Stirling engine multi-objective optimization using a genetic algorithm

Oumaima Taki, Kaoutar Senhaji Rhazi, Youssef Mejdoub
Laboratory of Networks, Computer Science, Telecommunication and Multimedia (RITM), CED Engineering Sciences, Higher School of Technology, Hassan II University, Casablanca, Morocco

ABSTRACT
With the growing demand of energy globally, the actual worrying state of the earth’s finite resources, namely fossil fuels, opens up the scope of energy researches to innovative and efficient solutions. Stirling engine has been an interesting subject of study since its invention, and many studies dealt with Stirling engine efficiency with attempts to optimize it in order to have a proper use of the engine in the real world, depending on the use cases. Stirling engine is an external combustion engine with a theoretical efficiency equivalent to that of Carnot. Alongside the global awareness to use efficient and less resource consuming solutions, there has been a spiking growth in the set of tools that are conceived to achieve that; specifically in the machine learning area. Among the various available algorithms, the one used in the hereby study is the non-sorted genetic algorithm II, which falls into the genetic algorithms category. This algorithm is well suited for multi-objective optimization problems; it consists of selecting the best design parameters that are contained in predefined upper and lower bounds, based on multiple objective functions.

Keywords:
Artificial intelligence
External combustion
Genetic algorithms
Heating
Machine learning
Stirling engine

1. INTRODUCTION
Stirling engine is an external combustion piston engine that transforms thermal energy into kinetic energy by heating and cooling the compressible fluid contained in the cylinders. The thermal efficiency of its cycle can reach that of a Carnot cycle, and that is the highest reachable efficiency theoretically [1]. It was first invented by Robert Stirling in 1816, and continued to investigate on its invention until 1850 when he built double and triple cylinders engines [2]. Nonetheless, those new engines were less performant than the first one. Many researchers continued to prosecute their researches on the Stirling engine; In 1860 with Lehman building a hot air expansion engine with a horizontal single cylinder [3]. In 1876, Alexander Rider built another hot air engine but with two cylinders put side by side and externally connected by a regenerator, without the need of valves, springs, levers or any delicate part [4].

The published results of the Philips Stirling Engines in 1947 were a significant improvement, they were 50 times lighter and 125 times smaller compared to the first engines [5]. A Stirling engine operates on a closed and regenerative thermodynamic cycle, with cycle compression and expansion of the compressible working fluid inside the cylinders, at different temperature levels [6], and thanks to its non-explosive combustion, a Stirling engine can achieve remarkable quietness.
The practical cycle is different from the theoretical Carnot one that consists of two constant temperatures and two constant volume processes, and it can be defined as a process that occurs in a closed space containing a working compressible fluid. The difference in volume inside the closed space generates changes in pressure of the fluid whereas the displacement in the closed space generates changes in cyclic temperatures of the fluid [7].

Schmidt was the first to thermodynamically model a Stirling cycle and he did an assumption that the working fluids in the cold and hot spaces have similar temperature of the cold and hot sources [8], and he then developed an isothermal model, which was based on the expansion and compression of an ideal gas, and that was used by others to develop Stirling engines. Urieli and Berchowitz used this work to develop an adiabatic model which divides the Stirling engine into five control volumes and assuming that the total mass of the working gas remains the same inside the engine so as to obtain an efficient heat transfer [9].

