A study and analysis of input output linearization control of permanent magnet synchronous motor based on fuzzy observer

Tahar Belbekri¹, Badreddine Bellali²
¹Department of Electrical Engineering, Faculty of Science and Technology, Ahmed Draia University Adrar, Adrar, Algeria
²Department of Electrical Engineering, Faculty of Technology, Mohamed Tahri University Béchar, Béchar, Algeria

Article Info
Article history:
Received Jan 4, 2024
Revised Mar 27, 2024
Accepted Apr 5, 2024

Keywords:
Fuzzy logic
Input output linearization
Luenberger observer
Nonlinear control
PMSM

ABSTRACT
This article describes the nonlinear control of input output linearization type without speed sensor of permanent magnet synchronous motor (PMSM). This machine which has several advantages such as simple structure and high efficiency. Its control requires speed determination by sensor. Due to several disadvantages of using the speed sensor, we replaced it with a so-called Luenberger observer. The simulation results show that the observer considerably improves the performance of the PMSM in particular in overshooting during start-up and during speed reversal. It also shows good robustness when applying load torque. To obtain high operating performance of the PMSM, it basically needs to replace the proportional integral (PI) controller with another controller based on fuzzy logic. This artificial intelligence technique allowed us to obtain more efficient results than the traditional method.

This is an open access article under the CC BY-SA license.

Corresponding Author:
Tahar Belbekri
Department of Electrical Engineering, Faculty of Science and Technology
Ahmed Draia University Adrar
National Road No. 06 Adrar 01000, Algeria
Email: belbekri23@gmail.com

1. INTRODUCTION
Over the past decade and due to advances in construction and development of permanent magnet materials, permanent magnet synchronous motor (PMSM) has become some of the most used machines in the industry. In addition, their power densities are very high and they have high efficiencies, high performances and they are easy to control [1]–[3]. The vector control makes the PMSM work like the direct current (DC) machine with separate excitation. However, this technique requires exact knowledge of the rotor position and flux. Among the non-linear controls, we chose the input-output linearization which will help resolve the previous precision problem [4].

Control techniques require speed information from a sensor placed in the shaft such as the encoder or resolver. Therefore, the elimination of the mechanical sensor in our model will make it possible to obtain solutions with low cost, weight, making the system simpler with very high reliability. Researchers have developed several techniques to overcome the drawbacks and improve the results and robustness of the system. Among these techniques we find, model reference adaptive system (MRAS) estimators and observers. In our case, we will use the Luenberger observer. Furthermore, these techniques were based on the use of stator voltages and currents. In addition, these estimation methods work with closed-loop control to remove dependence on synchronous motor parameters [5]–[7]. Due to improve the results based on the proportional integral (PI) controller, another proposal based on one of the artificial intelligence techniques that have become the most widespread solutions in scientific research [8].

Journal homepage: http://ijpeds.iaescore.com
In our article, we will use the fuzzy logic technique to make our control more efficient and robust. The remainder of this article is composed as: i) Section 2 presents the PMSM model, the details of input-output linearization and the association of the Luenberger observer with fuzzy logic technique; ii) Then the results and the discussion of our study in section 3; and iii) Finally, we end with a conclusion of this article.

2. INPUT OUTPUT LINEARIZATION

Controlling the speed of the PMSM machine is very complex, it requires very powerful techniques. One of these methods, we find the input-output linearization. It is a non-linear control method. So, before starting the detailed study of this technique, we will begin with the presentation of the PMSM motor model.

2.1. The nonlinear model of permanent-magnet synchronous motor

The modelling of physical systems and especially electrical machines is based on the development of a few simplifying hypotheses which facilitate the generation of system equations. These hypotheses are [9], [10]: i) The distribution is sinusoidal of the magnetic field in space; ii) Ignorance of saturation and variance of parameters; iii) The armature voltages are considered sinusoidal with balanced armature windings; iv) The Foucault current and the hysteresis effect are negligible; and v) The equations of the PMSM can be written in in the Park reference frame rotating in d, q by the system (1) [10]–[13].

\[
\begin{align*}
\frac{di_d}{dt} &= \frac{1}{l_{ds}} (u_d - r_i d + p l_{qs} i_q w_m) \\
\frac{di_q}{dt} &= \frac{1}{l_{qs}} (u_q - p l_{ds} i_d w_m - r_i q - p \phi_f w_m) \\
\frac{dw_m}{dt} &= \frac{1}{j} (p(l_{ds} - l_{qs}) i_d i_q + p \phi_f i_q - f w_m - c_r)
\end{align*}
\]

Where, the parameter in d and q axes: id, iq; stator currents; ud, uq: stator voltage; rs, lds, and lqs: the stator resistance and the inductance; j, p, and cr: are the system moment of inertia, the number of pole pairs, load torque, and viscous friction coefficient.

