<tr>
<td>Total torque $T_e$[Nm]</td>
<td>-0.059</td>
<td>-0.065</td>
</tr>
</tbody>
</table>
4. CONCLUSION

The paper describes a nonlinear three-phase 12/8 switched reluctance machine model suitable for use in a MATLAB/Simulink environment. To build the model, two-dimensional tables (look-up tables) obtained using the finite element analysis method and the Infolytica Motorsolve software simulator are used. The data for the look-up tables in Excel files are entered using the source code for reading data in the Simulink software simulator. In order to ensure the model’s reliability, 2D graphs of the simulation results from the FEM analysis and those generated from the MATLAB/Simulink are compared. A subsystem (block subsystem), where the user interface (mask) sets the necessary input parameters such as a phase resistance (Ω), a friction coefficient (N.m.s) and inertial value (kg.m.m) was created. Furthermore, an additional block has been added in the proposed model to monitor the inductance profile during the simulation study. A series of simulation and experimental results are derived to verify the accuracy of the built three-phase, 12/8 SRM (H55PWBKM-1844) nonlinear model. A very good parity between simulation and experimental results is observed. The relative error between simulation and experimental results is within 5-10%.

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