2. MATHEMATICAL MODELLING

The adiabatic model of Urieli and Berchowitz is used in this work to apply a multi-objective optimization. The engine is divided into five components connected in series as shown in Figure 1. The following assumptions were taken into account: i) Compression and expansion are adiabatic; ii) Uniform gas pressure inside the engine; iii) Sinusoidal movement of the piston and the displacer; iv) The used working fluid is an ideal gas; and v) Steady state, rotation speed is constant. The terms cited in this paper are presented in Table 1.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Symbol</th>
<th>Definition</th>
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</thead>
<tbody>
<tr>
<td>T_H</td>
<td>hot temperature</td>
<td>H</td>
<td>efficiency</td>
</tr>
<tr>
<td>T_C</td>
<td>cold temperature</td>
<td>ε_H</td>
<td>heat exchanger efficiency</td>
</tr>
<tr>
<td>W</td>
<td>Work</td>
<td>ε_R</td>
<td>regenerator efficiency</td>
</tr>
<tr>
<td>Q</td>
<td>heat</td>
<td>t_c</td>
<td>cycle time period</td>
</tr>
<tr>
<td>Q_H</td>
<td>released net heat</td>
<td>λ</td>
<td>volume ratio during processes</td>
</tr>
<tr>
<td>Q_L</td>
<td>absorbed net heat</td>
<td>M</td>
<td>proportionality constant of Stirling Engine</td>
</tr>
<tr>
<td>Q_0</td>
<td>lost heat due to thermal bridging</td>
<td>K_0</td>
<td>heat leak coefficient</td>
</tr>
<tr>
<td>C_H</td>
<td>hot source capacitance</td>
<td>δ</td>
<td>Stefan’s constant</td>
</tr>
<tr>
<td>C_L</td>
<td>heat sink capacitance</td>
<td>h</td>
<td>heat capacitance coefficient</td>
</tr>
</tbody>
</table>

2.1. Work done during the cycle

The ideal gas law is formulated as (1).

\[ PV = nRT \]

Considering the PV diagram of the Stirling cycle as graphed in Figure 2, we observe that during the compression process 4-1, the gas is cooled to maintain the constant cold temperature \( T_C \). The required work \( W_{4-1} \) to compress the gas inside the cylinder as (2).

\[ W_{4-1} = \int_P V = Const \int_4^1 dV = Const ln \left( \frac{V_1}{V_4} \right) \]

Using the general gas in (1).

\[ Const = P_1V_1 = P_4V_4 = n \cdot R \cdot T_C \]

In the isothermal expansion process 2-3, the heat source maintains the working fluid’s temperature constant at \( T_H \). The work of this process is formulated as (4),

\[ W_{2-3} = \int_P V = Const \int_2^3 dV = Const ln \]

and similar to the previous process.

\[ Const = P_2V_2 = P_3V_3 = n \cdot R \cdot T_H \]
In the processes 1-2 and 3-4, the volume is constant, the fluid is displaced from the cold space to the hot space, no work is done. We can then conclude that the network is expressed as the sum of work from process 4-1 and process 2-3:

\[ W_{\text{net}} = W_{4-1} + W_{2-3} \]  
(6)

2.2. Heat transferred during the cycle

According to the first law analysis of the ideal gas, the transferred heat is calculated by developing the equations to determine the internal energy change \( \Delta u \) in terms of the specific heat capacity [12]. The heat transferred from the gas, \( Q_R \), during the “cooling” process 3-4 in which the work is null, can be formulated as (7).

\[ Q_R = \Delta U = m \cdot C_v \cdot \Delta T \]  
(7)

During the compression and expansion processes, the temperature is constant [11] and thus, according to (7) the transferred heat is equal to the work. We have:

\[ -Q_{\text{out}} = Q_{4-1} = W_{4-1} = m \cdot R \cdot T_C \cdot \ln \left( \frac{V_4}{V_1} \right) \]  
(8)

\[ Q_{\text{in}} = Q_{2-3} = W_{2-3} = m \cdot R \cdot T_H \cdot \ln \left( \frac{V_2}{V_3} \right) \]  
(9)

\[ W_{\text{net}} = W_{4-1} + W_{2-3} = Q_{\text{in}} - Q_{\text{out}} \]  
(10)

The theoretical efficiency is then:

\[ \eta_{\text{th}} = \frac{W_{\text{net}}}{Q_{\text{in}}} = 1 - \frac{Q_{\text{out}}}{Q_{\text{in}}} = 1 - \frac{T_C}{T_H} \]  
(11)

If we want to calculate the practical efficiency, we have to take in consideration the regenerator efficiency \( \varepsilon_R \). Let’s reformulate the absorbed and the released working fluid heats:

\[ Q_h = nRT_h \ln(\lambda) + nC_v(1 - \varepsilon_R)(T_h - T_C) \]  
(12)

\[ Q_c = nRT_c \ln(\lambda) + nC_v(1 - \varepsilon_R)(T_h - T_C) \]  
(13)

Where \( n \) the number of the fluid’s moles, \( R \) the gas constant and \( \lambda \) the ratio of the volumes.