The calculation of the electromagnetic torque is necessary in the study of the PMSM model to complete the state equations of our machine. In our case, its expression is proportional to the armature current and the magnetic flux. It can be written by (2).

\[
C_e = p \left( (l_{ds} - l_{qs}) i_d i_q + \phi_f i_q \right)
\]

Finally, we will give the state representation of the model studied which will be used in the input output linearization to find the control law. The PMSM model is given by (3).

\[
\begin{align*}
\dot{x} &= f(x) + g(x) \\
y &= h(x)
\end{align*}
\]

With:

\[
\begin{align*}
x &= \begin{bmatrix} x_1 \\
x_2 \\
x_3 \end{bmatrix}, \quad u = \begin{bmatrix} u_d \\
u_q \end{bmatrix}, \quad g(x) = \begin{bmatrix} g_1(x) \\
g_2(x) \end{bmatrix} = \begin{bmatrix} \frac{1}{l_{ds}} (u_d) \\
0 \\
0 \end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
f(x) &= \begin{bmatrix} f_1(x) \\
f_2(x) \\
f_3(x) \end{bmatrix} = \begin{bmatrix} \frac{1}{l_{ds}} (-r_i x_1 + p l_{qs} x_2 x_3) \\
\frac{1}{l_{qs}} (-p l_{ds} x_1 x_2 - r_i x_2 - p \phi_f x_3) \\
\frac{1}{j} (p(l_{ds} - l_{qs}) x_1 x_2 + p \phi_f x_2 - f x_3 - c_r) \end{bmatrix}
\end{align*}
\]

2.2. Input output linearization

The method used in our study is input-output linearization system. This technique is based on obtaining a relation between the system output y and the control input u. However, this method presents apparent difficulties because the output is not always indirectly linked to the input u. In addition, the state equations of the systems are frequently non-linear and our goal is to control the output y(t) follows the desired trajectory [4], [14], [15].
2.2.1. Presentation of the technique

The number of steps of this technique depending on the value of the derivative. In the beginning, we will give the first steps of this method which is based on the differentiate of the output [16], [17].

\[ \dot{y} = \frac{\partial h(x)}{\partial x} \dot{x} = \frac{\partial h(x)}{\partial x} f(x) + \frac{\partial h(x)}{\partial x} g(x)u = L_f h(x) + L_g h(x)u \quad (5) \]

if \( L_g h(x) \neq 0 \) \( \Rightarrow u = \frac{1}{L_g h(x)} \left[ -L_f h(x) + v \right] \Rightarrow \dot{y} = v \quad (6) \)

if \( L_g h(x) = 0 \) \( \Rightarrow \dot{y} = L_f h(x) \quad (7) \)

We notice that the expression is independent of control \( u \). that is to say that the variable \( u \) does not appear in the last equation. In this case, we will continue the derivation of the output \( y(t) \).

\[ \ddot{y} = \frac{\partial^2 h(x)}{\partial x^2} f(x) = L_f^2 h(x) + L_g L_f h(x)u \quad (8) \]

if \( L_g L_f h(x) = 0 \) \( \Rightarrow \ddot{y} = L_f^2 h(x) \)

For the second derivative of the output \( y(t) \). We see that we obtained the same thing. The result is always independent of control \( u \). In this case, we will continue deriving the output until \( r < n \). Finally, from (9), we find the control described by (10).

\[ y^{(r)} = L_f^r h(x) + L_g L_f^{(r-1)} h(x)u \quad (9) \]

\[ u = \frac{1}{L_g L_f^{(r-1)} h(x)} \left[ -L_f^r h(x) + v \right] \quad (10) \]

With, \( L_g L_f^{(r-1)} h(x) \neq 0 \) and \( r \): The relative degree of system and it’s number of derivatives of \( y(t) \).

2.2.2. Application on the PMSM

The method studied in the previous subsection will be applied to the permanent magnet synchronous machine. Therefore, the essential point in this step is the choice of the outputs. In our model, we will take the following variable, as in (11).

\[ y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_3 \end{bmatrix} \]

We will start with the first output of vector (11) which represents the direct current \( i_d \). So, we will do the derivation until the command appears. That is, until we see the command in the solution.

\[ y_1 = x_1 \]

\[ \dot{y}_1 = \dot{x}_1 = \frac{1}{l_{ds}} (u_1 - r_2 x_1 + p l_{qs} x_2 x_3) = f_1(x) + \frac{u_1}{l_{ds}} \]

The command \( u_1 \) appears in (13). So, the relative degree \( r_1 = 1 \) for the first output. Likewise, the second output of the vector in (11) represents the speed \( \omega_m \). In this step, we are going to do the same thing as the first output.

\[ y_2 = x_3 \]

\[ \dot{y}_2 = \dot{x}_3 = \frac{1}{j} \left( p (l_{ds} - l_{qs}) x_1 x_2 + p \varphi_f x_2 - f x_3 - c_r \right) \]

\[ \dot{y}_2 = \frac{1}{j} \left( p (l_{ds} - l_{qs}) x_1 x_2 + \frac{1}{j} p (l_{ds} - l_{qs}) x_2 \dot{x}_1 + p \varphi_f \dot{x}_2 - f \dot{x}_3 \right) \]

\[ \dot{y}_2 = \frac{p x_1}{j l_{qs}} (l_{ds} - l_{qs}) f_2(x) + \frac{p x_2}{j l_{ds}} (l_{ds} - l_{qs}) f_1(x) + p \varphi_f f_2(x) - f f_3(x) \]

\[ \dot{y}_2 = \frac{p x_1}{j l_{ds}} (l_{ds} - l_{qs}) u_1 + \frac{p x_2}{j l_{qs}} (l_{ds} - l_{qs}) u_2 + p \varphi_f u_2 \]

\[ \dot{y}_2 = \frac{p x_1}{j l_{ds}} (l_{ds} - l_{qs}) u_1 + \frac{p x_2}{j l_{qs}} (l_{ds} - l_{qs}) u_2 + p \varphi_f u_2 \quad (17) \]
The estimated states are represented by the circumflex accent. That is, it will be used to represent all the variables that we want to estimate. Furthermore, we will find the observer model through the PMSM model which can be written by the following state model [21–23].

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= cx
\end{align*}
\]
With:

\[
A = \begin{bmatrix} -\frac{R_s}{L_{ds}} & p \frac{w_m}{L_{ds}} \\ -p \frac{w_m}{L_{qs}} & -\frac{R_s}{L_{qs}} \end{bmatrix}; \quad B = \begin{bmatrix} \frac{1}{L_{ds}} & 0 & 0 \\ 0 & \frac{1}{L_{qs}} & -\frac{p w_m}{L_{qs}} \end{bmatrix}; \quad C = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}
\]

\[
x = [\dot{x}_1 \dot{x}_2] = \begin{bmatrix} i_d \\ i_q \end{bmatrix}; \quad u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}; \quad y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} i_d \\ i_q \end{bmatrix}
\]

By use of the PMSM model study in subsection (2.1.), and based on the Luenberger observer principle, we can give its mathematical model by (23). We point that the value of \( I \) represents a gain of the observer

\[
\begin{align*}
\dot{x} &= A_2 \dot{x} + B_2 u + I(y - \hat{y}) \\
\dot{\hat{y}} &= \epsilon \dot{x}
\end{align*}
\]

\[
A_2 = \begin{bmatrix} -\frac{R_s}{L_{ds}} & p \frac{\hat{w}_m}{L_{ds}} \\ -p \frac{\hat{w}_m}{L_{qs}} & -\frac{R_s}{L_{qs}} \end{bmatrix}; \quad B_2 = \begin{bmatrix} \frac{1}{L_{ds}} & 0 & 0 \\ 0 & \frac{1}{L_{qs}} & -\frac{p \hat{w}_m}{L_{qs}} \end{bmatrix}; \quad C = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}; \quad I = \begin{bmatrix} I_{11} \\ I_{21} \\ I_{22} \end{bmatrix}
\]

The choice of these gains is based on obtaining a low error and good tracking of the reference speed. So, the goal is to make the error dynamics converge towards zero and the system becomes asymptotically stable. In this case, the eigenvalues will be with negative real parts and the observation mechanism is given by (24) [22]–[24].

\[
\hat{w}_m = k_p \left( i_d \dot{i}_q + i_d \dot{i}_q - \frac{\varphi_f}{L_{qs}} e_q \right) + k_i \int \left( i_d \dot{i}_q + i_d \dot{i}_q - \frac{\varphi_f}{L_{qs}} e_q \right) dt
\]

(24)

2.4. Fuzzy Luenberger observer

The objective of this study is to improve the response and reduce the error using new techniques. The PI controller will be replaced by one of the artificial intelligence techniques to obtain good performance. Among these techniques, we chose fuzzy logic. It is based on four important blocks [20], [25].