In the transfer of heat from the heat source to the sink is lost due to thermal bridging. It is calculated by (14) [12]:

\[ Q_0 = \frac{K_t}{2} [(2 - \varepsilon_H)T_H1 - (2 - \varepsilon_L)T_L1 + (\varepsilon_H T_H - \varepsilon_L T_C)]t_{\text{cycle}} \]  
(14)
Now with the consideration of the thermal bridging losses, we can put the released net heat \( Q_H \) and the absorbed net heat \( Q_L \) as (15) and (16):

\[
\begin{align*}
Q_H &= Q_0 + Q_h \\
Q_L &= Q_0 + Q_c
\end{align*}
\]  

(15)  

(16)  

The cyclic time period can be formulated as (17) [12]:

\[
t_{cycle} = \frac{nRT_h \ln(\lambda) + nC_v (1-\varepsilon_R)(T_h-T_L)}{C_H T_h (T_h-T_L)} + \frac{nRT_h \ln(\lambda) + nC_v (1-\varepsilon_R)(T_h-T_L)}{C_L T_L (T_h-T_L)} + \left( \frac{1}{M_1} + \frac{1}{M_2} \right) (T_h - T_L)
\]  

(17)  

We can then express the output power using the cyclic time period:

\[
P = \frac{\text{Work}}{\text{Time}} = \frac{Q_H - Q_L}{t_{cycle}}
\]  

(18)  

3. MULTI-OBJECTIVE OPTIMIZATION

3.1. Genetic algorithms

Genetic algorithms are optimization algorithms that simulate the natural selection and evolution of species populations. They were introduced by John Holland in the early 1970’s [13]. A genetic algorithm (GA) usually consists of two processes; we first choose individuals to compose our population, based on their contribution in our objective function, then we mix and match individuals through crossover and mutation of genes in order to generate an offspring that will represent the next generation. These two processes are then looped, until satisfaction of a predefined threshold, to get an optimal solution.

3.2. Non-sorted genetic algorithm

A problem that has multiple objectives does not have a single optimal solution, but rather a set of optimal solutions known as Pareto frontier [14]. One approach to deal with multiple objectives problem is to convert it to a single objective problem by using the weighting sum method for instance [15]. The reason of using a genetic algorithm to solve a multi-objective problem is that GAs use a population of solution, that evolves throughout processes, that contains multiple sets of optimal solutions of the objective functions. The first version of NSGA first appeared in 1995 [15], and was improved to a better version in 2002 [16] that is now commonly used. Let us first define what is meant by non-domination.

Suppose we have two objective functions \( X \) and \( Y \) that we would want to minimize and \( A(x_1, y_1), B(x_2, y_2) \) two solutions:

\[
\text{Adomnates} B \Leftrightarrow (x_1 \leq x_2 \land y_1 \leq y_2) \lor (x_1 < x_2 \lor y_1 < y_2)
\]  

(19)  

as we will see next, the solutions are sorted by their non-dominance; from the non-dominated ones to the most dominated (Figure 3).

The main loop of the algorithm is as follows:

- We choose randomly a starting population \( P_0 \), and with a tournament selection [17] the parents of the next generation are chosen.
- We apply a crossover between the chromosomes of the parents to generate an offspring \( Q_0 \). In addition to crossover, a mutation occurs randomly to chromosomes, with a rate of 10\% generally, then we obtain a new population with double size of the first one \( R_0 = P_0 \cup Q_0 \).
- A non-dominated sorting is then applied to \( R_0 \) to have the population sorted by frontiers.
- Let \( F_j \) the frontier such as \( |O| |1 - F_i| < |P_0| \) and \( |0| F_i| \geq |P_0| \). If there is equality of the cardinals, we put \( 0| F_i = P_1 \), and repeat from step 1 until we reach a predefined threshold \( N \) of maximum populations.
- If we are in the strict inequality of the cardinals’ case: \( |0| |F_i| > |P_0| \), we apply a crowding distance sorting is applied to the \( F_j \) frontier to retrieve only solutions that would fit to the new population. Then we restart from step 1 until threshold is satisfied.