2.4.1. Fuzzification

The first block scales the input data into the normalized universe of discourse [-1, 1]. It will be multiplied by the value of the scale factor to convert it to degrees of membership function. These membership functions will be identified by linguistic values [26]–[28]. The possible linguistic values for each input are 3, 5, 7, or 9. In this article, the linguistic value 9 is chosen, resulting in 81 rules as seen in Table 1 [8], [20].

<table>
<thead>
<tr>
<th>( \Delta e )</th>
<th>NB</th>
<th>NM</th>
<th>NS</th>
<th>NVS</th>
<th>ZE</th>
<th>PVS</th>
<th>PS</th>
<th>PM</th>
<th>PB</th>
</tr>
</thead>
<tbody>
<tr>
<td>NB</td>
<td>NB</td>
<td>NB</td>
<td>NB</td>
<td>NB</td>
<td>NM</td>
<td>NS</td>
<td>NVS</td>
<td>ZE</td>
<td>PVS</td>
</tr>
<tr>
<td>NM</td>
<td>NB</td>
<td>NB</td>
<td>NB</td>
<td>NB</td>
<td>NM</td>
<td>NS</td>
<td>NVS</td>
<td>ZE</td>
<td>PVS</td>
</tr>
<tr>
<td>NS</td>
<td>NB</td>
<td>NB</td>
<td>NB</td>
<td>NB</td>
<td>NM</td>
<td>NS</td>
<td>NVS</td>
<td>ZE</td>
<td>PVS</td>
</tr>
<tr>
<td>NVS</td>
<td>NB</td>
<td>NB</td>
<td>NM</td>
<td>NS</td>
<td>NVS</td>
<td>ZE</td>
<td>PVS</td>
<td>PS</td>
<td>PB</td>
</tr>
<tr>
<td>ZE</td>
<td>NB</td>
<td>NM</td>
<td>NS</td>
<td>NVS</td>
<td>ZE</td>
<td>PVS</td>
<td>PS</td>
<td>PM</td>
<td>PB</td>
</tr>
<tr>
<td>PS</td>
<td>NM</td>
<td>NS</td>
<td>NVS</td>
<td>ZE</td>
<td>PVS</td>
<td>PS</td>
<td>PS</td>
<td>PM</td>
<td>PB</td>
</tr>
<tr>
<td>PM</td>
<td>NVS</td>
<td>ZE</td>
<td>PVS</td>
<td>PS</td>
<td>PM</td>
<td>PB</td>
<td>PB</td>
<td>PB</td>
<td>PB</td>
</tr>
<tr>
<td>PG</td>
<td>ZE</td>
<td>PVS</td>
<td>PS</td>
<td>PM</td>
<td>PB</td>
<td>PB</td>
<td>PB</td>
<td>PB</td>
<td>PB</td>
</tr>
</tbody>
</table>

The inputs of this block are two, the first entry is the error \( e \) and the second is the derivative of the error \( \Delta e \). The fuzzy sets that will be used are as follows: negative large (NB), negative medium (NM), negative small (NS), negative very small (NVS), zero equals (ZE), very small positive (PVS), positive little (PS), moderately positive (PM), and large positive (PB).
2.4.2. Rule base

This block represents knowledge based on human operators. It allows us to make the requested variation at the output of the fuzzy logic block to have a minimal error with a rapid response. The increase or decrease of the output of this block is based on is based on the observation of errors [29]. In our study, this rule base takes the form of IF-THEN instructions.

2.4.3. Inference mechanism

The inference mechanism is a very essential part of the fuzzy controller. This mechanism can work with several methods. In our control model, we will use the Mamdani “min-max” method. Table 1 shows the 9*9 rules used in this study.

2.4.4. Defuzzification

This final part of the fuzzy controller converts the fuzzy variable sets into sharp, clear variable sets [28]. This defuzzification step can use diverse techniques. In our work, we will use the center of gravity method which is used in several scientific works.