The Pareto frontier, also known as Pareto set or Pareto front, is a crucial concept in multi-objective optimization. It refers to the optimal set of solutions where the improvement of one objective comes at the cost of another. Vilfredo Pareto first observed this phenomenon in his work on economics [18]. Today, the Pareto frontier is widely used in various fields such as engineering and economics. It helps to identify the best possible options for a system or process and can be visualized as a curve in a two-dimensional space or higher.
dimensions [19]. To find the Pareto frontier, researchers use a variety of methods such as genetic algorithms, simulated annealing, and linear programming. It provides a useful tool for decision-making, and the Pareto frontier can be used to optimize multiple objectives simultaneously [20].

![NSGA-II algorithm from step t to step t+1](image)

**Figure. 1. NSGA-II algorithm from step t to step t+1 [14]**

### 3.3. Previous uses of genetic algorithms in Stirling engine optimization

Genetic algorithms (GAs) have been increasingly applied to optimize Stirling engines due to their flexibility and robustness. The Stirling engine, named after Robert Stirling, is a type of external combustion engine that uses a cyclic compression and expansion of a working fluid, typically air or helium, at different temperature levels. The engine's efficiency depends on several factors such as the heat exchanger geometry, regenerator design, and engine parameters. GAs provide a powerful optimization technique to optimize these factors and achieve maximum performance.

Researchers have utilized GAs in different aspects of Stirling engines. A study by Suleiman et al. [21] optimized the heat exchanger geometry of a Stirling engine using GAs to maximize the engine's thermal efficiency. Similarly, Sheykhi and Mehran [22] employed GAs to optimize the regenerator of a Stirling engine to improve its performance. The regenerator is a critical component in the engine that recovers the heat from the hot working fluid and transfers it to the cold working fluid, improving the engine's efficiency. The regenerator's design, such as the matrix material, matrix thickness, and channel size, affects the engine's performance significantly.

GAs can also be used to determine the optimal engine parameters, including the displacer's amplitude, frequency, and phase angle. Rahmati et al. [23] used GAs to optimize the engine's parameters, which resulted in a significant improvement in the engine's performance. Another study Abuelyamen and Ben-Mansour [24] applied GAs to optimize the geometry of the regenerator and engine's parameters simultaneously, resulting in a better performance. Overall, GAs have proved to be a reliable optimization tool for Stirling engines, providing valuable insights into the engine's design and operation. In addition, GAs can handle the nonlinear and complex optimization problems that arise in Stirling engines, making them a powerful tool for researchers in this field.

### 3.4. The algorithm application

The gamma Stirling engine was chosen for our study due to its high efficiency, reliability, low maintenance requirements, high power-to-weight ratio, and adaptability to different fuel sources. Previous studies have shown that the gamma configuration results in higher thermal efficiency and power output compared to other configurations [25]. Additionally, gamma Stirling engines have been found to have the highest thermal efficiency and lowest specific fuel consumption compared to other types of Stirling engines [26].

In this study, the multi-objectives problem we are trying to solve is to maximize two objective functions: output power and thermal efficiency. The design parameters that we would optimize are as follows, and they are limited by lower and upper bounds [12]:

\[
0.4 \leq \varepsilon_R \leq 0.9 \\
0.4 \leq \varepsilon_H \leq 0.8 \\
0.4 \leq \varepsilon_L \leq 0.9 \\
300 \leq C_H \leq 1800
\]
The remaining constant parameters are found in Table 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_v (J.mol^{-1}.K^{-1})$</td>
<td>15</td>
</tr>
<tr>
<td>$\xi$</td>
<td>$2 \times 10^{-10}$</td>
</tr>
<tr>
<td>$C(W.K^{-1})$</td>
<td>1300</td>
</tr>
<tr>
<td>$R(J.mol^{-1}.K^{-1})$</td>
<td>4.3</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>0.9</td>
</tr>
<tr>
<td>$T_H(K)$</td>
<td>1300</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>2</td>
</tr>
<tr>
<td>$K_c(W.K^{-1})$</td>
<td>2.5</td>
</tr>
<tr>
<td>$T_C(K)$</td>
<td>290</td>
</tr>
<tr>
<td>$n$</td>
<td>1</td>
</tr>
<tr>
<td>$T_0(K)$</td>
<td>288</td>
</tr>
<tr>
<td>$\delta(W.m^{-2}.K^{-4})$</td>
<td>$5.67 \times 10^{-8}$</td>
</tr>
<tr>
<td>$h(W.m^{-2}.K^{-1})$</td>
<td>20</td>
</tr>
<tr>
<td>$1 + \frac{1}{M_1 + M_2 (s.K^{-1})}$</td>
<td>$2 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