3. RESULTS AND DISCUSSION

In this part of our work, we will give detailed presentations of all the simulation results as well as their discussions. The simulated control model is based on sensorless input-output linearization of the PMSM using the Luenberger observer. Two adaptation mechanisms of this observer are simulated. the first is based on a PI and the second is a fuzzy controller.

3.1. Luenberger observer based on PI controller

Figures 2 and 3 show the simulation of the speed response (estimated and actual) and error of speed response by using the Luenberger observer with application of + 10 N.m load disturbance at 1 sec and 1.5 sec respectively. The linearization input output of the permanent magnet synchronous motor using the sensor or the observer gives us almost the same results and performances even if we apply the load torque. The comparison error between the actual speed and the value of estimated speed takes the value 7.2 (rad/sec) when starting the PMSM motor and it decreases to almost zero at t = 0.32 sec, which presents an overshoot by 4.8%. The applied load torque is rejected. When reversing the speed, the error takes the value -19.01 (rad/sec) and it decreases to a zero value after a duration of time equal to t = 0.37 sec.

![Figure 2. Speed response by using the Luenberger observer (estimated and actual)](image)

3.2. Luenberger observer based on fuzzy controller

The Figures 4 and 5 show the simulation of the speed response (estimated and actual) and error of speed response by using the fuzzy Luenberger observer with application of + 10 N.m load disturbance at 1 sec and 1.5 sec respectively. Like the previous analysis, the linearization input output of the PMSM using the sensor or the observer gives us almost the same results and performances even if we apply the load torque. The comparison error between the actual speed and the value of the estimated speed takes the value 5.9
(rad/sec) when starting the PMSM motor and it decreases to almost zero at $t = 0.31$ sec, which presents an overshoot by 3.93%. The applied load torque is rejected. When reversing the speed, the error takes the value -12.45 (rad/sec) and it decreases to a zero value after a duration of time equal to $t = 0.36$ sec.

![Figure 3. Error of speed response by using the Luenberger observer](image3.png)

![Figure 4. Speed response by using the Luenberger observer (estimated and actual)](image4.png)

![Figure 5. Error of speed response by using the fuzzy Luenberger observer](image5.png)
4. CONCLUSION

In our paper, we presented a study and analysis of the input-output linearization of a PMSM based on one of the artificial intelligence techniques. Two types of controllers were used in the adaptation mechanism, one is based on the PI regulator and the other on fuzzy logic. A simulation using MATLAB software was carried out and its results were analyzed and commented.

The input-output linearization of the PMSM makes the operation of the system nonlinear like a linear system. In addition, the comparison of the results obtained with these two types of controllers showed the good properties of the artificial intelligence method, it confirmed the robustness with more precise control by comparing with a PI regulator, a low overshoot at the start of the control and when reversing the rotation speed with a faster response time.

The objectives of the reference speed and the disturbance rejection are better, which allows for minimal speed observation errors. Overall, the results obtained from the input-output linearization of PMSM based on fuzzy logic are very satisfactory. They show the robustness and stability of the system in the operating modes by the Luenberger observer based on closed-loop fuzzy logic.

REFERENCES


A study and analysis of input output linearization control of permanent magnet synchronous motors (PMSM) through Luenberger observers and fuzzy logic-based speed observers for optimal performance.

**References**


**Biographies of Authors**

**Tahar Belbekri** is a lecturer in Electrical Engineering Department at the University of Ahmada Draia, Adrar, Algeria. He received his engineering degree, magister degree, and Ph.D. degrees in Electrical Engineering from University Mohamed Tahri, Béchar Algérie, in 2010, 2014, and 2022, respectively. His field of scientific research concerns: electric machines control, power electronics, and artificial intelligence. He can be contacted at emails: belbekri23@gmail.com or tahar.belbekri@univ-adrar.edu.dz.

**Badreddine Bellali** is a lecturer in Electrical Engineering Department at the University of Ahmed Draia, Béchar, Algeria. He received his state engineer degree in Electronic from USTOP (University of Sciences and Technology of Oran), Algeria in 2001, the M.S. degree from University Center of Béchar in 2007, and the Ph.D. degree in Electrical Engineering from the University of Béchar in 2014. His areas of interest are the electric drives control. He can be contacted at email: bellali.badreddine@univ-bechar.dz.