4. RESULTS AND DISCUSSION

In this study we used an initial population size of 1000 and maximum number of generations of 100. We obtain, in an average of 31 seconds over 50 attempts, a curve (Figure 4) representing the Pareto frontier of a simultaneous optimization of the output power and the thermal efficiency. We notice that the curve does not have a clean logarithmic shape; it contains a considerable amount of noise.

![Figure 4. Gamma Stirling configuration [21]](image)

![Figure 5. Pareto frontier of the optimized design patterns](image)
After many attempts, we have found that after setting all design patterns to constant values, keeping only the temperature variants, we can obtain a fitter curve with less noise (Figure 5). We then decided to let only the temperature and the capacitance of the cold sink as variable design parameters. As a result, we obtain a linear curve, which is not representative of our multi-optimization problem, and we cannot choose a pareto frontier (Figure 6 and Figure 7).

Figure 6. Pareto frontier with less design parameters

Figure 7. Simplified problem with two design parameters

5. CONCLUSION

In this study, we were faced to a multi-optimization problem; optimizing the output power and the thermal efficiency of the Stirling engine. We chose as design parameters the effectiveness of the regenerator and the exchangers, the heat capacitances of the heat source and heat sink and the low and high temperature of the working fluid. From a toolkit of available algorithms to solve such problems, we chose the NSGA-II algorithm, that we have found suitable for our case. We also concluded that if we keep only the temperature of the working fluid variable, the Pareto frontier tends to have a fit logarithmic curve shape. As it was debated in its apparition the work of Amrit Pratap et al., it is definitively a fast and elitist genetic algorithm. We got an optimal thermal efficiency between 37% and 74%, and an optimal output power between 1200 W and 11300 W. These are the ranges where the optimal realistic solutions are located, we must apply a decision-making algorithm in a future work to locate them. The remaining design parameters can be looked in detail at as well in other studies, in order to achieve a more global optimization of the engine, such as the nature of the working fluid, the angle of the crank wheel, and the rotational speed.

REFERENCES


### BIOGRAPHIES OF AUTHORS

**Oumaima Taki** is an electrical engineer who was born in 1995 in Morocco. She graduated from the National School of Arts and Crafts (ENSAM) Casablanca in 2018. She is currently pursuing her Ph.D. in the Electrical Engineering Department at the Superior School of Technology (EST) Casablanca, where she is focused on researching 'Stirling engine' and 'engine optimization'. She can be contacted at email: oumaima.taki@ensem.ac.ma.

**Kaoutar Senhaji Rhazi** is a qualified professor in Electrical Engineering; at the School of Technology in Casablanca. Morocco. A graduate engineer in electrical engineering from the Mohammadia School of Engineers (EMI) in Rabat, Morocco (in 1991). Had the research preparation certificate (CPR) in telecommunications Ph.D. in July 2006 (in electromagnetic compatibility). Passed academic qualification in the same field in 2014. Became higher education teacher in 2020. Current research interests are: 'power electronics' and 'electromagnetic compatibility'. She can be contacted at email: senhaji.ksr@gmail.com.

**Youssef Mejdoub** was born in Morocco, in 1980. He received his Ph.D. Thesis on Modeling of Multiconductor Transmission Lines, in 2014 from Cadi Ayyad University, Marrakech Morocco. Since 2016, he has been a Professor at the Superior school of technology (EST), University of Hassan II of Casablanca. He currently works at the Electrical Engineering Department, Superior school of technology. His current research interests are 'antennas', 'electromagnetic compatibility' and 'MTL lines'. He can be contacted at email: ymejdoub@yahoo.fr or youssef.mejdoub@univh2c.ma.